# Experimental study of thick-target bremsstrahlung spectrum of  $32P$   $\beta$  radiation

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The bremsstrahlung spectrum due to complete absorption of <sup>32</sup>P  $\beta$  rays in thick aluminum absorbers in a sandwich geometry is measured in an energy range of 30-1300 keV. The spectral distribution after correction for detector response and target self-absorption is compared with various theoretical calculations. It is found to agree with the Bethe and Heitler theory corrected for Coulomb effects within 15%, but disagrees with calculations of Wyard and Rezanka et al.

## I. INTRODUCTION

A number of workers' have studied bremsstrahlung emitted by electrons passing through matter, but there are only a small number of measurements of spectral distribution of this radiation in the case of  $\beta$  radiation completely absorbed in thick-target materials. Liden and Starfelt<sup>2</sup> studied bremsstrahlung from  $^{32}P \beta$  rays completely absorbed in various targets. Recently, Prasad Babu  $et al.^3$  have repeated these measurements. However, in the geometry used by latter workers, targets were placed halfway between the source and detector, and effects of angular distribution. of bremsstrahlung and backscattering of  $\beta$  rays were ignored while making a comparison with theory. Again, both of these measurements are restricted to photon energy below 1000 keV. We have modified the geometry into a sandwich type and have extended the energy region up to 1300 kev. In addition to this radiation called external bremsstrahlung (EB), there is always present in these measurements internal bremsstrahlung (IB) associated with  $\beta$  decay in the field of the nucleus. In the case of low-S targets, contributions of internal and external bremsstrahlung are comparable. We have corrected our measurements of EB for the contribution of IB, with the latter measured in a geometry which is most sensitive to this radiation. Thus our results are expected to be more accurate, particularly at the high-energy end of the EB spectrum, than the previous measurements.

## II. THEORETICAL CONSIDERATIONS

A number of authors have developed the theory of external bremsstrahlung produced by fast-moving electrons in different approximations. The most widely used Born-approximation theory was developed independently by Bethe and Heitler, <sup>4</sup> developed independently by Bethe and Hertler,<br>Sauter,<sup>5</sup> and Racah.<sup>6</sup> They have evaluated the cross. section  $\sigma(W_e, k)$  for the production of the bremsstrahlung photon of energy  $k$  by a fast-moving electron of total energy  $W_e$  (including the rest-mass

energy  $mc^2$ ) in the Coulomb field of nucleus of atomic number Z. Several correction factors have been suggested to modify this theory to make it more realistic. Koch and Motz' have reviewed these attempts. Coulomb corrections for the lowenergy region have been suggested by Elwert<sup>8</sup> and Guth<sup>9</sup> in terms of a factor called the Elwert factor  $f<sub>E</sub>$ , which is a function of target atomic number and initial and final energies of the electron.

When an electron is completely stopped in a target, its bremsstrahtung spectrum expressed as photons per unit interval of photon energy  $k$ , integrated over all outgoing photon angles, per incident electron of total energy  $W_e$ , is given by  $n(k, W<sub>e</sub>)$  such that

$$
n(k, W_e) = \int_{1+k}^{W_e} \frac{N\sigma(W, k)}{(-dW/dx)} dW,
$$

where N is the number of atoms per cm<sup>3</sup> and  $-dW/$  $dx$  is the energy loss per cm. In the case of continuous  $\beta$  radiation, the bremsstrahlung spectrum expressed as the number of EB photons per  $mc^2$ per  $\beta$  disintegration is given by

$$
S(k) = \int_{1+k}^{W_{\text{max}}} P(W_e) n(k, W_e) dW_e,
$$

where  $W_{\text{max}}$  is the end-point energy of  $\beta$  particles and  $P(W_e)$  is the  $\beta$  spectrum of the radio isotope under study.

Wyard" has used an approximate expression for the bremsstrahlung cross section in which the shape of the intensity distribution is roughly independent both of the Z value of the absorber and of electron energy. He obtained an intensity distribution curve that is independent of both the target material and the  $\beta$  emitter.

aterial and the  $\beta$  emitter.<br>Rezanka *et al*.<sup>11</sup> have argued that since the measured cross sections for bremsstrahlung from monoenergetic electrons deviate considerably from those given by Bethe and Heitler, the EB spectrum should be obtained from emperically corrected cross sections. They have used an emperical cor-

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rection factor  $\chi(W_e - 1)$ , given by Koch and Motz,<sup>7</sup> and have calculated the bremsstrahlung spectrum of  ${}^{32}P\beta$  rays for complete absorption in aluminum targets.

