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Electron bremsstrahlung energy spectra above 2 MeV

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With a partial-wave interpolation method we calculate the electron bremsstrahlung energy spectra from neutral atoms with atomic number $Z = 13$ and 92 for incident electrons of kinetic energy $T_1 = 5$ and 10 MeV. This is an attempt to bridge the gap between the results we have previously obtained for $T_1 \leq 2$ MeV and the results which can be predicted in the high-energy limit. We conclude that for light elements Born approximation modified by the Elwert factor and a form factor remains accurate, while for heavier elements the Davies, Bethe, Maximon, Olson approach is of comparable 5%–10% accuracy at these energies and continues to improve with increasing energy.

We wish to present data on the electron bremsstrahlung spectrum from incident electrons of kinetic energy 5 and 10 MeV, obtained with an extension of our previous numerical partial-wave calculation techniques which utilizes interpolation in partial-wave cross sections.[1,2] Our purpose is to obtain some guidance as to when (for how low incident energies) high-energy limit forms for the electron bremsstrahlung spectrum may be used.

Under the circumstances that the kinetic energies of the incident and recoil electron $T \gg 1 m_e c^2$, Davies, Bethe, Maximon, and Olsen (DBMO)[3] obtained an analytic expression for the bremsstrahlung spectrum,

$$\sigma(k) \equiv \beta_1^2 \frac{k}{Z^2}\frac{d\sigma}{dk} = 4\alpha^3\beta_1^2 \left\{ \left[1+\left(\frac{E_2}{E_1}\right)^2\right]\left(\int_\delta^1 (q-\delta)^2[1-F(q)]^2\frac{dq}{q^3} + 1 - f(Z)\right) - \frac{2}{3}\frac{E_2}{E_1}\left(\int_\delta^1 \left(q^3 - 6\delta^2 q \ln\frac{q}{\delta} + 3\delta^2 q - 4\delta^3\right)[1-F(q)]^2\frac{dq}{q^4} + \frac{5}{6} - f(Z)\right)\right\},$$

where E_1 and E_2 are the *total* energy (including rest mass energy) of the incident and recoil electron, k the radiated photon energy, β_1 the incident electron velocity, Z the atomic number, and $F(q)$ is the atomic form factor. Also

$$\delta = k/(2E_1E_2),$$

$$f(Z) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n+[n^2+(Z\alpha)^2]}.$$

It is believed that this calculation is valid for $T_1 > 15$–50 MeV. The purpose of this work is an attempt to bridge the gap between the results we have previously obtained for $T_1 = 1$ keV to 2 MeV and these predictions for higher energies.

Recently we presented a tabulation[1] of the electron bremsstrahlung energy spectra from neutral atoms with atomic number $Z = 2$–92 for incident electron kinetic energies T_1 in the range from 1 keV to 2 MeV. This tabulation was based on a calculation with partial-wave methods[2] in which the process is described as a single-electron transition in a relativistic self-consistent screened potential. Electron wave functions were obtained in partial-wave series by numerically solving the radial Dirac equation. Photon wave functions were also expanded in partial-wave series; the angular integrals of the bremsstrahlung matrix elements were performed analytically, while the radial integrals were calculated numerically and then summed numerically over the partial series. The unpolarized bremsstrahlung cross section, differential in photon energy k, has the form

$$\sigma(k) = \sum_{l_1, l_2} \sigma_{l_1, l_2}(k) = \sum_{l_1} \sigma_{l_1}(k),$$

where l_1 and l_2 are the orbital angular momentum quantum numbers of the incident and final electrons, respectively.

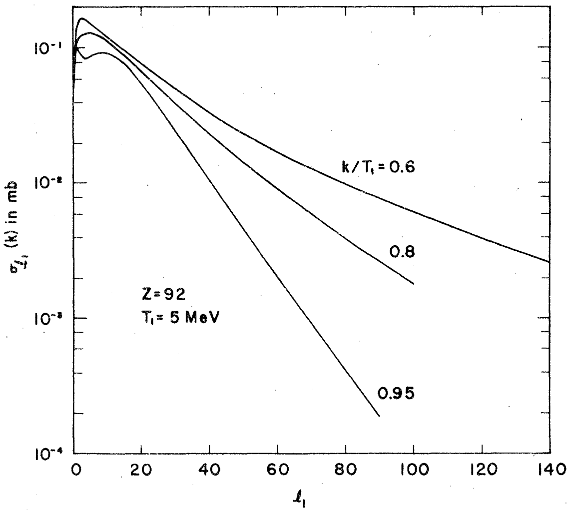

FIG. 1. Variation of the partial-wave cross section $\sigma_{l_1}(k)$ as a function of l_1 for the cases with $Z=92$, $T_1=5$ MeV, and $k/T_1=0.6$, 0.8, 0.95.

