Radiative collapse of a relativistic electron-positron plasma to ultrahigh densities

F. Winterberg
Desert Research Institute, University of Nevada System, Reno, Nevada 89507
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It is shown that a relativistic electron-positron plasma formed by the coalescence of two counterstreaming intense relativistic electron and positron beams can collapse into a very-small-diameter filament. The collapse is accompanied by the emission of intense coherent γ radiation. The radius of the collapsed state is determined by the quantum-mechanical uncertainty principle. At its maximum contraction the electron-positron plasma can approach nuclear densities. The predicted effect may have many interesting applications, some of which are briefly mentioned. The generation of the intense beams seems to be possible with existing techniques.

The transient generation of matter possessing very high densities is of great interest in many areas of physics. One example are the densities in white dwarf and neutron stars, until now unattainable in the laboratory. It is clear that if nuclear densities could be attained this would have far-reaching consequences in applied nuclear physics. However, so far no way to reach such ultrahigh densities has ever been conceived but we propose here a method by which this could possibly be achieved. The basic idea behind our proposal is explained in Fig. 1. Two relativistic beams, one consisting of electrons and the other of positrons, each having the current $\frac{1}{2}I$ and each having equal energy per particle, that is equal γ value [$\gamma=(1-\beta^2)^{-1/2}$, where $\beta=v/c$, v is the drift velocity of electrons and positrons, c is the velocity of light] are set up in the toroidal chambers I and II of a double storage ring. Both beams are confined by an external toroidal magnetic field H_0, and are either produced by well-established storage-ring techniques, or by the method that has been proposed for the generation of intense relativistic electron beams.[1] In this latter case, the electron and positron beams would be produced by particle injection from the periphery of the toruses I and II, and compressed towards the axis of these toroidal sections by the rising external toroidal magnetic field H_0 in conjunction with the time-varying magnetic flux produced by the transformer T with the core C going through both principal torus axis of Secs. I and II. By this time-varying flux the electrons and positrons are accelerated in equal and opposite directions, that is, with their respective currents in the same direction. Since the electron and positron velocities in their respective toroidal Secs. I and II are equal and opposite, both beams attract each other by electric and magnetic forces. Therefore, if in the vicinity of the torus Secs. I and II the externally confining magnetic field is increased, but not in the intermediate toroidal Sec. III, the beams will be pushed towards the intermediate Sec. III, and where they coalesce into a relativistic space-charge-neutralized electron-positron plasma with a total toroidal current I. In order to prevent the premature coalescence of the electron and positron rings during their buildup, the magnetic field in between Secs. I and II and Sec. III must be stronger than in the sections themselves. This can be done by a coil having the same geometry as the sections. If this coil is then surrounded by a second coil that has an elliptic cross section and that can increase the magnetic field in Secs. I and II above that in Sec. III, both rings will be pushed towards this section resulting in their coalescence. We will show that above a certain critical range of the current I and particle energy, this electron-positron plasma will rapidly shrink down to a very small radius, determined by Heisenberg's uncertainty principle.

The time dependence of the plasma is ruled by two processes, one enhancing its expansion and the other its shrinkage. The process enhancing its expansion is the internal heating by Coulomb scattering taking place between the electrons and positrons colliding head on. The other process, enhancing its shrinkage, is cooling by emission of radiation from transverse oscillations of the particles confined in the magnetic field of the plasma current. If the radiation losses exceed the transverse energy gain by Coulomb collisions the plasma will shrink.

The energy gain by Coulomb collisions is most easily calculated in a local reference frame in which either the moving electrons or positrons are at rest. If the particle number density, either for the electrons or positrons, in a laboratory system is n, then in a co-moving system the number density of that particle species colliding head on is equal to $n'=\gamma n$. Furthermore, if the time element in the laboratory system is dt, it is in a co-

