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Hydrodynamic theory near the nematic–smectic-*A* transition

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The hydrodynamic theory of the smectic and the nematic phase is generalized to the vicinity of the nematic–smectic-*A* phase transition by including the motion of the three slowly relaxing quantities. The sound spectrum is recalculated and found to be in agreement with recent experimental data on the dispersion and damping of first sound. In addition, the theory predicts two interesting effects for the nematic phase: (i) a smectic layer ordering induced by a stationary shear flow, and (ii) the existence of high-frequency shear waves.

I. INTRODUCTION

Quite recently, a strong anomaly in the attenuation and velocity of longitudinal sound waves was observed[1] near the nematic–smectic-*A* (*N*-*A*) transition. Part of the data was found to be in clear contradiction with existing theoretical results.[2,3] The purpose of this paper is to show that, with the help of a properly generalized hydrodynamic theory analogous to the Landau-Khalatnikov theory of order-parameter relaxation in helium,[4-6] one can understand the behavior of sound waves close to T_c, especially their anisotropy, remarkably well. This is despite the fact that the simple theory presented here neglects critical fluctuations. A set of generalized hydrodynamic equations is derived, which includes the relaxations of the magnitude ψ of the smectic order parameter and the equation of motion of the director $\vec{n}$. In the case of helium, the motion of only one soft variable, the superfluid density, has to be considered; in addition, contrary to the Landau-Khalatnikov results for helium, the hydrodynamic theory predicts a nonvanishing contribution from the order-parameter relaxation even above the *N*-*A* transition. Finally, far away from T_c, with only minor modifications, the proposed equations can be used to describe noncritical dispersive behavior of hydrodynamic propagating modes. In the context of liquid crystals, this type of theory was first employed by De Gennes,[7] who arrived at a number of interesting and experimentally verified predictions for the nematic-isotropic transition.

Due to the intricate interplay between the smectic's translational and orientational orders, being one- and three-dimensional respectively, the static theory of the *N*-*A* transition still suffers from serious difficulties. However, it must be stressed that most of the results presented here are not influenced by these difficulties. This is especially true for the structure of the dynamic equations and their eigenmodes. Only when dealing with the critical behavior of the elastic coefficients in order to *explicitly* evaluate the dynamic results must one be aware of the unresolved static questions.

I shall proceed as follows. In Sec. II the complete set of the generalized hydrodynamic equations will be derived systematically and compared to the equations of Ref. 3. They are then employed in Sec. III to calculate the dispersion and damping of longitudinal sound waves both below and above T_c. In addition, two interesting effects related to the anisotropy of the dispersion and damping are found: First, analogous to the case waves[8] may exist above T_c, and has the same angular dependence as the hydrodynamic shear waves in the smectic phase. Second, a smectic layer ordering is induced by a stationary shear flow in the nematic phase. At the end of Sec. III, the noncritical dispersion found experimentally in the nematic[5] and smectic-*A* phases[1] and the critical one expected in the vicinity of the smectic-*A*-smectic-*C* transition are discussed. In Sec. IV the results are summarized.

II. GENERALIZED HYDRODYNAMIC EQUATIONS

The thermodynamics near the *N*-*A* transition are determined by the conserved quantities (the densities of mass ρ, energy ϵ, and momentum $\vec{g}$, respectively), and in addition by the displacement u of the layers, the director $\vec{n}$, and the magnitude ψ of the smectic order parameter. So the change in entropy density is given by

$$T\,ds = d\epsilon - \mu\,d\rho - v_i\,dg_i - \phi_i\,d\nabla_i u + h_i\,dn_i - \mu_\psi\,d\psi\,. \tag{1}$$

Starting from Eq. (1) and the conservation laws, one can determine the structure of the hydrodynamic equations by a standard procedure[9,10] which is based on the following three points: (i) The energy current is a redundant quantity, and the energy conservation must be consistent with and implied by the form of all the other fluxes; (ii) the entropy production is positive definite; and (iii) the variables u and n_i are canonically conjugated to $\vec{g}\cdot\vec{n}$ and the intrinsic angular momentum,[11,12] respectively. (There are a number of equivalent notations in the literature.[13] Here, that of Ref. 14 is employed.) The resulting equations are the continuity equations for mass and entropy,