The straightforward application of this partial-wave method is feasible for obtaining fairly accurate theoretical predictions for the electron bremsstrahlung spectrum when $T_1 \lesssim 2$ MeV, except near the soft-photon limit of the spectrum, where the partial-wave series does not converge fast enough to make the partial-wave method feasible. In the soft-photon region of the spectrum the low-energy theorem[4] can be used to obtain bremsstrahlung predictions from elastic scattering cross sections.[5]

We wish to see whether it is possible to obtain accurate bremsstrahlung cross sections for $T_1 > 2$ MeV with the partial-wave method, even though it becomes prohibitive to calculate all partial-wave cross sections. We find that the partial cross section σ_l is a smoothly varying function of l. To illustrate this point, we show in Fig. 1 the variation of σ_{l_1} as a function of l_1 for the cases $Z=92$, $T_1=5$ MeV, and $k/T_1=0.6, 0.8, 0.95$. We see that after $l_1=10$ σ_{l_1} is a smoothly decaying function of

TABLE I. Comparisons of bremsstrahlung cross sections $\sigma(k)$ in mb for $Z=13$ and 92, $T_1=1$, 2, 5, and 10 MeV between results obtained from simpler theories, such as Born approximation (BH), Born approximation modified with form-factor screening and Elwert factor (EBF), and high-energy theory (DBMO).

T_1	Z	k/T_1	0.0	0.2	0.4	0.6	0.8	0.95
1		BH	∞	7.84	4.84	3.01	1.65	0.67
	13	ES	12.60	7.54	4.93	3.22	1.91	1.00
		EBF/ES	0.99	0.99	0.98	0.96	0.95	0.94
		DBMO/ES	0.95	0.98	1.05	1.19	1.53	2.42
	92	ES	11.62	8.30	6.16	4.74	3.67	2.95
		EBF/ES	0.89	0.83	0.77	0.68	0.58	0.50
		DBMO/ES	0.78	0.71	0.67	0.63	0.59	0.57
2		BH	∞	8.73	5.52	3.55	2.03	0.84
	13	ES	13.21	8.24	5.49	3.67	2.21	1.08
		EBF/ES	0.98	0.99	0.98	0.97	0.96	0.95
		DBMO/ES	0.98	1.01	1.06	1.16	1.43	2.32
	92	ES	11.45	8.10	5.92	4.45	3.34	2.43
		EBF/ES	0.95	0.92	0.87	0.80	0.69	0.60
		DBMO/ES	0.86	0.82	0.79	0.75	0.70	0.71
5		BH	∞	10.47	6.95	4.80	3.05	1.33
	13	ES	13.74	9.53	6.80	4.85	3.16	1.47
		EBF/ES	0.97	0.97	0.97	0.97	0.97	0.99
		DBMO/ES	0.97	0.98	1.01	1.06	1.20	1.86
	92	ES	11.14	8.21	6.13	4.62	3.42	2.33
		EBF/ES	1.01	1.00	1.00	0.98	0.90	0.74
		DBMO/ES	0.92	0.90	0.90	0.88	0.85	0.81
10		BH	∞	12.00	8.26	6.00	4.13	1.96
	13	ES	13.81	10.24	7.75	5.91	4.20	2.04
		EBF/ES	0.98	0.97	0.97	0.97	0.97	1.00
		DBMO/ES	0.98	0.98	0.99	1.01	1.08	1.51
	92	ES	10.81	8.39	6.59	5.21	3.87	2.53
		EBF/ES	1.05	1.04	1.04	1.03	1.03	0.87
		DBMO/ES	0.95	0.93	0.92	0.92	0.92	0.87

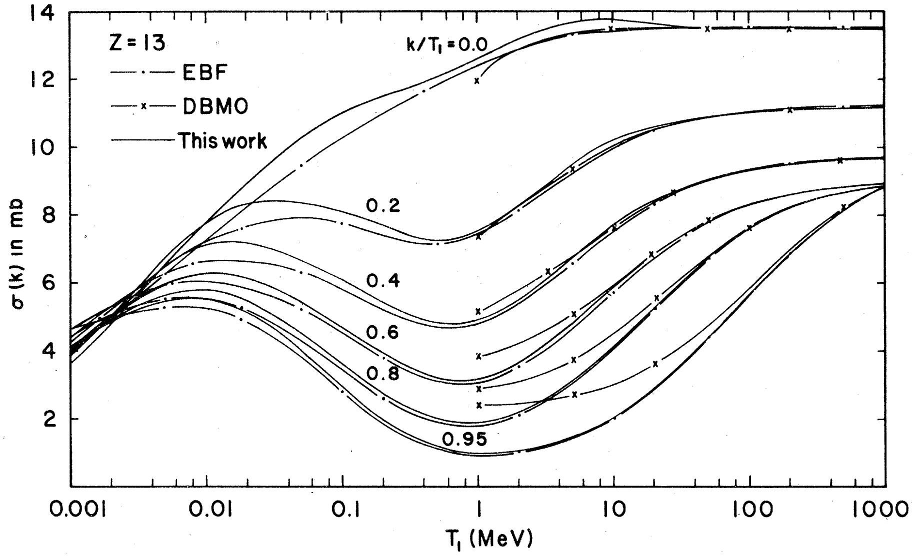

FIG. 2. Comparisons of bremsstrahlung cross section $\sigma(k)$ for $Z=13$, $T_1=1$ keV to 1000 MeV between results obtained from partial-wave method (solid lines), results obtained from Born approximation modified with form-factor screening and the Elwert factor (EBF, dotted broken lines), and results obtained from high-energy theory (DBMO, crossed broken lines).

l_1 as l_1 increases. Thus not all partial-wave cross sections σ_{l_1} have to be computed directly; a modified partial-wave method is possible in which one calculates a finite set of σ_{l_1}-values on a grid in l_1, of increasing spacing, and then interpolates the intermediate terms.