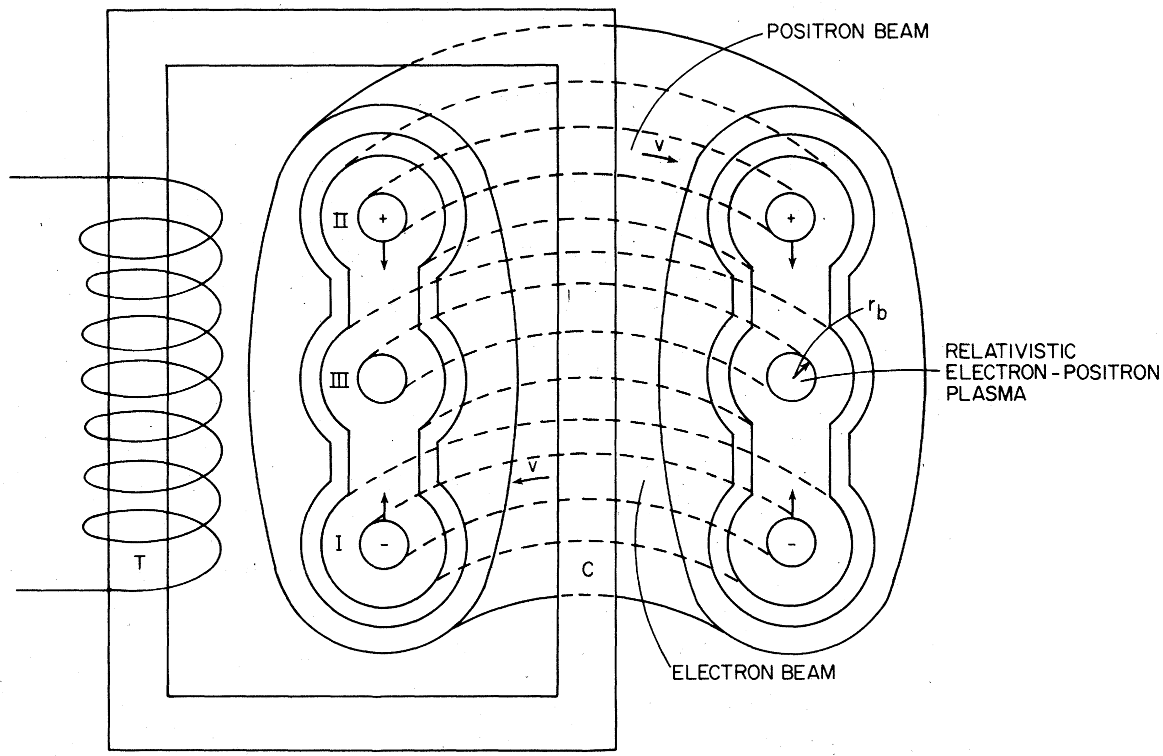

FIG. 1. Cut through double storage ring for relativistic electron and positron beams and for the formation of the relativistic electron-positron plasma.

moving system equal to $dt' = dt/\gamma$. The transverse energy gain of one electron or positron in a co-moving system, assuming the relative drift velocity is $v \simeq c$, is then given by[2]

$$\frac{dE}{dt'} = 4\pi n' \frac{e^4}{mc} \ln\Lambda' , \tag{1}$$

and hence in a laboratory frame

$$\frac{dE}{dt} = 4\pi n \frac{e^4}{mc} \ln\Lambda . \tag{2}$$

Here $\ln\Lambda'$ is the Coulomb logarithm with $\Lambda' = b_{max}/b_{min}$. One has to put $b_{max} = r_b$, where r_b is the plasma radius and furthermore[2] $b_{min} = e^2/\gamma mc^2 = r_0/\gamma$ ($r_0 = e^2/mc^2$ is the classical electron radius). In going to a laboratory system the value of b_{min} has to be multiplied by γ and one has $\Lambda = r_b/r_0$. The total current in the plasma (using electrostatic cgs units) is $I \simeq 2nec\pi r_b^2$. One then finds that

$$\frac{dE}{dt} = \frac{2ce^2}{r_b^2} \frac{I}{I_A} \ln\Lambda , \quad I_A = \frac{mc^3}{e} = 1.7 \times 10^4\,\text{A}. \tag{3}$$

The azimuthal magnetic field inside the plasma column $r < r_b$ is given by

$$H_\phi = (2I/r_b c)(r/r_b) , \tag{4}$$

and the radial restoring force acting on a beam particle therefore given by

$$F = -e\beta H_\phi = -\frac{2e\beta I}{cr_b^2} r \simeq -\frac{2eI}{cr_b^2} r . \tag{5}$$