$$\dot{\rho}+\vec{\nabla}\cdot(\rho\vec{v})=0\,, \tag{2}$$

$$\dot{s}+\vec{\nabla}\cdot(s\vec{v}+\vec{f}^D)=R/T\,, \tag{3}$$

the momentum conservation,

$$\dot{g}_i+\nabla_i p-n_i\nabla_k\phi_k-\lambda_{kji}\nabla_j h_k+\beta_{ji}\nabla_j\mu_\psi+\nabla_j\pi^D_{ij}=0\,, \tag{4}$$

the equation of motion for the displacement and the director,

$$\dot{u}-\vec{n}\cdot\vec{v}+z^D=0\,, \tag{5}$$

$$\dot{n}_i+\lambda_{ijk}\nabla_j v_k+Y^D_i=0\,, \tag{6}$$

and finally the relaxation of the smectic order parameter

$$\dot{\psi}+\beta_{ij}\nabla_i v_j+X^D=0\,. \tag{7}$$

Some of the nonlinear terms such as the Erickson stress tensor have been neglected here. In accordance with the uniaxiality of both the nematic and the smectic-A phases, the transport coefficients λ_{ijk} and β_{ij} have the form

$$\begin{aligned}\lambda_{ijk}&=\tfrac{1}{2}(1-\lambda)\delta^T_{ij}n_k-\tfrac{1}{2}(1+\lambda)\delta^T_{ik}n_j\,,\\ \beta_{ij}&=\beta_\parallel n_i n_j+\beta_\perp\delta^T_{ij}\,,\end{aligned} \tag{8}$$

where $\delta^T_{ij}\equiv\delta_{ij}-n_i n_j$. The rate R of entropy production [Eq. (3)] is given by the product of the dissipative fluxes (denoted by superscript D) and the thermodynamic forces,

$$R=-f^D_i\nabla_i T-\pi^D_{ij}\nabla_j v_i-z^D\nabla_i\phi_i-Y^D_i h_i+X^D\mu_\psi\,,$$

where the fluxes f^D_i, π^D_{ij}, ... are obtained by expansion in all the forces $\nabla_i T$, $\nabla_i v_j$, ... To the lowest order in the wave vector q, only

$$Y^D_i=-h_i/\gamma_1\quad\text{and}\quad X^D=\eta\mu_\psi \tag{9}$$

do not vanish. To the next order,

$$\begin{aligned}f^D_i&=-\kappa_{ij}\nabla_j T-\zeta n_i\nabla_l\phi_l\,,\\ \pi^D_{ij}&=-\nu_{ijkl}\nabla_k v_l\,,\\ Z^D&=-\lambda_p\nabla_l\phi_l-\zeta n_i\nabla_i T\end{aligned} \tag{10}$$

retain their form as given by the usual hydrodynamics of the smectic and nematic phases.[5,14]

Equations (2)–(10) represent the complete set of generalized hydrodynamic equations. However, they are only useful in conjunction with the constituent relations between the conjugate variables and the variables. One can calculate them easily by suitably extending De Gennes's free-energy expression[15]:

$$\vec{h}=-D(\delta\vec{n}+\vec{\nabla}_\perp u)+\vec{h}^{\text{nem}}\,, \tag{11}$$

$$d\psi=\chi\,d\mu_\psi+(\partial\psi/\partial\rho)\,d\rho+(\partial\psi/\partial\nabla_\parallel u)\,d\nabla_\parallel u\,, \tag{12}$$

$$dP=(\partial P/\partial\rho)\,d\rho+C\rho\,d\nabla_\parallel u-\rho(\partial\psi/\partial\rho)d\mu_\psi\,, \tag{13}$$

$$\begin{aligned}\vec{\nabla}\vec{\phi}&=(B\nabla^2_\parallel+D\nabla^2_\perp)u+D\vec{\nabla}\cdot\vec{n}\\ &\quad+C\nabla_\parallel\rho-(\partial\psi/\partial\nabla_\parallel u)\nabla_\parallel\mu_\psi\,.\end{aligned} \tag{14}$$

