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Heating of a collisionless turbulent plasma by multiphoton absorption

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The heating of electrons in the stochastic electric field of a collisionless turbulent plasma by the multiphoton absorption of a laser beam is described from a quantum-mechanical viewpoint. A kinetic equation is derived and the change in kinetic energy of the electrons is calculated. The effective interaction frequency is found and compared to the effective collision frequency of multiphoton inverse bremsstrahlung for the strong laser beam. It is found that the rate of energy absorption is proportional to the inverse square root of the intensity of the laser beam and the average of the square of the turbulent electric field when the laser frequency is approximately equal to the electron plasma frequency.

I. INTRODUCTION

The heating of a collisionless turbulent plasma by a laser beam is of prime interest to laser fusion, but the exact nature of this mechanism has never been quite clear. In the collisional absorption of laser energy, an electron must interact with the Coulomb field of an ion ("inverse bremsstrahlung"). As a result of this absorption, the electron energy and temperature increase and are partially transferred to the ions through Coulomb collisions. This collisional absorption decreases with increasing electron temperature and increasing laser energy because the mean free temperature and increasing laser energy because the mean free path of the electron gets longer with increasing electron energy. At laser intensities where collisional absorption decreases, collective effects referred to as "anomalous" absorption become important. In contrast to the inverse bremsstrahlung, these processes are collisionless and arise from a large number of instabilities, which are predicted by theory and numerical simulation.[1-4]

Because of these instabilities, turbulence develops and the electrons interact with photons of the laser beam in the turbulent electric field. One of the aims of this paper is to clarify the relationship betewen the rate of change of the kinetic energy of the electron and the frequencies and intensities of the laser beam and turbulent electric field.

The interaction of the system of the electron and laser field with the turbulent electric field is treated by means of first-order perturbation theory in a manner similar to those of other authors.[5-7] In this paper, however, the perturbing potential is the collective potential produced by the long-range nature of the Coulomb field of an extremely large number of plasma particles. This collective potential is expanded in Fourier series.[8] Transition probabilities are calculated and a kinetic equation for the electrons derived from them. It is important to point out that the validity of a kinetic equation for the inverse bremsstrahlung derived by Seely and Harris[7] has been questioned. In their theory,[7] the perturbing potential is a Coulomb potential of an ion, and the probability of collision between an electron and a test nucleus is not considered in the derivation of a kinetic equation for the inverse bremsstrahlung.[9]

The rate of change of the kinetic energy of the electron is calculated under the assumption that the spectrum $\langle |E_k|^2\rangle$ of the turbulent electric field takes an approximate k^{-2} shape, where k is the wave number of the turbulent electric field.[10] This assumption is appropriate when the laser frequency approximately equals the electron plasma frequency, the turbulence has been sufficiently developed before the heating phase of the collisionless regime,[10] and the intensity of the laser is weaker than the threshold intensity for electromagnetic instabilities.

II. TRANSITION PROBABILITIES

The time-dependent Schrödinger equation describing the wave function $\psi(\vec{r},t)$ of the free electron (of mass m and charge e) in the potentials $\vec{A}(\vec{r},t)$ and $\Phi(\vec{r},t)$ of the transverse radiation (laser) and longitudinal turbulent fields is, in Gaussian units,

$$i\hbar\,\frac{\partial\psi(\vec{r},t)}{\partial t} = \left(\frac{1}{2m}\left|\frac{\hbar}{i}\vec{\nabla}-\frac{e}{c}\vec{A}(\vec{r},t)\right|^2+e\Phi(\vec{r},t)\right)\psi(\vec{r},t)\,. \quad (1)$$

The scalar potential $\Phi(\vec{r},t)$ of the stochastic field

$\vec{E}(\vec{r},t)$ can be expanded in the complex Fourier series[8,11]

$$\Phi(\vec{r},t)=\sum_{\vec{k}}\Phi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{r}}, \tag{2}$$

where

$$\vec{k}=2\pi\left(\frac{\nu_x}{L},\frac{\nu_y}{L},\frac{\nu_z}{L}\right),\quad \nu_{xyz}=0,\pm1,\pm2,\ldots . \tag{3}$$

