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Stochastic incoherences of optical Bloch equations

K. Wódkiewicz
Institute of Theoretical Physics, Warsaw University, Warsaw 00-681, Poland
and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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The theory of optical Bloch equations with stochastic coefficients is discussed in detail. It is shown that Bloch equations with stochastic fields lead to multiplicative stochastic processes for which exact solutions can be obtained. The steady state and the transient regimes of the phase-diffusion model of the laser bandwidth are treated, and exact solutions of Bloch equations are obtained. The case of fluctuation due to collisions is also discussed. A simple stochastic model of radiation-induced collisions is presented. For each stochastic model of incoherences formulas for the longitudinal and transverse lifetimes are derived exactly. It is shown that the finite bandwidths of light or the collision incoherences do not, in general, have a simple additive effect in atomic equations of motion.

I. INTRODUCTION

In almost all problems when the resonance interaction of light with an atom is investigated the starting point of many physical discussions is a set of optical Bloch equations[1]:

$$\dot{u} = -Dv - u/T_2', \tag{1.1a}$$

$$\dot{v} = Du - v/T_2' + \lambda \mathcal{A}_0 w, \tag{1.1b}$$

$$\dot{w} = -(w+1)/T_1 - \lambda \mathcal{A}_0 v, \tag{1.1c}$$

where $D = \omega_0 - \omega_L$ is the detuning of the driving-field frequency ω_L from the resonance frequency ω_0, and T_2' and T_1 are phenomenological transverse and longitudinal homogeneous lifetimes. The amplitude $\mathcal{A}_0$ describes the external driving field which is coupled to the atomic system with strength λ.

When coherent effects are discussed it turns out that the simple set of equations (1.1) is not sufficient to describe more complicated phenomena, for example, the resonance-fluorescence power spectrum[2] or the photon correlation experiments[3] where a fourth-order correlation function of the fluorescent light is required.

For this reason the starting point of many of these considerations has been a set of optical Bloch equations derived from the nonrelativistic quantum electrodynamics[4,5]:

$$\dot{\sigma} = (-i\omega_0 - 1/2\tau_0)\sigma + \lambda\sigma_3 A^{(+)}, \tag{1.2a}$$

$$\dot{\sigma}^+ = (i\omega_0 - 1/2\tau_0)\sigma^+ + \lambda A^{(-)}\sigma_3, \tag{1.2b}$$

$$\dot{\sigma}_3 = -1/\tau_0 - 1/\tau_0\sigma_3 - 2\lambda(\sigma^+ A^{(+)} + A^{(-)}\sigma). \tag{1.2c}$$

In these equations the creation $A^{(-)}$ and the annihilation $A^{(+)}$ parts of the electromagnetic field operators evolves freely from its initial value at $t=0$ when we have assumed that the atomic system is decoupled from the radiation field.

For a typical fluorescence problem when the atomic beam is crossed perpendicularly by the driving laser,[6,7] the initial condition $t=0$ corresponds to the moment when the atom enters the laser beam. The damping $1/\tau_0$ is equal to the Einstein A coefficient of spontaneous emission. There is a close connection between the operator-valued equations (1.2) and the phenomenological equations (1.1).

If we assume that the driving source of light is described by a single-mode coherent state,

$$A^{(+)}(t)\,|\mathcal{A}_0\rangle = e^{-i\omega_L t}\mathcal{A}_0\,|\mathcal{A}_0\rangle, \tag{1.3}$$

we obtain from Eq. (1.2) the phenomenological equations (1.1) with

$$v(t) + iu(t) = \mathrm{Tr}\{\sigma(t)\, e^{i\omega_L t}\,|\mathcal{A}_0\rangle\langle\mathcal{A}_0| \otimes \rho_A\}, \tag{1.4a}$$

$$w(t) = \mathrm{Tr}\{\sigma_3(t)\,|\mathcal{A}_0\rangle\langle\mathcal{A}_0| \otimes \rho_A\}. \tag{1.4b}$$

where ρ_A is an arbitrary density matrix for the atom. From this relation it is also clear that $T_2' = 2\tau_0$ and $T_1 = \tau_0$. As far as monochromatic-laser interaction with the atom is concerned, the set of Eqs. (1.2) with a single-mode coherent state of the electromagnetic field has been an excellent tool for the derivation of all relevant physical properties of the system.[8-10]

In many high-resolution spectroscopy experiments, the deviations from the ideal situation of a single atom coupled to a perfectly monochromatic source of light could become important. In a typical fluorescence experiment when an atomic beam interacts with a pulsed or a cw laser we have several sources of incoherence due to collisions, fluctuating static fields, laser linewidth, etc. An old way of proceeding in such a case has been a phenomenological modification of the homogeneous lifetimes in Eq. (1.1) in order to take into account the incoherence effects in the system, atom plus radiation.

A justification of such modifications of T_2' and

T_1 for a given physical situation would require a nonperturbative solution of Eq. (1.2) with a realistic density matrix for the field and for the atom. In most applications such a program of microscopic calculations is extremely difficult to perform.[11-13] In order to avoid these difficulties a simpler stochastic approach to the incoherences has been developed. For example, in order to take into account the incoherences of the driving field, the free-field operators $A^{(+)}$ and $A^{(-)}$ in Eq. (1.2) have been replaced by random complex functions $\mathcal{A}(t)$ and $\mathcal{A}^*(t)$ with given statistics to model a finite bandwidth of the laser.[14,15] In the last year or so, many papers have been published concerning the influence of the laser linewidth on different physical phenomena.[14-19] A nonperturbative theory of atomic relaxation in the presence of intense partially coherent radiation has been developed and applied to the resonance-fluorescence problem. Since then, resonance fluorescence with a finite laser bandwidth has been treated by several authors using different techniques and different decorrelation assumptions in order to find nonperturbative solutions of the problem.[16-18] The stochastic approach to the incoherence properties of light interacting with an atom has an obvious advantage being that without going into complicated microscopic details of the incoherence it can be easily incorporated in the proper atomic equations of motion. A clear disadvantage is of course the fact that in a stochastic approach one is not taking into account all the very complicated microscopic effects leading, for example, to a finite laser bandwidth. In this sense the stochastic description of the electromagnetic field compared to a full microscopic treatment should be understood as a phenomenological approach.