Generally, the variables ψ, P, and $\vec{\nabla}\cdot\vec{\phi}$ also depend on s/ρ. This has been neglected, because $\partial(s/\rho)/\partial t=0$ to the lowest orders of q, the accuracy of our calculation in Sec. III. The two elastic coefficients $D\equiv(q_0\psi)^2/M_T$ and $B\equiv(q_0\psi)^2M_V$ vanish above T_c. (q_0 is 2π over the interlayer distance, and M_V and M_T are the two components of the effective-mass tensor introduced by De Gennes.[15]) The molecular field of the director[13] in the nematic phase is denoted by $\vec{h}^{\text{nem}}$, the susceptibility $\chi=\partial\psi/\partial\mu_\psi$ is given by the first coefficient in the free-energy expansion $f=a\psi^2+b\psi^4+\cdots$, and we have

$$\chi=1/a\quad\text{for } T>T_c,\quad \chi=-1/2a\quad\text{for } T<T_c.$$

The subscripts $\parallel$ and $\perp$ refer to the preferred direction $\vec{n}$, and finally[3] $C=\partial\phi_\parallel/\partial\rho$ and $B=\partial\phi_\parallel/\partial\nabla_\parallel u$.

With

$$\delta\psi_0\equiv(\partial\psi/\partial\rho)\delta\rho+(\partial\psi/\partial\nabla_\parallel u)\delta\nabla_\parallel u\,,\quad \delta\vec{n}_0\equiv-\vec{\nabla}_\perp u\,,$$

Eqs. (6) and (7) can be rewritten

$$\dot{n}_i+\lambda_{ijk}\nabla_j v_k-h^{\text{nem}}_i/\gamma_1=-(\delta\vec{n}-\delta\vec{n}_0)/\tau_1\,, \tag{15}$$

$$\dot{\psi}+\beta_{ij}\nabla_i v_j=-(\delta\psi-\delta\psi_0)/\tau\,, \tag{16}$$

where we have defined the relaxation times

$$\tau=\chi/\eta\quad\text{and}\quad\tau_1=\gamma_1/D\,. \tag{17}$$

The director is a hydrodynamic variable above T_c; therefore τ_1 is infinite throughout the nematic phase (while τ diverges only at the N-A transition). The two elastic coefficients B and D have been found[16] to vanish with different exponents. This fascinating result appears to be a manifestation of the difference between the long-range orientational order and the algebraic decay of the positional order.[17] Consequently, it is unlikely that τ and τ_1 would diverge with the same exponent. However, as will be shown in Sec. III, the divergence of τ is solely responsible for the sound-wave anomaly, and thus the data in Ref. 1 do not yield information on D.

In Ref. 3 Jähnig has also extended the hydrodynamic equations to include order-parameter relaxation. Unfortunately, despite the correctness of his basic concept, he has neglected both the static and the dynamic couplings of ψ to the hydrodynamic variables. The static ones are the thermodynamic cross derivatives $\partial\psi/\partial\rho$ and $\partial\psi/\partial\nabla_{\parallel}u$ in Eqs. (12) and (13); the dynamic ones are represented by the transport coefficients $\beta_{\parallel}$ and $\beta_{\perp}$ in Eqs. (4) and (7) [cf. Eqs. (2.5a) and (3.2) of Ref. 3]. Because these are the very terms which in conjunction with the divergence of τ lead to the observed dispersion and damping, this part of his results is in disagreement with the experimental data of Ref. 1. Two other points of criticism, less important in the context of ultrasonic measurements, are (i) the conjugate variable of u is $-\nabla_i\phi_i$, which in the notation of Ref. 3 is given as $\nabla_j(\pi^s_{ij}+\pi^a_{ij})$. Therefore Eq. (3.1) of Ref. 3 is correct only far away from T_c (and with $\vec{\nabla}T\equiv 0$); in the vicinity of the N-A transition the motion of u is given by Eqs. (5) and (9). (ii) The equation of motion for $\vec{n}$ is generally characterized by two transport coefficients γ_1 and γ_2 (or, as in our notation, γ_1 and $\lambda=-\gamma_2/\gamma_1$). In this respect, Eq. (5.4d) of Ref. 3 seems to be deficient.