This restoring force in conjunction with the excitation by the Coulomb collisions leads to the transverse radial particle oscillations determined by the equation of motion

$$\gamma m \ddot{r} = F , \tag{6}$$

or

$$\ddot{r} + \omega^2 r = 0 , \tag{7}$$

with $\omega^2 = 2eI/\gamma mcr_b^2 = (2/\gamma)(c/r_b)^2(I/I_A)$. Note that the relativistic transverse mass γm enters into Eq. (6). These transverse oscillations result in intense emission of radiation. The energy loss for one particle due to this radiation is given by[2]

$$P_e = \tfrac{2}{3}(e^2\overline{\dot{v}_\perp^2}/c^3)\gamma^4 , \tag{8}$$

where $v_\perp$ is the perpendicular velocity component, in our case $\dot{v}_\perp = \ddot{r}$. One thus finds that $\overline{\dot{v}_\perp^2} = \overline{\ddot{r}^2} = \frac{1}{3}\omega^4 r_b^2$ and obtains

$$P_e = \frac{8}{9}\frac{e^2c}{r_b^2}\left(\frac{I}{I_A}\right)^2\gamma^2 . \quad (9)$$

Since $\omega^2 = (2/\gamma)(c/r_b)^2(I/I_A) \equiv 4\pi ne^2/\gamma m$, it follows that the plasma remains optically transparent for all frequencies of the emitted radiation, regardless of its radius r_b or particle number density n, and for this reason the emitted radiation cannot be reabsorbed by it. Therefore, if $P_e > dE/dt$, the plasma will ultimately shrink down to a radius r_{min} determined by Heisenberg's uncertainty principle:

$$\gamma mc r_{min} \simeq \hbar . \quad (10)$$

The condition $P_e > dE/dt$ implies that

$$\gamma^2 I/I_A > \tfrac{9}{4}\ln\Lambda . \quad (11)$$

The condition against the plasma to pinch itself off by action of its own magnetic field is given by

$$H_\phi^2/4\pi < 2\gamma nmc^2 , \quad (12)$$

which results in

$$I < \gamma I_A , \quad (13)$$

and which can be combined with the inequality (11) to give

$$\gamma > I/I_A > \tfrac{9}{4}\ln\Lambda/\gamma^2 . \quad (14)$$

The maximum current is thus given by

$$I_{max} = \gamma I_A = 1.7\times 10^4\gamma \text{ A} . \quad (15)$$

To satisfy inequality (14) requires that

$$\gamma^3 > \tfrac{9}{4}\ln\Lambda , \quad (16)$$

setting a minimum γ value which is $\gamma_{min} = (\frac{9}{4}\ln\Lambda)^{1/3}$. The minimum current I_{min} required to lead to collapse is thus given by

$$I_{min} = \frac{9\ln\Lambda}{4\gamma^2} I_A = \frac{\gamma_{min}^3}{\gamma^2} I_A = \left(\frac{\gamma_{min}}{\gamma}\right)^3 I_{max} . \quad (17)$$

Assuming that $\frac{9}{4}\ln\Lambda \simeq 10^2$, which is valid for $r_b \simeq 1$ cm one has $\gamma_{min} \simeq 4.5$ and $I_{min} \simeq 1.7\times 10^6\gamma^{-2}$ A. If, for example, $\gamma = 10^2$, corresponding to beams with a particle energy of ~50 MeV, one has $I_{min} = 170$ A. For $\gamma \simeq 3\times 10^3$, which is typical for electron intersecting storage rings, one has $I_{min} = 0.17$ A. Electron beams of this magnitude can be easily produced and also seem to be principally attainable for positrons with the state of the art in storage-ring technology.

After its collapse to the radius r_{min} given by Eq. (10), the number of particles per unit beam length, counting both electrons and positrons, is equal to $(1/r_0)(I/I_A)$. The particle number density in the plasma is therefore calculated to be

$$n_{max} = (\gamma^2/\pi r_0 \lambda_e^2)(I/I_A) , \quad (18)$$

with $\lambda_e = \hbar/mc = 3.8\times 10^{-11}$ cm and hence $n_{max} \simeq 8\times 10^{30}(I/I_A)\gamma^2$. The maximum density in the collapsed plasma is given by $\rho_{max} = \gamma mn_{max} \simeq 7.3\times 10^3(I/I_A)\gamma^3$. Assume for example that $I = I_A = 17$ kA, which seems technically feasible, and $\gamma \simeq 10^2$, it follows that $\rho_{max} \simeq 10^{10}$ g/cm^3. With $I = 170$ kA, $\gamma = 2\times 10^3$, which seems at the limit of technical feasibility, $\rho_{max} \sim 10^{15}$ g/cm^3. This is in the range of nuclear densities.