In this paper we are going to investigate the influence of various statistical models of incoherences on the solutions of optical Bloch equations. We show that Bloch equations with stochastic random variables are examples of multiplicative stochastic processes,[20] studied recently by the author.[21] We solve Bloch equations exactly, using the previously developed methods for multiplicative stochastic processes described by the following general operator-valued equation[21]:

$$\frac{d\Psi}{dt}=(M_0+x(t)M_1+x^*(t)M_2)\Psi, \qquad (1.5)$$

where M_0, M_1, and M_2 are arbitrary matrices and $x(t)$ and $x^*(t)$ are the random variables of the stochastic process. We show that all the discussed stochastic models of the incoherences lead to equations of the type (1.5) with proper M matrices and with $x(t)$ and $x^*(t)$ given random variables. In Sec. II, for completeness of our discussion, we obtain the standard Torrey equation for a perfectly coherent source of light described by Eq. (1.3). This basic equation will be generalized in the next sections for various statistical theories of incoherences.

In Sec. III we discuss the phase-diffusion model of the laser linewidth. We obtain proper generalizations of the Torrey equation and we show under what assumptions the results obtained by previous workers[14-18] are exact or approximate.[22]

In Sec. IV we discuss exact solutions of Bloch equations when the electromagnetic field is a random stochastic process with a fluctuating amplitude.

In Sec. V we solve the Bloch equations exactly introducing a model description of collisions assuming that the energy separation of the two-level atom performs random fluctuations around ω_0.[23]

In Sec. VI we present a statistical model of radiation-induced collisions assuming that the phase fluctuations of the electromagnetic field are correlated with the frequency fluctuations of ω_0. Finally some concluding remarks are presented.

II. PERFECTLY COHERENT TORREY EQUATION

For the purpose of this paper we write the optical Bloch equations (1.2) instead of the standard spin-precession form in the following matrix form [see Eq. (1.5)]:

$$\frac{d\Psi}{dt}=(M_0+M_1x(t)+x^*(t)M_2)\Psi, \qquad (2.1)$$

where the vector operator Ψ is defined as follows:

$$\Psi(t)=(e^{i\omega_L t}\sigma(t), e^{-i\omega_L t}\sigma^+(t), \sigma_3(t), 1), \qquad (2.2)$$

and the matrices M_0, M_1, and M_2 are given by the following formulas:

$$M_0=\begin{bmatrix} -iD-1/2\tau_0 & 0 & 0 & 0 \\ 0 & iD-1/2\tau_0 & 0 & 0 \\ 0 & 0 & -1/\tau_0 & -1/\tau_0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (2.3a)$$

$$M_1=\begin{bmatrix} 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (2.3b)$$

$$M_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ -2\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{2.3c}$$

The time-dependent coefficients $x(t)$ and $x^*(t)$ are the free-field operators of the electromagnetic field multiplied by simple phase factors:

$$x(t) = A^{(+)}(t)e^{i\omega_L t}, \quad x^*(t) = A^{(-)}(t)e^{-i\omega_L t}. \tag{2.4}$$

Equation (2.1) has the form of an equation of a multiplicative stochastic process for which the random variables are the electromagnetic field operators $A^{(+)}$ and $A^{(-)}$. For the single-mode coherent state (1.3) we have the following correlation function of the random variables $x(t)$ and $x^*(s)$:

$$\langle x(t)x^*(s)\rangle = \mathcal{A}_0\mathcal{A}_0^* . \tag{2.5}$$

It is also a well known fact that this equation can be solved for the quantum expectation value of Ψ exactly:

$$\frac{d\langle\Psi\rangle}{dt} = (M_0 + \mathcal{A}_0 M_1 + \mathcal{A}_0^* M_2)\langle\Psi\rangle . \tag{2.6}$$

The solution of Eq. (2.6) can be written in the Laplace-transform form:

$$\langle\Psi(t)\rangle = \int_C \frac{dz}{2\pi i} N^{-1}(z)\langle\Psi(0)\rangle , \tag{2.7}$$

where the matrix $N(z)$ is given by the following formula:

$$N(z) = zI - M_0 - \mathcal{A}_0 M_1 - \mathcal{A}_0^* M_2 . \tag{2.8}$$

In formula (2.7) the contour of integration C lies parallel to the imaginary axis in the complex z plane, to the right of all singularities of the integrands. In order to find the time behavior of $\langle\Psi(t)\rangle$ we have to invert the matrix $N(z)$. The determinant of this matrix plays an essential role because its zeros are the poles of the integration in Eq. (2.7). Taking into account the definitions of the matrices M_0, M_1, and M_2 [see Eq. (2.3)], we obtain the following algebraic equation for the zeros of the determinant:

$$z\left\{\left(z+\frac{1}{T_2'}\right)\left[\left(z+\frac{1}{T_2'}\right)\left(z+\frac{1}{T_1}\right)+\Omega^2\right]+D^2\left(z+\frac{1}{T_1}\right)\right\}=0 . \tag{2.9}$$

The cubic equation inside the brackets is the well-known Torrey equation with $\Omega^2 = 4\lambda^2\mathcal{A}_0\mathcal{A}_0^*$ the on-resonance Rabi frequency, $T_1 = \tau_0$ the longitudinal and $T_2' = 2\tau_0$ the transverse homogeneous lifetimes.[1]

In the next sections we show how incoherences affect the form and the meaning of this important relation.

III. PHASE-DIFFUSION MODEL OF THE LASER BANDWIDTH

A. Stochastic model of phase diffusion

The main goal of the stochastic approach is to generate the following correlation function for the random electromagnetic field $\mathcal{A}(t)$ and $\mathcal{A}^*(s)$,

$$\langle\mathcal{A}(t)\mathcal{A}^*(s)\rangle_{\text{av}} = \mathcal{A}_0\mathcal{A}_0^* e^{-i\omega_L(t-s) - \Gamma_L|t-s|} , \tag{3.1}$$

in order to have a power spectrum of the laser (a Lorentzian in this case) with a finite bandwidth. The angular brackets in Eq. (3.1) indicate a statistical average over the random variables of the stochastic process.