We have compared the result of our measure. ments with spectra calculated from (i) the Bethe and Heitler theory using cross section  $\sigma(W_e, k)$ , (ii) the Bethe and Heitler theory corrected with the Elwert factor  $f_E(Z, W_e, W_e - k)$ , (iii) calculations of Rezanka *et al*.,<sup>11</sup> and, finally, (iv) Wyard's approx. Rezanka *et al*.,  $^{I_1}$  and, finally, (iv) Wyard's approx imation.<sup>10</sup>

# III. EXPERIMENTAL DETAILS

We show in Fig. 1 the geometrical setup used in our measurements. The radioactive  $^{32}P$  source was obtained from Bhabha Atomic Research Center, Trombay, Bombay, as H, PO, in dilute HCl was prepared on a thin polythene film of  $1 \text{ mg cm}^{-2}$ thickness by evaporation. The source size was



FIG. 1. Cross-sectional view of the geometry of the experimental setup. (1) Source of  $^{32}P$  on polythene film; (2) aluminum targets; (3) perspex ring; (4) perspex stand; (5) perspex annular disk; (6) lead collimator, the inside surface being lined with aluminum; (7) brass annular disk; (8) 4-cm-thick lead shield, inside surface being lined with aluminum; (9) aluminum can containing NaI(T1) crystal; (10) black adhesive tape; (11) photomultiplier; (12) cathode follower; (13) adjustable support; (14) iron cylinder.

limited to less than 0.5 cm in diameter, and source material was uniformly distributed with the help of plain insulin solution. Its strength was 0.6 mCi and it was placed at a distance of 20 cm from a 7D8 Nal(Tl) crystal, of a 4.45-cm diameter and a 5.1-cm thickness, coupled to a 6292 Dumount photomultiplier assembly was shielded in a hollow lead cyclinder of 4-cm thickness, the inside surface of which was lined with aluminum. A conical lead collimater, whose inside surface was lined with aluminum and placed above the detector on annular brass plate, prevented scattered photons from reaching the crystal, and limited the background to a low level. The source was sandwiched between two aluminum targets each of 1255.2 mg cm<sup>-2</sup> thickness. The bremsstrahlung spectrum was recorded with a  $\nu$ -ray spectrometer coupled to a multichannel pulse-height analyzer (Nuclear Data, ND 1100). The data for target and room background were recorded for 50000 sec each, and counting statistics at 1000 keV were better than 2%.

#### IV. CORRECTIONS

The bremsstrahlung spectrum was corrected for room background, source decay, and detector response. The factors involved in the detector response were detector resolution, iodine  $K$  x-ray escape peak correction, backscattered peak contribution, Compton continuum contribution, absorption in air and the aluminum can of the detector, and photopeak efficiency of the detector. Values of the resolution correction, backscattered peak contribution, Compton continuum contribution, and photofraction were obtained with the help of standard  $\gamma$  sources of  ${}^{57}Co$ ,  ${}^{141}Ce$ ,  ${}^{51}Cr$ ,  ${}^{203}Hg$  $^{137}Cs$ ,  $^{54}Mn$ ,  $^{22}Na$ , and  $^{60}Co$ . Geometrical and intrinsic efficiency was taken from the data tabulated by Crouthamel<sup>12</sup> for a crystal of the same dimensions. Iodine  $K$  x-ray escape peak correction was obtained by the method used by  $Power^{13}$  and the correction factor for energy resolution was obcorrection factor for energy resolution was ob-<br>tained by the method given by Novey.<sup>14</sup> These correction factors and the overall detector response function are shown in Fig. 2. Correction for selfabsorption of bremsstrahlung in target was calculated by usingattenuation coefficients given by Hublated by using attenuation coefficients given by Hub.<br>bell.<sup>15</sup> This was also determined experimentally in the given geometrical setup, and found to agree with calculated values within 5%. The self-absorption correction factor is also shown in Fig. 2.

The bremsstrahlung pulse-height distribution was corrected for the various factors mentioned above. The corrected bremsstrahlung spectrum contained contributions of both external and internal bremsstrahlung. The contributions of internal bremsstrahlung obtained by Powar and Singh<sup>16</sup> in a geo-



FIG. 2. Pulse-height distribution of  $^{32}P$  bremsstrahlung in aluminum and various corrections.  $\bullet\bullet\bullet$ , experimentally measured pulse-height distribution;  $\times\times\times$ , background spectrum; --.--, Compton electron distribution (1); ----, backscattering distribution (2); - $\cdots$ , correction factor for the iodine  $K$  x-ray escape and resolvrection factor for the iodine  $K$  x-ray escape and resolving power (3); ..., overall detector response function with geometrical efficiency  $(4)$ ; ..., correction factor with geometrical efficiency  $(4)$ ;  $-$  for absorption in the radiator  $(5)$ .

metry which minimizes the contribution of EB relative to that of IB, and agreeing with KUB Nilsson theory<sup>17</sup> within  $10\%$ , were subtracted from the measured spectrum to obtain the EB spectrum of aluminum.