III. APPLICATION TO EXPERIMENTS

Employing the hydrodynamic equations (2)–(14), one can easily calculate the critical dispersion and damping of the propagating modes, if attention is paid to one subtle point: The usual way of solving the equations to the two lowest orders of q with $\omega\sim q$ will only lead to three relaxation modes and no dispersion at all, because it overlooks the fact that $\omega\tau$ or $\omega\tau_1$ are, close enough to T_c, actually large quantities. The more proper way is to go on treating the ω which appear alone as proportional to q while taking the two relaxation factors

$$R=i\omega\tau/(1-i\omega\tau) \quad\text{and}\quad R_1=i\omega\tau_1/(1-i\omega\tau_1) \tag{18}$$

as arbitrary parameters of the equations, which are then solved to the desired order of q. Although this method does not yield the explicit form of the eigenmodes, it gives an adequate description of the dispersive behavior. We shall restrict our calculation to the lowest order in q, i.e., we shall neglect Eqs. (9) and set $\vec{h}^{\text{nem}}=0$ in Eq. (11). Without loss of generality, the wave vector can be put into the xz plane: $\vec{q}=(q_x,0,q_z)$. Taking $\vec{n}$, μ_ψ, ρ, u, and $\vec{v}$ as the independent variables [cf. Eqs. (11)–(14)] and eliminating the first four in favor of $\vec{v}$, we get one equation for v_y (case 2) and two coupled ones for v_x and v_z (case 1). Only case 1 gives rise to propagating modes. The two coupled equations are

$$\begin{aligned}
&\left[-\frac{\omega^2}{q^2}+\left(\frac{\partial P}{\partial\rho}-\frac{\beta_\perp^2}{\chi\rho}R\right)\sin^2\theta-\left(\frac{1+\lambda}{2}\right)^2\frac{D}{\rho}R_1\cos^2\theta\right]v_x\\
&\quad+\left[\frac{\partial P}{\partial\rho}-C-\left(\frac{1+\lambda}{2}\right)^2\frac{D}{\rho}R_1-\frac{\beta_\parallel\beta_\perp}{\chi\rho}R\right]\cos\theta\sin\theta\, v_z=0\,,\\
&\left[\frac{\partial P}{\partial\rho}-C-\left(\frac{1+\lambda}{2}\right)^2\frac{D}{\rho}R_1-\frac{\beta_\parallel\beta_\perp}{\chi\rho}R\right]\cos\theta\sin\theta\, v_x\\
&\quad+\left[-\frac{\omega^2}{q^2}+\left(\frac{\partial P}{\partial\rho}-2C+\frac{B}{\rho}-\frac{\beta_\parallel^2}{\chi\rho}R\right)\cos^2\theta-\left(\frac{1+\lambda}{2}\right)^2\frac{D}{\rho}R_1\sin^2\theta\right]v_z=0\,.
\end{aligned}\tag{19}$$

Since the inverse compressibility $\partial P/\partial\rho$ is by far the largest quantity[5,18] in Eqs. (19), we need only solve for the first-order correction to $\partial P/\partial\rho$, yielding the velocities $c_1^2=\omega_1^2/q^2$ and $c_2^2=\omega_2^2/q^2$ of first and second sound, respectively, as

$$c_1^2=c_{10}^2-\frac{R}{\chi\rho}\left[\left(\beta_\parallel+\frac{\partial\psi}{\partial\nabla_\parallel u}\right)\cos^2\theta+\beta_\perp\sin^2\theta-\rho\frac{\partial\psi}{\partial\rho}\right]-\frac{R_1D}{\rho}(\lambda+1)^2\cos^2\theta\sin^2\theta\,, \tag{20}$$

$$c_2^2=c_{20}^2-\frac{R}{\chi\rho}\left(\frac{\partial\psi}{\partial\nabla_\parallel u}+\beta_\parallel-\beta_\perp\right)^2\cos^2\theta\sin^2\theta-\frac{1}{4}\frac{R_1D}{\rho}(1+\lambda)^2(\cos^2\theta-\sin^2\theta)^2\,, \tag{21}$$

where

$$c_{10}^2=\frac{\partial P}{\partial\rho}-2C\cos^2\theta+\frac{B}{\rho}\cos^4\theta\,,\quad c_{20}^2=\frac{B}{\rho}\cos^2\theta\sin^2\theta$$

are the respective zero-frequency velocities.[5,14] As pointed out by Bhattacharya *et al.*, c_{10} displays a cusp which, in analogy to helium,[19] is likely to be caused by the divergence of the specific heat in the first term $\partial P/\partial\rho$. In addition to this, the elastic coefficients B (which has been shown[20] to vanish with an exponent of approximately $\frac{1}{4}$) and also the somewhat similar coefficient C will enhance the cusp and increase its asymmetry, since both are identically zero above T_c.