In this paper we assume only Gaussian stochastic processes, for which all higher correlation functions can be obtained from the second-order one [Eq. (3.1)] by permutations and multiplications.[24]

A stochastic theory leading to the field correlation function (3.1) has been known for a long time as the phase-diffusion model of laser light and it is based on the formal analogy with the theory of the Brownian motion.[25]

In the phase-diffusion approach the following decomposition of the stochastic electromagnetic field is performed:

$$\mathcal{A}(t) = \mathcal{A}_0 e^{-i\omega_L t - i\chi(t)}, \quad \mathcal{A}^*(t) = \mathcal{A}_0^* e^{i\omega_L t + i\chi(t)} , \tag{3.2}$$

where ω_L is the optical frequency of the laser source and the random variable of the field is a real function $\chi(t)$ that represents the instantaneous stochastic phase of the electromagnetic field.

The phase-diffusion model is based on the formal analogy between the position of a particle performing a Brownian motion and the random phase $\chi(t)$ of the electromagnetic field. The velocity of the particle has its counterpart in $\varphi(t) = \dot{\chi}(t)$.

We can now make the proper choice for the correlation function of the phase $\chi(t)$ based on the Brownian-motion analogy.

It is known that the suitable correlation function has the following form[25]:

$$\langle\chi(t)\chi(s)\rangle_{\text{av}} = \Gamma_L(t+s-|t-s|) + e^{-\Gamma_L t} + e^{-\Gamma_L s} - 2 . \tag{3.3}$$

In the limit $\Gamma_L t \gg 1$ we have $\langle\chi^2(t)\rangle_{\text{av}} = 2\Gamma_L t$ which is known as the Einstein-Smoluchowski relation in the theory of Brownian motion with Γ_L being the inverse of the diffusion time. In the limit $\Gamma_L t \ll 1$ we obtain from Eq. (3.3) $\langle\chi^2(t)\rangle_{\text{av}} = \Gamma_L^2 t^2$.

In the theory of Brownian motion the first limit is called the irreversible limit and the second the reversible one.[24,25]

For lasers operating in steady-state the irreversible limit is the proper one to use. For transient times the reversible limit can become important.[22]

From the correlation function (3.3), we can compute the following correlations between the "velocity" $\varphi(t)=\dot{\chi}(t)$ and the phase:

$$\langle\chi(t)\varphi(s)\rangle_{av}=2\Gamma_L\Theta(t-s)-\Gamma_L e^{-\Gamma_L s}, \tag{3.4a}$$

$$\langle\varphi(t)\varphi(s)\rangle_{av}=2\Gamma_L\,\delta(t-s), \tag{3.4b}$$

where $\Theta(t-s)$ is the unit step function. In the limit of $\Gamma_L t(\Gamma_L s)\gg 1$, we have

$$\langle\chi(t)\chi(s)\rangle_{av}=\Gamma_L(t+s-|t-s|), \tag{3.5a}$$

$$\langle\chi(t)\varphi(s)\rangle_{av}=2\Gamma_L\Theta(t-s), \tag{3.5b}$$

$$\langle\varphi(t)\varphi(s)\rangle_{av}=2\Gamma_L\delta(t-s). \tag{3.5c}$$

Let us note that the correlation function $\langle\varphi(t)\varphi(s)\rangle_{av}$ remains unchanged in the reversible and irreversible limits. This shows that the phase-diffusion model based only on the correlation function (3.5c) cannot take into account the transient effects of the phase autocorrelations given by Eq. (3.3). The stochastic process characterized by the correlation functions (3.5a)–(3.5c) (the limit $\Gamma_L t>1$) has its own interpretation in the framework of the random-walk theory. A stochastic process with such correlation functions is called the Wiener-Lêvy process.[24] We see that the Wiener-Lêvy process is the irreversible limit of the general diffusion process characterized by the correlation function (3.3).

Contrary to the Brownian-motion theory, the phase $\chi(t)$ and its "velocity" $\varphi(t)$ are not directly observable physical quantities. What we are really after in laser theory are the electromagnetic field correlation functions:

$$\langle\mathfrak{A}(t)\mathfrak{A}^*(s)\rangle_{av}=\mathfrak{A}_0\mathfrak{A}_0^*\langle\exp[-i\chi(t)+i\chi(s)]\rangle_{av}\times\exp[-i\omega_L(t-s)], \tag{3.6a}$$

$$\langle\mathfrak{A}(t)\mathfrak{A}(s)\rangle_{av}=\mathfrak{A}_0\mathfrak{A}_0^*\langle\exp[i\chi(t)+i\chi(s)]\rangle_{av}\times\exp[-i\omega_L(t+s)]. \tag{3.6b}$$

To compute these statistical averages we use the following characteristic functions for the Gaussian processes[24]:

$$\left\langle\exp\left(i\int_{-\infty}^{\infty}J(\tau)\chi(\tau)d\tau\right)\right\rangle_{av}=\exp\left(-\frac{1}{2}\int_{-\infty}^{\infty}d\tau_1\int_{-\infty}^{\infty}d\tau_2\,J(\tau_1)\langle\chi(t_1)\chi(t_2)\rangle_{av}J(\tau_2)\right), \tag{3.7}$$

where $J(\tau)$ is an arbitrary function. In order to get the expectation values (3.6) we have to put simply in the formula (3.7) $J(\tau)=\epsilon\delta(t-\tau)+\delta(s-\tau)$ with $\epsilon=-1$ for Eq. (3.6a) and $\epsilon=1$ for Eq. (3.6b). After simple calculations we obtain

$$\langle\mathfrak{A}(t)\mathfrak{A}^*(s)\rangle_{av}=\mathfrak{A}_0\mathfrak{A}_0^*e^{-i\omega_L(t-s)-\Gamma_L|t-s|}, \tag{3.8a}$$

$$\langle\mathfrak{A}(t)\mathfrak{A}(s)\rangle_{av}=\mathfrak{A}_0^2\exp[-2\Gamma_L(t+s)+4+\Gamma_L|t-s|-2e^{-\Gamma_L t}-2e^{-\Gamma_L s}-i\omega_L(t+s)]. \tag{3.8b}$$

In the limit of the Wiener-Lêvy process, we obtain from Eq. (3.8) the following correlation functions:

$$\langle\mathfrak{A}(t)\mathfrak{A}^*(s)\rangle_{av}=\mathfrak{A}_0\mathfrak{A}_0^*e^{-i\omega_L(t-s)-\Gamma_L|t-s|}, \tag{3.9a}$$

$$\langle\mathfrak{A}(t)\mathfrak{A}(s)\rangle_{av}=\mathfrak{A}_0^2e^{-i\omega_L(t+s)}\times\begin{cases}e^{-\Gamma_L(t+3s)}, & t>s\\ e^{-\Gamma_L(s+3t)}, & t<s.\end{cases} \tag{3.9b}$$

We see that the correlation function given by Eq. (3.6a) has the same form in both limits $\Gamma_L t\gg 1$ and $\Gamma_L t\ll 1$. The autocorrelation function (3.6b) is different in these limits and depends explicitly on both time arguments, i.e., is not stationary.