# V. ERRORS

The major sources of error in the present measurements are from the detector response function which, in turn, is chiefly contributed by geometrical and intrinsic efficiency of detector. The error quoted in the values of intrinsic efficiencies is  $2\%$ -3%, and the experimentally determined photofractions are uncertain by less than 4%. These two factors contribute an overall error in detection efficiency of not more than 5%. Uncertainties due to statistics in recording the data are better than 2% at 1000 keV. Errors involved in the backscattering correction, energy resolution, and iodine escape correction are very small, as these contributions are themselves very small. The error involved in approximating the Compton electron

distribution to a straight line for the purpose of calculating the Compton contribution is estimated to be of the order of 5% by comparing the straightline approximation for the Compton electron distribution for a  $\gamma$ -ray energy to the actual Compton electron distribution obtained from experimental spectrum. Also, error in energy calibration was small. The overall error in the present measurements is estimated to be  $10\%$  at 1000 keV, which is the rms value of the various errors involved. The error at lower energies is lower because of better counting statistics.

#### VI. RESULTS AND DISCUSSION

Results of the present measurements of external bremsstrahlung produced by complete absorption of  $^{32}P \beta$  rays in aluminum targets are shown in Figs. 3 and 4. These are compared with Bornapproximation theories of Bethe and Heitler (curve a), Elwert (curve b), calculations of Rezanka et al. (curve c), and Wyard's approximation (curve d). Figure 3 depicts the spectral distribution of EB photons, with the experiment having been normalized to Elwert theory at  $300 \text{ keV}$ . As there are wide variations among the various theories, we



FIG. 3. Comparison of the external bremsstrahlung spectral distribution with various theoretical distributions. (a) Bethe-Heitler theory, ---; (b) Bethe-Heitler (Elwert) theory,  $\longrightarrow$ ; (c) Rezanka et al. calculation,  $-\cdots$ ; (d) Wyard's approximation,  $-\cdots$  [Experimental distribution is normalized at 300 keV with theory at (b)].



FIG. 4. Shape factor plots {experiment/theory) of EB in aluminum (normalized with various theories at 300 keV). (a) Bethe-Heitler theory,  $OOO$ ; (b) Bethe-Heitler (Elwert) theory,  $\bullet \bullet$ ; (c) Rezanka et al. calculations,  $\forall x \times$ ; (d) Wyard's approximation,  $\triangle \triangle$ . (Total rms errors are shown at several points for orientation. )

show in Fig. 4 the shape factor plots of experiment/theory for various cases. The experiment has been normalized with theory at 300 keV in all

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these plots. Errors are shown at several points for orientation. These are the total rms errors which increase with energy because statistical uncertainties become large due to substraction of contribution of IB and background, particularly at the high-energy end of the spectrum.

It is observed that the experimental spectral distribution agrees with that predicted by the Bethe and Heitler and the Elwert theories to within 15%. However, there are wide departures from Wyard's approximation and the calculations of Rezanka  $et$ al. Both of these theories give values that are higher than the experimental values by a factor of 3 at the high-energy end of the spectrum. It may be pointed out that while Wyard's theory is an approximate one and is liable to be in error at the high-energy end of the EB spectrum, the calcula<br>tions of Rezanka *et al*,<sup>11</sup> are claimed to be more tions of Rezanka et  $al.^{11}$  are claimed to be more accurate. The disagreement of experimental spectrum with these calculations, however, points to some serious algebraic or conceptual errors in them.

Measurements of Prasad Babu  ${et}$   $al. ,^3$  which are limited to 950 keV, are higher than our measurements by  $30\%$  above  $400$  keV. The results of Liden and Starfelt' agree with our measurements, but extend up to 10OO keV only, while our measurements go up to 1300 keV. Again, neither of these workers have compared their results with other workers have compared their results with other<br>theories, especially that of Rezanka *et al*.<sup>11</sup> which is claimed to be more accurate.

We conclude that our measurements on the EB spectral distribution from aluminum targets with  $^{32}P \beta$  rays extending up to 1300 keV energy, which is higher than the previous measurements, show good agreement with the Bethe and Heitler and Elwert theories, but disagree with calculations of Rezanka et al. and with Wyard's approximation.

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