The turbulent fluctuations have frequencies ω' concentrated in a narrow band around the plasma frequency Ω for high densities and low electron temperatures in the heating phase of laser fusion.[12] Here we assume that the laser frequency is approximately equal to the electron plasma frequency, and the intensity of the laser is weaker than the threshold intensity for electromagnetic instabilities.[10]

Hence

$$\Phi_k(t)=\sum_{\vec{k}}\Phi_{k\omega'}e^{i\omega' t}=\Phi_{\vec{k}\Omega}e^{i\Omega t}+\Phi_{\vec{k}-\Omega}e^{-i\Omega t}$$
$$=U(\vec{k})\cos(\Omega t+\Phi_{\vec{k}}), \tag{4}$$

where $U(\vec{k})$ and $\phi_{\vec{k}}$ are the random amplitudes and phases, respectively. Since the $\Phi(\vec{r},t)$ is real, we obtain[8]

$$U(\vec{k})=U(-\vec{k})^*,\quad \phi_{\vec{k}}=\phi_{-\vec{k}}. \tag{5}$$

The wavelength of the laser radiation is assumed to be much longer than that of the turbulent electric field, since the laser frequency is equal to the electron plasma frequency and c is much greater than the phase velocity of the turbulent electric field. Hence the spatial dependence of the laser field can be neglected. Accordingly, a spatially independent and circularly polarized electromagnetic wave propagating in the z direction is assumed as the laser field, i.e.,

$$\vec{A}(\vec{r},t)=A_0(\hat{x}\cos\omega t+\hat{y}\sin\omega t). \tag{6}$$

The electric field strength of the laser radiation, E_0, is much greater than the turbulent electric field strength, E, for the laser fusion, i.e.,

$$E_0=(8\pi I/c)^{1/2}\gg E, \tag{7}$$

where I is the intensity of the laser light. Hence the $e\Phi$ term in the large parentheses of Eq. (1) is considered a small perturbing term compared to the first term, and the first-order perturbation calculation (considering only the one-quantum transition) for $e\Phi$ is a sufficiently precise calculation.

Using the first-order perturbation theory as in Ref. 7 with Eqs. (1)–(6), we find the transition probability per unit time averaged over the phase, the direction, and the amplitude of the turbulent electric field to be

$$\left\langle\frac{|a(1-2)|^2}{2T}\right\rangle=\frac{e^2}{2\hbar}\left\langle\left|U\left(\frac{\vec{p}_2-\vec{p}_1}{\hbar}\right)\right|^2\right\rangle\times\sum_{n=-\infty}J_n^2\left(\frac{\lambda}{\hbar\omega}\right)[\delta(Q-n\hbar\omega+\hbar\Omega)+\delta(Q-n\hbar\omega-\hbar\Omega)], \tag{8}$$

where J_n is the Bessel function of order n, and

$Q=(p_2^2-p_1^2)/2m,\quad \lambda=(eA_0/mc)\Delta p_\perp=(eE_0/m\omega)\Delta p_\perp.$

III. KINETIC EQUATION

From Eq. (8), we see that the observable transition probability per unit time for the transition from state 1 to states in the momentum range between $\vec{p}_2$ and $\vec{p}_2+d\vec{p}_2$ with absorption of n photons for $n>0$ (or emission of n photons for $n<0$) and absorption or emission of a quantum of the turbulent electric field is

$$T(n,\vec{p}_1-\vec{p}_2)\,d^3p_2=\frac{\pi e^2}{2\hbar}\left\langle\left|U\left(\frac{\vec{p}_2-\vec{p}_1}{\hbar}\right)\right|^2\right\rangle J_n^2\left(\frac{\lambda}{\hbar\omega}\right)[\delta(Q-n\hbar\omega+\hbar\Omega)+\delta(Q-n\hbar\omega-\hbar\Omega)]\,\frac{V d^3p_2}{(2\pi\hbar)^3}, \tag{9}$$

where V is the volume of plasma. Here, the probability of collision between the test electron and the turbulent electric field is self-contained in $\langle|U(\vec{k})|^2\rangle$. The δ function implies that the energy of the system consisting of an electric, photon, and quantum of the turbulent electric field is conserved, as is the momentum of the system. However, we should note that the momentum of the photon is neglected from the beginning, when the spatial dependence of the laser field was neglected (the wave number of the laser field is zero).