B. Exact solutions of Bloch equations

The atomic equations of motion (1.2) with stochastic fields (3.2) can be solved exactly. Using the definitions (3.2) and Eq. (1.2) we obtain a matrix equation (1.5) for a multiplicative stochastic process with

$$\Psi(t)=(e^{i\omega_L t}\sigma(t),\sigma_3(t)e^{-i\chi(t)},e^{-i\omega_L t-2i\chi(t)}\sigma^+(t),e^{-i\chi(t)}) \tag{3.10}$$

and

$$M_0=\begin{bmatrix}-iD-1/2\tau_0 & \lambda\mathfrak{A}_0 & 0 & 0\\ -2\lambda\mathfrak{A}_0^* & -1/\tau_0 & -2\lambda\mathfrak{A}_0^* & -1/\tau_0\\ 0 & \lambda\mathfrak{A}_0 & iD-1/2\tau_0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{3.11a}$$

$$M_1=\begin{bmatrix}0&0&0&0\\0&-1&0&0\\0&0&-2&0\\0&0&0&-1\end{bmatrix}, \tag{3.11b}$$

$$M_2=0. \tag{3.11c}$$

The random variable of the stochastic process is equal to $x(t)=\varphi(t)$.

If we assume that the atomic density matrix ρ_A is such that $\mathrm{Tr}[\rho_A\sigma(0)]=0$ and $\mathrm{Tr}[\rho_A\sigma_3(0)]=-1$, i.e., at $t=0$ the atom is in its ground state; we obtain an initial condition dependent from the random field variables through the factor $e^{-i\chi(0)}$. For such a multiplicative process it has been shown that the following equation is an exact result following from Eq. (1.5)[21]:

$$\frac{d\langle\Psi\rangle_{av}}{dt}=(M_0-\Gamma_L M_1^2-\Gamma_L M_1 e^{-\Gamma_L t})\langle\Psi\rangle_{av}\,, \tag{3.12}$$

with a statistically independent initial condition

$$\langle\Psi(0)\rangle_{\rho_A}=(0,-1,0,1)\,. \tag{3.13}$$

In the limit of the Wiener-Lévy process Eq. (3.12) can be solved exactly in the same way as it was done in Sec. I, i.e., in terms of the Laplace transform (2.7). Taking the definitions of the matrices M_0 and M_1 [see Eq. (3.11)], we obtain the following generalization of the perfectly coherent Torrey equation (2.9):

$$(z+\Gamma_L)\left[\left(z+iD+\frac{1}{2\tau_0}\right)\left(z-iD+\frac{1}{2\tau_0}+4\Gamma_L\right)\times\left(z+\frac{1}{\tau_0}+\Gamma_L\right)+4\lambda^2\mathfrak{a}_0\mathfrak{a}_0^*\left(z+\frac{1}{2\tau_0}+2\Gamma_L\right)\right]=0\,. \tag{3.14}$$

In the limit $\Gamma_L t<1$ the algebraic equation governing the time behavior of the solutions of Eq. (3.12) is the following:

$$z\left[\left(z+iD+\frac{1}{2\tau_0}\right)\left(z-iD+\frac{1}{2\tau_0}+2\Gamma_L\right)\left(z+\frac{1}{\tau_0}\right)+4\lambda^2\mathfrak{a}_0\mathfrak{a}_0^*\left(z+\frac{1}{2\tau_0}+\Gamma_L\right)\right]=0\,. \tag{3.15}$$

As we can see from Eqs. (3.14) and (3.15) the influence of the laser linewidth is different for transient times than for the steady-state regime. In the limit of a perfectly coherent laser, i.e., $\Gamma_L=0$ the generalized Torrey equations (3.14) and (3.15) reduce to the standard form given by Eq. (2.9). At exact resonance $D=0$ and in the limit of high-intensity field $\Omega\tau_0>1$ and $\Omega>\Gamma_L$, we can compute the approximate roots of Eqs. (3.14) and (3.15). After simple calculations, we obtain

$$z_1=-\Gamma_L\,,\quad z_2=-\frac{1}{2\tau_0}-2\Gamma_L\,,\quad z_{3,4}=-\frac{3}{4\tau_0}-\frac{3}{2}\Gamma_L\pm i\Omega \tag{3.16}$$

for the Wiener-Lévy process and

$$z_1=0,\quad z_2=-\frac{1}{2\tau_0}-\Gamma_L\,,\quad z_{3,4}=-\frac{3}{4\tau_0}-\frac{\Gamma_L}{2}\pm i\Omega \tag{3.17}$$

for the transient regime. These results indicate that for high intensity-fields and at exact resonance we should modify the homogeneous lifetimes in the following way:

$$1/T_2'=1/2\tau_0+2\Gamma_L\,,\quad 1/T_1=1/\tau_0+\Gamma_L \tag{3.18a}$$

for the Wiener-Lévy process and

$$1/T_2'=1/2\tau_0+\Gamma_L\,,\quad 1/T_1=1/\tau_0 \tag{3.18b}$$

in the transient regime, in order to take into account the laser linewidth Γ_L.