With this partial-wave interpolation method we have calculated the bremsstrahlung energy spectra for $Z=13$ and 92, $T_1=5$ and 10 MeV, $k/T_1=0.6$, 0.8 and 0.95. For $k/T_1=0$ results were again obtained from elastic scattering data using the low-energy theorem. Our exact screened (ES) results are shown in Table I, together with results obtained from simpler theories, such as the Born approximation [Bethe and Heitler (BH)],[6] the Born approximation modified with the form factor and with the Elwert factor (EBF),[7] and the high-energy approximation (DBMO).[3] The ES results for $k/T_1=0.2$ and 0.4 were obtained by smooth interpolation to the ratios $\sigma_{\mathrm{EBF}}/\sigma_{\mathrm{ES}}$ of the cases with $k/T_1=0.0$, 0.6, 0.8, and 0.95. To understand how these simpler theories are converging we also show similar comparisons for $T_1=1$ and 2 MeV. We can then estimate the ES results for $T_1>10$ MeV by smooth interpolation between the ES results for $T_1\leq 10$ MeV and the predictions of the high-energy theory at $T_1=1000$ MeV. In Figs. 2 and 3 we present estimates of the bremsstrahlung cross sections $\sigma(k)$ for $Z=13$ and

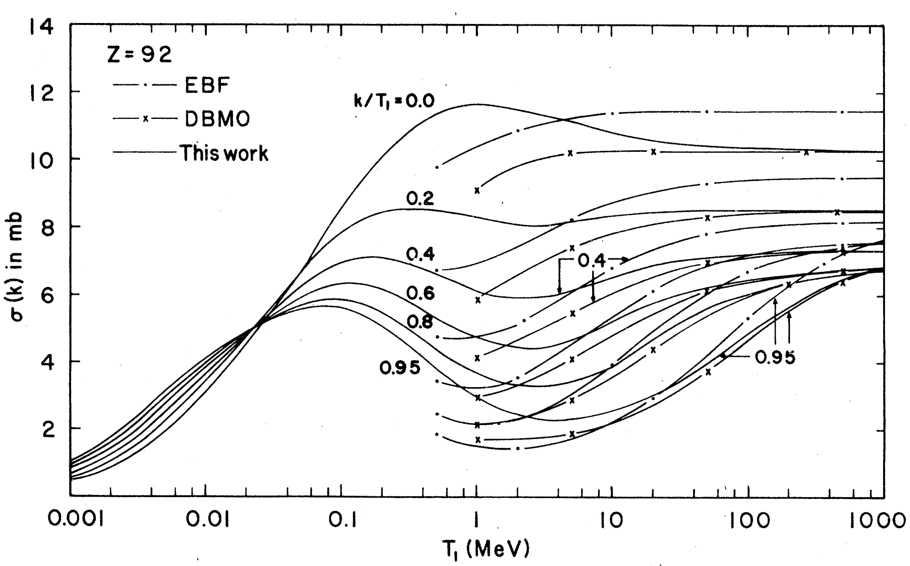

FIG. 3. Same as Fig. 2 except that $Z=92$.

92, $T_1 = 1$ keV to 1000 MeV, obtained in this way. It appears that for $Z = 13$ EBF is good within about 5% for $T_1 \gtrsim 2$ MeV and 12% below 2 MeV, while results of high-energy theory are good within about 5% for $T_1 \gtrsim 5$ MeV and $k/T_1 \lesssim 0.6$, for $T_1 \gtrsim 15$ MeV up to $k/T_1 = 0.8$, and for $T_1 \gtrsim 70$ MeV up to $k/T_1 = 0.95$. For $Z = 92$, EBF is good within 12% for $T_1 \gtrsim 5$ MeV and $k/T_1 \lesssim 0.8$, while for $T_1 \lesssim 2$ MeV the difference between EBF and ES data becomes large. For $Z = 92$, results of high-energy theory are good within about 5% for $T_1 \gtrsim 25$ MeV and $k/T_1 \lesssim 0.6$, for $T_1 \gtrsim 40$ MeV up to $k/T_1 = 0.8$, and for $T_1 \gtrsim 70$ MeV up to $k/T_1 = 0.95$. These estimates can be further confirmed if our interpolation in partial waves can be pushed to higher energies. We believe this may prove feasible once further economies are introduced in our numerical codes.

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