Under the same methodology and the same assumption for the electron distribution as Ref. 7 with Eqs. (9) and (5), the kinetic equation for the laser-irradiated electrons in a collisionless turbulent plasma is

$$\frac{\partial f(\vec{v}_2)}{\partial t}=\frac{\pi V e^2 m^3}{2\hbar(2\pi\hbar)^3}\left(\frac{m}{2\pi KT}\right)^{3/2}\int d^3v_1\left\langle\left|U\left(\frac{m(\vec{v}_2-\vec{v}_1)}{\hbar}\right)\right|^2\right\rangle\left[\exp\left(-\frac{mv_2^2}{2KT}\right)-\exp\left(-\frac{mv_1^2}{2KT}\right)\right]\times\sum_{\substack{n=-\infty\\ n\neq0}}^{\infty}J_n^2\left(\frac{\lambda}{\hbar\omega}\right)[\delta(Q-n\hbar\omega+\hbar\Omega)+\delta(Q-m\hbar\omega-\hbar\Omega)], \tag{10}$$

where $f(\vec{v})$ is the electron-distribution function.

IV. EFFECTIVE INTERACTION FREQUENCY

In a weak electromagnetic field, the rate of energy absorption by the multiphoton inverse bremsstrahlung is more dominant than that by the multiphoton absorption in the influence of the turbulent electric field. However, as the intensity of the laser beam increases collective instabilities develop and the latter predominates.[13] The expression for the effective interaction frequency for the case of the strong laser field can also be obtained by using the same process and approximations as Ref. 7. Thus the change in average kinetic energy of the electrons is

$$\frac{d\langle\epsilon\rangle}{dt}=\frac{\pi^2 Vem^4\omega}{\hbar(2\pi\hbar)^3E_0}\cosh\left(\frac{\hbar\Omega}{KT}\right)\left\langle\left|U\left(\frac{2eE_0}{\hbar\omega}\right)\right|^2\right\rangle\left(\frac{2eE_0}{m\omega}\right)^4. \tag{11}$$

Here we assume that the plasma turbulence develops sufficiently before heating by the strong laser beam, and that the directional distribution of the turbulent electric field is isotropic.

When the laser frequency equals the electron plasma frequency, only the oscillating two-stream instability occurs. In this case, computer simulations show that, after saturation, the plasma-wave spectrum $\langle|U(\vec{k})|^2\rangle$ assumes an approximate k^{-2} shape for wave numbers greater than the most unstable linear mode.[10] Hence we assume

$$\langle|U(\vec{k})|^2\rangle=A/k^4. \tag{12}$$

Then, from Eqs. (2), (4), and $\vec{E}=-\nabla\Phi$

$$A=(4\pi^2\langle\vec{E}^2\rangle/k_{max}V), \tag{13}$$

where $\langle\vec{E}^2\rangle$ is the average of the square of the turbulent electric field.[8,14] Here k_{max} is approximately equal to $k_D=(4\pi ne^2/kT)^{1/2}$. Substitution of Eqs. (12) and (13) into Eqs. (11) yields

$$\frac{d\langle\epsilon\rangle}{dt}=\frac{\pi e\Omega\langle\vec{E}^2\rangle}{2E_0k_D}\cosh\left(\frac{\hbar\Omega}{KT}\right). \tag{14}$$

The effective interaction frequency is defined by

$$\frac{d\langle\epsilon\rangle}{dt}=\frac{e^2E_0^2}{2m\omega^2}\nu_{eff}. \tag{15}$$

Comparison of Eq. (14) with Eq. (15) gives the effective interaction frequency

$$\nu_{eff}=\frac{\pi m\Omega^3\langle\vec{E}^2\rangle}{eE_0^3k_D}\cosh\left(\frac{\hbar\Omega}{KT}\right). \tag{16}$$

Equation (14) indicates that the rate of energy absorption by the interaction of the electrons with the turbulent electric field and the laser field is proportional to $I^{-1/2}$ and the average of the square of the turbulent electric field, $\langle\vec{E}^2\rangle$. The effective interaction frequency has the same dependence on the frequency and intensity of the laser beam as the effective collision frequency for the inverse bremsstrahlung, which has been found by Seely and Harris[7] and by Silin.[15] This is due to the fact that the spatial dependency of the perturbation due to the turbulent electric field differs from the perturbation of Ref. 7 [Eq. (7)] by the constant factor.

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