As it is clear from definition (3.10) these rules can be applied only for $\sigma(t)$ or $\sigma_3(t)e^{-i\chi(t)}$. In order to obtain the expression for the time evolution of the atomic inversion $\sigma_3(t)$, we can derive an equation of the type (1.5) with the following definitions of the vector operator Ψ and matrices:

$$\Psi(t)=(\sigma_3(t),\ \sigma^+(t)e^{-i\omega_L t-i\chi(t)},\ \sigma(t)e^{i\omega_L t+i\chi(t)},\ -1/\tau_0), \tag{3.19}$$

$$M_0=\begin{bmatrix}-1/\tau_0 & -2\lambda\mathfrak{a}_0 & -2\lambda\mathfrak{a}_0^* & 1\\ \lambda\mathfrak{a}_0^* & iD-1/2\tau_0 & 0 & 0\\ \lambda\mathfrak{a}_0 & 0 & -iD-1/2\tau_0 & 0\\ 0&0&0&0\end{bmatrix}, \tag{3.20a}$$

$$M_1=\begin{bmatrix}0&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}, \tag{3.20b}$$

$$M_2=0 \tag{3.20c}$$

with $x(t)=\varphi(t)=\dot\chi(t)$. A straightforward application of the theory of multiplicative stochastic processes leads to the following equation for $\langle\Psi\rangle_{av}$:

$$\frac{d\langle\Psi\rangle_{av}}{dt}=(M_0-\Gamma_L M_1^2)\langle\Psi\rangle_{av} \tag{3.21}$$

with the initial condition

$$\langle\Psi(0)\rangle_{av}=(-1,0,0,-1/\tau_0)\,. \tag{3.22}$$

Equation (3.21) is valid for all values of $\Gamma_L t$. The difference between results (3.12) and (3.21) comes from the fact that the operator [Eq. (3.10)] evaluated at $t=0$ depends on the stochastic phase through the factor $e^{-i\chi(0)}$. The operator Ψ [Eq. (3.19)] evaluated with the same density matrix ρ_A at $t=0$ is statistically independent from the laser field. The corresponding determinantal equation for the solution of Eq. (3.21) is the following:

$$z\left\{\left(z+\frac{1}{\tau_0}\right)\left[\left(z+\frac{1}{2\tau_0}+\Gamma_L\right)^2+D^2\right] +4\lambda^2\alpha_0\alpha_0^*\left(z+\frac{1}{2\tau_0}+\Gamma_L\right)\right\}=0. \quad (3.23)$$

In the limit when $\Gamma_L=0$ Eq. (3.23) reduces to the perfectly coherent Torrey equation. In contrast to Eq. (3.14) and (3.15) the polynomial (3.23) can be otained from the Torrey equation (2.9) by simple replacement rules:

$$1/T_2'=1/2\tau_0+\Gamma_L\,, \quad 1/T_1=1/\tau_0\,. \quad (3.24)$$

In other words the incoherences affects only the transverse lifetime T_2'. The rules given by Eq. (3.24) are the same as the rules (3.19) for the transient-laser regime and high laser power with the important difference that Eq. (3.24) holds for all values of $\Gamma_L t$ and for all field intensities.

C. Atomic correlation function: Exact solution

The last quantity to be discussed in this section is the following two-point atomic correlation function:

$$G_u(\tau)=\sigma^+(u)\sigma(u+\tau)\,, \quad \tau>0\,. \quad (3.25)$$

This correlation function plays an important role in the definition of the power spectrum of the scattered light by the two-level system.[26] From the basic equations of motion (1.2) with stochastic fields (3.2) we obtain an equation of the form (1.5) with the time derivative computed with respect to τ. The matrices M_0 and M_1 are the same as in Eq. (3.11). The random variable is given by the following relation: $x(\tau)=\varphi(u+\tau)$ and the operator-valued vector Ψ is given as follows:

$$\Psi(t)=\left(e^{i\omega_L\tau}\sigma^+(u)\sigma(u+\tau),\, e^{-i\omega_L u-i\chi(u+\tau)}\sigma^+(u)\sigma_3(u+\tau),\, e^{-i\omega_L(2u+\tau)-i2\chi(u+\tau)}\sigma^+(u)\sigma^+(u+\tau)\,,\, e^{-i\omega_L u-i\chi(u+\tau)}\sigma^+(u)\right). \quad (3.26)$$

For the Wiener-Lévy stochastic process we obtain, applying again the known technique for multiplicative stochastic processes, the following exact equation for $\langle\Psi\rangle_{av}$:

$$\frac{d}{d\tau}\langle\Psi\rangle_{av}=(M_0-\Gamma_L M_1^2)\langle\Psi\rangle_{av} \quad (3.27)$$

with the following initial condition:

$$\langle\Psi\rangle_{av}\big|_{\tau=0}=\langle(\sigma^+(u)\sigma(u),\, e^{-i\omega_L u-i\chi(u)}\sigma^+(u)\sigma_3(u),\, 0,\, e^{-i\omega_L u-i\chi(u)}\sigma^+(u))\rangle_{av}\,. \quad (3.28)$$

The τ dependence of the correlation function is thus governed by roots of Eq. (3.14). The u dependence is given by Eq. (3.14) and (3.23) as well.

D. Discussion

In this section we have presented the phase-diffusion model of the laser linewidth based on the Brownian-motion analogy. The specification of the "velocity" correlation function (3.4b) as it was done in many papers is not enough to define the phase-diffusion model. Only in the steady-state limit when the phase-diffusion model is described by the Wiener-Lévy stochastic process the decorrelation assumptions used in the previous works are exact and have not to be imposed additionally.[14-18] The time evolution of the dipole operator is governed by the roots of the generalized Torrey equation (3.14). In the limit of high laser intensity, i.e., $\Omega\tau_0>1$, Eq. (3.14) reduces to the standard Torrey equation with the following formulas for the phenomenological lifetimes:

$$\frac{1}{T_2'}\simeq\frac{1}{2\tau_0}+2\Gamma_L\,, \quad \frac{1}{T_1}\simeq\frac{1}{\tau_0}+\Gamma_L+\frac{2D\Gamma_L}{\Omega}\,. \quad (3.29)$$

The longitudinal lifetime T_1 depends on the detuning, laser linewidth, and Rabi frequency as well.

For transient times the initial condition dependence on the statistical properties of the field becomes important and the proper solution differs from the steady-state situation. The techniques of the decorrelation assumptions do not work in this case.

The method of multiplicative stochastic processes that we have developed previously produces the exact solution for the transient times in a straightforward way. The proper generalized Torrey equation (3.15) leads to the following rules for the homogeneous lifetimes in the limit of high-intensity fields:

$$\frac{1}{T_2'}=\frac{1}{2\tau_0}+\Gamma_L\,,\quad \frac{1}{T_1}\simeq\frac{1}{\tau_0}+\Gamma_L+\frac{D\Gamma_L}{2\Omega}\,. \qquad (3.30)$$

At exact resonance the laser linewidth shifts uniformly the longitudinal and the transverse lifetimes.

The atomic inversion operator is governed by a much simpler Torrey-type equation (3.23) which is valid for all values of $\Gamma_L t$. The rules given by Eq. (3.24) are exact for all values of the field intensity and detuning.