Turning now to the dispersion of the sound velocity, we note first that the contribution of the director's motion is negligible: above T_c, $\vec{n}$ is a hydrodynamic variable and R_1 vanishes identically; below T_c, both D and $1+\lambda$ vanish with T approaching T_c, making the term preceded by R_1 much smaller than the one preceded by R. (The transport coefficient $1+\lambda$ is zero at T_c because $\lambda=-\gamma_2/\gamma_1$, and, as has been shown by Jähnig and Brochard,[2] γ_2 and γ_1 diverge in the same way.) We can therefore rewrite Eq. (20) as

$$c_1^2-c_{10}^2=[-i\omega\tau/(1-i\omega\tau)](c_\infty^2-c_{10}^2), \qquad (22)$$

where

$$c_\infty^2-c_{10}^2=\left[\left(\beta_\parallel+\frac{\partial\psi}{\partial\nabla_\parallel u}\right)\cos^2\theta+\beta_\perp\sin^2\theta-\rho\frac{\partial\psi}{\partial\rho}\right]^2\Big/\chi\rho .$$

It is evident from Eq. (22) that the dispersion of sound waves is caused by the static ($\partial\psi/\partial\rho$, $\partial\psi/\partial\nabla_\parallel \boldsymbol{u}$) and dynamic ($\beta_\parallel, \beta_\perp$) couplings of ψ to the hydrodynamic variables. Due to the density dependence of T_c, $\partial\psi(T-T_c)/\partial\rho$ diverges as $-(\partial\psi/\partial T)(\partial T_c/\partial\rho)$, with the exponent $\beta-1$. Note that the same argument is not valid for $\partial\psi/\partial\nabla_\parallel \boldsymbol{u}$, because T_c can only depend (through the free energy) quadratically on $\nabla_\parallel u$. A linear dependence would erroneously imply a higher T_c, at which the nematic liquid crystal would undergo a transition to a strained smectic phase. Above T_c, both $\partial\psi/\partial\rho$ and $\partial\psi/\partial\nabla_\parallel u$ are identically zero and the transport coefficients $\beta_\parallel$ and $\beta_\perp$ make the only contribution. Though their temperature dependence is completely unknown, the anisotropic part $\Delta\beta\equiv\beta_\perp-\beta_\parallel$ can be measured in a simple flow-alignment experiment: Setting $\dot\psi=0$ in Eq. (16), a velocity field of the form $\vec{v}=vq(z-x)(\hat{x}+\hat{z})$ will lead to some smectic layer ordering, where the magnitude of the smectic order parameter is given as

$$\psi=(qv\tau)\Delta\beta . \qquad (23)$$

A similar effect is known above the nematic-isotropic transition.[7]

Detailed quantitative agreement with every aspect of the experiment, especially the frequency dependence of the damping for $\omega\tau\gg1$, cannot be expected, because the simple theory presented here neglects fluctuations which are known to be important. However, the equations do provide a convenient framework for discussion of the data and give an intuitive understanding of many of the results. Notably, the angular dependence of the experimental curves in Fig. 1 of Ref. 1 is in agreement with Eq. (22): The anisotropy of the sound velocity very close to T_c is given by that of c_∞, where the additional angular dependence, that of $\Delta c=c_\infty-c_{10}$, is monotonically decreasing:

$$\Delta c(0°)>\Delta c(45°)>\Delta c(90°), \qquad (24)$$

if

$$\left(\beta_\parallel+\frac{\partial\psi}{\partial\nabla_\parallel \boldsymbol{u}}-\rho\frac{\partial\psi}{\partial\rho}\right)^2>\left(\beta_\perp-\rho\frac{\partial\psi}{\partial\rho}\right)^2,$$

and monotonically increasing otherwise. And indeed the data points closest to T_c show how the more complicated dependence $c_1(0°)>c_1(90°)>c_1(45°)$ further away from T_c changes to the monotonicity of Eq. (24). In the same figure, the data for $T>T_c$ suggest $\Delta\beta\approx0$ in TBBA, the liquid crystal used in the experiment. However, since this is not required by the symmetry, we may expect other materials to display a greater anisotropy above T_c, implying a larger value of $\Delta\beta$.

The behavior of second sound in the smectic phase is described by Eq. (21). In contrast to the helium case, where $c_2\sim\xi/\tau$ (ξ is the temperature-dependent correlation length), Brochard[21] has shown that experimental data require $c_2\gg\xi/\tau$ in smectics. This of course means that the two inequalities $\omega\tau\gtrsim1$ and $q\xi\ll1$ can be satisfied simultaneously, and gives rise to the possibility, not encountered in helium, of detecting the critical dispersions of second sound.