For the Levy-Wiener process describing the steady-state phase diffusion we obtain an exact solution for the atomic correlation function (3.25). It turns out that the proper generalized Torrey equation has the same form as in the case of the time evolution of the atomic dipole moment [see Eq. (3.14)]. Because the power spectrum of resonance fluorescence is related to this correlation function, the detuning and the power broadening of the lifetime T_1 leads for a finite bandwidth of the laser to a slight asymmetry in the predicted fluorescence spectrum.[17,27] This observation, without any relation to power and detuning broadening of T_1 has been already made in the literature. We note that the solution for the correlation function (3.25) has been obtained without the regression theorem used in previous discussions of this problem.[15]

IV. RANDOM ELECTROMAGNETIC FIELD AMPLITUDE

In Sec. III we have found exact solutions of Bloch equations if a random-phase description of the electromagnetic field is introduced. In this section we continue the same investigation but without introducing the amplitude-phase decomposition given by Eq. (3.2). We rewrite the electromagnetic field amplitudes in the following way:

$$e^{i\omega_L t}\mathcal{A}(t)=\sqrt{I_0}+\mathcal{E}(t)\,, \qquad (4.1a)$$

$$e^{-i\omega_L t}\mathcal{A}^*(t)=\sqrt{I_0}+\mathcal{E}^*(t)\,, \qquad (4.1b)$$

where I_0 is a constant intensity and $\mathcal{E}(t)$ is a random fluctuating variable with the following Gaussian statistics:

$$\langle\mathcal{E}(t)\,\mathcal{E}^*(s)\rangle_{av}=(\Gamma_1/2\lambda^2)\,\delta(t-s)\,, \qquad (4.2a)$$

$$\langle\mathcal{E}(t)\,\mathcal{E}(s)\rangle_{av}=(\Gamma_2/2\lambda^2)\,\delta(t-s)\,, \qquad (4.2b)$$

$$\langle\mathcal{E}(t)\rangle_{av}=\langle\mathcal{E}^*(t)\rangle_{av}=0\,. \qquad (4.2c)$$

We have introduced here two different arbitrary linewidths Γ_1 and Γ_2 in order to investigate contributions from correlation functions (4.2a) and (4.2b) separately. The coupling constant factor λ^{-2} was introduced in Eq. (4.2) in order to have Γ_1 and Γ_2 of the dimension of the linewidth. The stochastic fields introduced by Eqs. (4.1) and (4.2) can have different physical justification. The fluctuations can be associated with a different stochastic model of laser linewidth based on amplitude fluctuations or with fluctuating external fields leading, for example, to collisions.

The atomic Bloch equations (1.2) with the fluctuating fields (4.1) can be written in the form of Eq. (1.5) with

$$\Psi(t)=(e^{i\omega_L t}\sigma(t),\,e^{-i\omega_L t}\sigma^+(t),\,\sigma_3(t),\,-1/\tau_0) \qquad (4.3)$$

and the matrices M_0, M_1, and M_2 are then the following:

$$M_0=\begin{bmatrix} -iD-1/2\tau_0 & 0 & \lambda\mathcal{A}_0 & 0\\ 0 & iD-1/2\tau_0 & \lambda\mathcal{A}_0^* & 0\\ -2\lambda\mathcal{A}_0^* & -2\lambda\mathcal{A}_0 & -1/\tau_0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}, \qquad (4.4a)$$

$$M_1=\begin{bmatrix} 0 & 0 & -i & 0\\ 0&0&0&0\\ 0&2i&0&0\\ 0&0&0&0\end{bmatrix}, \qquad (4.4b)$$

$$M_2=\begin{bmatrix} 0&0&0&0\\ 0&0&-i&0\\ 2i&0&0&0\\ 0&0&0&0\end{bmatrix}. \qquad (4.4c)$$

The random variables of the stochastic process (4.1) are the following: $x(t)=\mathcal{E}(t)$ and $x^*(t)=\mathcal{E}^*(t)$. From Eq. (1.5) with definitions (4.3) and (4.4) we obtain the following exact equation for the stochastic average of the vector (4.3):

$$\frac{d}{dt}\langle\Psi\rangle_{av}=[M_0-\tfrac{1}{4}\Gamma_2(M_1^2+M_2^2)-\tfrac{1}{4}\Gamma_1(M_1M_2+M_2M_1)]\langle\Psi\rangle_{av} \qquad (4.5)$$

with the initial condition

$$\langle\Psi(0)\rangle_{\rho_A}=(0,0,-1,-1/\tau_0)\,. \tag{4.6}$$

The determinant of the proper $N(z)$ matrix for Eq. (4.5) leads to the following algebraic equation:

$$z\left\{\left[\left(z+\frac{1}{2\tau_0}+\frac{\Gamma_1}{2}\right)^2+D^2\right]\left(z+\frac{1}{\tau_0}+\Gamma_1\right)-2\lambda^2 I_0\Gamma_2 +4\lambda^2 I_0\left(z+\frac{1}{2\tau_0}+\frac{\Gamma_1}{2}\right)-\frac{\Gamma_2^2}{4}\left(z+\frac{1}{\tau_0}+\Gamma_1\right)\right\}=0\,. \tag{4.7}$$

Eq. (4.7) shows how differently the autocorrelation (4.2b) contributes to the generalized Torrey equation comparing to the cross-correlation (4.2a). If the autocorrelation of the sotchastic field vanishes i.e., $\Gamma_2=0$ the solution of Eq. (4.5) can be obtained from the perfectly coherent case by simple replacements:

$$1/T_1=1/\tau_0+\Gamma_1\,, \tag{4.8a}$$

$$1/T_2'=1/2\tau_0+\tfrac{1}{2}\Gamma_1\,. \tag{4.8b}$$

Thus the linewidth Γ_1 affects both the transverse and the longitudinal lifetimes but in a different way. We have obtained then the same rules as in the high-power limit (3.18) with the exception that off-resonance the lifetime T_1 is not power broadened.

If the linewidth Γ_2 of the autocorrelation (4.2b) is not equal to zero we can obtain the following generalization of the relations (4.8) for high-intensity laser field:

$$\frac{1}{T_1}\simeq\frac{1}{\tau_0}+\Gamma_1+\Gamma_2\,,\quad \frac{1}{T_2'}\simeq\frac{1}{2\tau_0}+\frac{\Gamma_1}{2}-\frac{\Gamma_2}{2}\,. \tag{4.9}$$

If the linewidth of the correlation functions (4.2) are equal, i.e., $\Gamma_1=\Gamma_2$ we recover from Eq. (4.9) the known result for amplitude fluctuations of the laser field.[14,15]

V. ATOMIC FREQUENCY FLUCTUATIONS

As already mentioned in the Introduction, due to collisions the energy separation ω_0 of the two-level atom can also fluctuate around its fixed value ω_0. This means that in the atomic equations (1.1) we should replace ω_0 by $\omega_0+\delta\omega(t)$ and assume that $\delta\omega(t)$ is a random variable of a stochastic process. The simplest assumption about the statistical properties of $\delta\omega$ is the following set of Gaussian correlation functions:

$$\langle\delta\omega(t)\rangle_{\text{av}}=0\,,\qquad \langle\delta\omega(t)\,\delta\omega(s)\rangle_{\text{av}}=2\Gamma_B\delta(t-s)\,. \tag{5.1}$$

In order to find the physical interpretation of Γ_B let us assume for the moment that there are no damping and external fields in the Bloch equations (1.2). We can then write down immediately the solution for the $\sigma(t)$ operator and form the following correlation function:

$$\langle\sigma^+(t)\sigma(s)\rangle_{\text{av}}=\left\langle\exp\left(i\int_s^t d\tau\,\delta\omega(\tau)\right)\right\rangle_{\text{av}} \times\exp[i\omega_0(t-s)]\,\sigma^+(0)\sigma(0)\,. \tag{5.2}$$

The stochastic average of the phase can be computed using the general formula (3.7) for Gaussian processes and the definitions (5.1):

$$\langle\sigma^+(t)\sigma(s)\rangle_{\text{av}}=\exp[i\omega_0(t-s)-\Gamma_B|t-s|]\,\sigma^+(0)\sigma(0)\,. \tag{5.3}$$

If we recall that the operator $\sigma(t)$ is the two-level truncation of the momentum operator the interpretation of Eq. (5.3) is clear. Due to collisions the correlation function of the velocities is damped with a characteristic coherence time Γ_B^{-1}. This simple statistical model can describe the influence of collisions on the solutions of Bloch equations. With such a statistic we can write again a general Eq. (1.5) $x(t)=\delta\omega(t)$ and Ψ given by Eq. (4.3). A simple repetition of the used procedure leads in this case to a Torrey equation (2.9) with the incoherence affecting only the transverse lifetime

$$1/T_2'=1/2\tau_0+\Gamma_B\,.$$

Due to the definition of the vector-operator Ψ [see Eq. (4.3)] this conclusion is valid both for the dipole operator $\sigma(t)$ and the population inversion operator $\sigma_3(t)$.

VI. COLLISION INDUCED BY RADIATION

In Sec. V we have discussed a simple stochastic model leading to a finite bandwidth Γ_B of the velocity correlation function. In this section we generalize the previous discussion to the case when the random fluctuations are not statistically independent from the phase fluctuations of the electromagnetic field. This simple stochastic assumption leads to the effect in which the random phase of the laser is coupled to the phase of the atomic dipole leading to modifications of the velocity correlation function coherence time due to radiation. This model can describe a collision induced by radiation. This model is the simplest stochastic description of this effect. In order to describe these collisions induced by radiation we introduce the following random variables: $\delta\omega_R$ for the dipole phase and χ_R for the laser phase.

Following the results from previous sections we assume the following correlation functions for the frequency fluctuation $\delta\omega_R$ and the laser phase χ_R which is induced by collisions:

$$\langle \chi_R(t)\chi_R(s)\rangle_{av}=\gamma_1(t+s-|t-s|), \tag{6.1a}$$

$$\langle \delta\omega_R(t)\,\delta\omega_R(s)\rangle_{av}=2\gamma_2\delta(t-s), \tag{6.1b}$$

$$\langle \delta\omega_R(t)\,x_R(s)\rangle_{av}=2\gamma_{12}\Theta(s-t). \tag{6.1c}$$

Formula (6.1c) couples the random variable $\delta\omega_R$ to χ_R. From the theory of Gaussian stochastic processes we obtain easily the following characteristic function:

$$\Big\langle \exp\Big(i\int J_1(\tau)\,\delta\omega_R(\tau)d\tau+i\int J_2(\tau)\chi_R(\tau)d\tau\Big)\Big\rangle_{av}$$
$$=\exp\Big(-\frac{1}{2}\int J_1(\tau_1)\langle\delta\omega_R(\tau_1)\,\delta\omega_R(\tau_2)J_2(\tau)\rangle_{av}-\frac{1}{2}\int J_2(\tau_1)\langle\chi_R(\tau_1)\chi_R(\tau_2)\rangle_{av}J_2(\tau_2)$$
$$-\frac{1}{2}\int J_1(\tau_1)\langle\delta\omega_R(\tau_1)\,\chi_R(\tau_2)\rangle_{av}J_2(\tau_2)-\frac{1}{2}\int J_2(\tau_1)\langle\chi_R(\tau_1)\,\delta\omega_R(\tau_1)J_2(\tau_2)\rangle_{av}\Big), \tag{6.2}$$

where J_1 and J_2 are two arbitrary functions. In order to understand the physical role of the cross-correlation function (6.1) we compute the following atom-field correlation:

$$\langle\sigma^+(t)\,\mathcal{A}(s)\rangle_{av}=e^{i\omega_0 t-i\omega_L s}\Big\langle\exp\Big(i\int_0^t d\tau\,\delta\omega(\tau)-i\chi(s)\Big)\Big\rangle_{av}\sigma^+(0)\,\mathcal{A}_0, \tag{6.3}$$

where the time-evolution of the dipole operator is given by the free oscillation ($\lambda=0$) of the fluctuation $\delta\omega_R$ around ω_0. With the proper choice of functions J_1 and J_2 in the formula (6.2) we compute

$$\Big\langle\exp\Big(i\int_0^t d\tau\,\delta\omega_R(\tau)-i\chi_R(s)\Big)\Big\rangle_{av}=\begin{cases}\exp(-\gamma_1 t-\gamma_2 s+2\gamma_{12}s), & t>s\\ \exp(-\gamma_1 t-\gamma_2 s+2\gamma_{12}t), & s>t.\end{cases} \tag{6.4}$$