Equation (21) is also valid above T_c. With B and R_1 identically zero, we have

$$\omega^2=-R(\Delta\beta^2/\rho\chi)\cos^2\theta\sin^2\theta\, q^2\equiv-Rc_\psi^2\, q^2 . \qquad (25)$$

To the lowest order in q (i.e., $\omega\sim q^0$), it gives the usual relaxation mode $\omega=-i/\tau$. For the more interesting high-frequency range $R\approx-1$, however, Eq. (25) yields a pair of propagating modes, which are easily recognized as shear waves and indeed have the same angular dependence as the hydrodynamic ones in the smectic phase. In close analogy to this, a pair of high-frequency shear waves has been predicted[8] for the superfluid A phase of ^{3}He, though only below T_c. One can hope to detect these unusual shear waves only if

$$\omega\tau\gg1\gg q\xi \quad\text{or}\quad c_\psi\tau\gg q^{-1}\gg\xi . \qquad (26)$$

This implies that c_ψ should not be much smaller than c_{20}, which, if possible, will result in an enhanced anisotropy of the dispersion of longitudinal sound wave above T_c, and also facilitate the flow-alignment experiment.

Equations (2)–(7) describe the dynamics of any system in which one translational and two rotational symmetries are spontaneously broken, and in addition a unit vector and a scalar relax slowly. When the appropriate elastic coefficients are set equal to zero, they become valid for simpler systems. I shall conclude this section with a few remarks about similar situations away from the N-A transition.

Nematic liquid crystals often display noncritical and anisotropic dispersion. With the assumptions first that this is due to the internal molecular relaxations, characterized by a simple time constant τ, and second that the *isotropic* elastic tensor becomes *uniaxial* in the high-frequency regime $\omega\tau \gg 1$, Jähnig[22] was able to establish a relation between the anisotropic parts of the damping and of the velocity. He has not shown how uniaxiality can be realized. It is easy to see that by setting $\vec{h} \equiv \vec{h}^{nem}$ and $\vec{\phi} \equiv 0$ and redefining ψ as the slowly relaxing molecular quantity, Eqs. (2)–(7) will quite naturally give rise to the uniaxial (smecticlike) stress tensor assumed by Jähnig. And we may conclude that there will be a pair of high-frequency shear waves in the nematic phase with the velocity c_ψ as given by Eq. (25).

The situation is quite similar in the smectic-A phase. And the reported[1] detection of dispersion indicates the existence of the high-frequency shear wave

$$\omega^2 = [B/\rho + (\partial\psi/\partial\nabla_{\parallel} u + \beta_{\parallel} - \beta_{\perp})^2]\cos^2\theta \sin^2\theta q^2 , \tag{27}$$

which may be much faster than the hydrodynamic one, $\omega^2 = c_{20}^2 q^2$. This will explain the fact that second sound is more readily observable by Brillouin scattering[23] than by ultrasonic methods.[5] Approaching the smectic-A-smectic-C transition from above, the director $\vec{n}$ and ψ (which now stands for the smectic-C order parameter) are again the soft quantities, and the hydrodynamic equations (2)–(7) and the spectrum equations (20) and (21) remain valid.[24] The only differences are, of course, that $1+\lambda$ is no longer necessarily a small quantity, and that both relaxation factors R and R_1 need to be considered. The analogy, however, does not extend below T_c, where the symmetry is much more complicated.

IV. SUMMARY

The hydrodynamic equations were generalized to include the slow relaxation of the director $\vec{n}$ and the magnitude ψ of the smectic order parameter. Their effect on sound dispersion and damping, especially on the angular dependence, has been calculated. The contribution of the motion of $\vec{n}$ turns out to be negligible in comparison with that of ψ. The disagreement between experiment and previous theories was traced to the omission of both the static and the dynamic couplings of ψ to the hydrodynamic variables. I have found that a shear flow will induce a smectic layer ordering in the nematic phase. If this effect is large enough, longitudinal sound waves become anisotropic and a pair of high-frequency shear waves will propagate above T_c. Analogies are drawn to the smectic-A–C transition and the noncritical dispersion of smectic and nematic liquid crystals.

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