The formula (6.4) leads to a damped correlation function for all values of t and s only if

$$2\gamma_{12}<\min(\gamma_1,\gamma_2). \tag{6.5}$$

This restriction for the cross-correlation linewidth assures the proper behavior of the atom-field correlation function for t or s going to infinity. The requirement of having a stationary correlation function (6.3) leads to the following formula for γ_{12}:

$$\gamma_{12}=\tfrac{1}{2}(\gamma_1+\gamma_2). \tag{6.6}$$

We will not assume any particular formula for γ_{12} in the following calculations. We remember only, that a physical requirement restricts γ_{12} to satisfy condition (6.5). With this kind of statistic we can easily obtain a generalization of the proper equation of motion satisfied by the vector-operator (3.10) with random variables $x(t)=\chi_R$ and $x^*(t)=\delta\omega_R$. The remaining definitions of the matrices (3.11a) and (3.11b) are the same with an additional definition

$$M_2=\begin{bmatrix}-1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}. \tag{6.7}$$

After a straightforward application of the theory of multiplicative stochastic processes we obtain the following generalization of Eq. (3.12) for radiation-induced collisions:

$$\frac{d\langle\Psi\rangle_{av}}{dt}=(M_0-\gamma_1M_1^2-\gamma_2M_2^2-2\gamma_{12}M_1M_2)\langle\Psi\rangle_{av}. \tag{6.8}$$

The proper generalization of the Torrey equation for Eq. (6.8) is the following:

$$(z+\gamma_1)\left[\left(z-iD+\frac{1}{2\tau_0}+4\gamma_1+\gamma_2-4\gamma_{12}\right)\left(z+iD+\frac{1}{2\tau_0}+\gamma_2\right)\left(z+\frac{1}{\tau_0}+\gamma_1\right)+4\lambda^2\mathcal{a}_0\mathcal{a}_0^*\left(z+\frac{1}{2\tau_0}+2\gamma_1+\gamma_2-2\gamma_{12}\right)\right]=0\,. \qquad (6.9)$$

In the limit of high-intensity field Eq. (6.9) leads to the following formulas for the homogeneous lifetimes:

$$\frac{1}{T_1}\simeq\frac{1}{\tau_0}+\gamma_1+\frac{2D}{\Omega}(\gamma_1-\gamma_{12})\,, \qquad (6.10a)$$

$$\frac{1}{T_2'}\simeq\frac{1}{2\tau_0}+2\gamma_1+\gamma_2-2\gamma_{12}\,. \qquad (6.10b)$$

We see again that the time evolution of the dipole operator is governed by a power broadened T_1. As in Sec. III the time evolution of the inversion operator $\sigma_3(t)$ [Eq. (3.19)] is given by different matrices (3.20). With $x^*(t)=\delta\omega_R(t)$, we have for the collision an extra contribution due to the fact that

$$M_2=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{bmatrix}\,. \qquad (6.11)$$

A repetition of the general method leads to the following exact relations for the lifetimes valid for all values of the field intensity:

$$1/T_1=1/\tau_0\,, \qquad (6.12a)$$

$$1/T_2'=1/2\tau_0+\gamma_1+\gamma_2-2\gamma_{12}\,. \qquad (6.12b)$$

We see from results (6.10) and (6.12) that the stochastic model of radiation-induced collisions leads to solutions for which there are no simple universal rules governing the lifetimes of the dipole moment and the population inversion. Each of these operators has its own homogeneous lifetime [(6.10) or (6.12)].

It is impossible to incorporate the results of this section by a universal introduction of phenomenological damping terms in the semiclassical Bloch equations (1.1) as it was done in many references.[28,29]

If the fluorescence spectrum of light is investigated the proper equation to work with is the polynomial (6.9) [see Sec. III C for the discussion of the atomic correlation function]. For this case we can reproduce the results of a microscopic theory of collisions induced by radiation at exact resonance ($D=0$), which are given by the following relations:

$$\frac{1}{T_1}=2\Gamma_I+\frac{1}{\tau_0}\,,\quad \frac{1}{T_2'}=\frac{1}{2\tau_0}+\Gamma_E+\Gamma_I\,, \qquad (6.13)$$

assuming in formulas (6.9) that $\gamma_1=2\Gamma_I$, $\gamma_2=\Gamma_E$, and $2\gamma_{12}=\gamma_1$. The usual interpretation of the lifetimes is the following. The width Γ_I is associated with inelastic collisions and Γ_E with elastic one. Due to the flexibility of the results (6.5) and (6.6), we can adjust γ_1, γ_2, and γ_{12} to model different physical situations of collisions induced by radiation.

VII. CONCLUSIONS

In this paper we have presented a general stochastic approach to incoherence properties induced by laser linewidth and collisions. Our treatment was based on operator-valued optical Bloch equations with stochastic coefficients modeling different sources of incoherence. The operator-valued equations of motion with stochastic coefficients have been studied recently by the author and exact solutions for Gaussian stochastic processes have been obtained. Applying the previously developed techniques, we have obtained exact solutions of optical Bloch equations for the phase-diffusion model of laser light. We have shown that the phase-diffusion theory has two different time regimes. The first one, the steady-state, is modeled by the Wiener-Lévy stochastic process. The other regime, the transient, has not been discussed thus far in the literature. We have shown that the decorrelation assumptions used in previous calculations cannot be used in the transient regime. According to results of Ref. 14 the incoherences do not act in an additive way to all phenomenological lifetimes. The results of our discussion support this opinion. More than that, we have shown that in some cases the incoherence effects cannot be included in the Bloch equations by simple phenomenological damping terms. We have obtained exact solutions of Bloch equations with detuning-dependent and power-broadened decay constants. These damping terms can be obtained from the homogeneous lifetimes T_2' and T_1 by special rules.

We have also obtained exact solutions for simple models of collisions and collision induced by radiation. Again, there are no simple additive rules in the Bloch equations to take into account the incoherences due to collisions. Different formulas for T_2' and T_1 have been obtained for different stochastic processes. A full treatment of incoherences would require a detailed microscopic discussion of physical processes leading to the laser linewidth or to collisions. It is interesting to establish to which microscopic assumptions correspond the stochastic description of the incoherence.

These problems in the framework of resonance fluorescence will be discussed in a forthcoming paper.

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