The collapse time is given by $\tau_c = E_\perp/P_e$, with the perpendicular kinetic particle energy $E_\perp \simeq \frac{1}{2}\gamma mv_\perp^2 \simeq \frac{1}{2}\gamma m\omega^2 r_b^2 = mc^2(I/I_A)$, hence

$$\tau_c = \tfrac{9}{8}(r_b^2/r_0 c\gamma^2)(I_A/I) . \quad (19)$$

Assume that initially $r_b \simeq 1$ cm and that $I = I_A$, $\gamma = 10^2$, it follows $\tau_c \simeq 10^{-2}$ sec. This time is large compared with the time needed to push both beams together by increasing the magnetic field in Secs. I and II of the double storage ring. The two beams can therefore coalesce without difficulty into the electron-positron plasma.

In the last stage of the collapse the maximum photon energy, estimated from the lowest harmonic of the emitted radiation, is

$$\hbar\omega_{max} = E_\perp = mc^2(I/I_A) . \quad (20)$$

For example if $I = I_A$ one has $\hbar\omega_{max} = mc^2$. Because of the small plasma diameter this presents a highly coherent γ radiation. Furthermore, since the collapse time τ_c depends on the plasma radius r_b according to $\tau_c \propto r_b^2$, most of the dissipated beam energy is released in the last moment of the collapse, resulting in a burst of very intense γ radiation. The maximum power of this final burst can be computed by putting $r_b = r_{min}$ into Eq. (9) with the result

$$P_e^{max} = \tfrac{8}{9}(c/r_B)(I/I_A)^2\gamma^4 mc^2$$
$$\simeq 2.1\times 10^8 (I/I_A)^2\gamma^4 \text{ erg/sec} , \quad (21)$$

where $r_B = \hbar^2/me^2$ is the Bohr radius. If for example $I = I_A$, $\gamma = 10^2$, one finds $P_e^{max} \simeq 2.1\times 10^{16}$ erg/sec. The energy of this pulse is delivered in the time τ_c^{min} obtained from putting $r_b = r_{min}$ into (19) and one finds

$$\tau_c^{min} = \tfrac{9}{8}(r_B/c\gamma^4)(I_A/I) . \quad (22)$$

During the time the plasma collapses down to the radius r_{min}, it decays by electron-positron annihilation with a cross section equal to[3] $\sigma \simeq (\pi r_0^2/2\gamma^2)\ln(2\gamma^2)$. The decay time for annihilation is therefore given by

$$\tau_a \simeq (\sigma nc)^{-1} = \frac{4\gamma^2 r_b^2}{r_0 c\ln(2\gamma^2)}\frac{I_A}{I} . \quad (23)$$

The smallest time τ_a^{min} is obtained by putting $r_b = r_{min}$ with the result

$$\tau_d^{\min} = \frac{4r_B}{c\ln(2\gamma^2)}\frac{I_A}{I}. \tag{24}$$

From Eqs. (19) and (23) we find that

$$\tau_d/\tau_c = \tfrac{32}{9}\gamma^4/\ln(2\gamma^2). \tag{25}$$

Since the collapse must occur prior to the radiative annihilation of the plasma, it is required that $\tau_d \gg \tau_c$. This condition implies that $\frac{32}{9}\gamma^4 \gg \ln(2\gamma^2)$, and which is always satisfied. Furthermore, in order for the collapsed plasma to form a quasistatic quantum mechanical state, τ_d has to be large in comparison with the "orbital" time scale $\tau_0 \simeq r_{\min}/c \simeq \hbar/\gamma mc^2$. For $r_b = r_{\min}$ one finds that

$$\frac{\tau_d}{\tau_0} = \frac{4\gamma}{\ln(2\gamma^2)}\frac{\hbar c}{e^2}\frac{I_A}{I}. \tag{26}$$

If $\gamma = 10^2, I = I_A$ one obtains $\tau_d/\tau_0 \sim 10^4$, which satisfies the condition for a quasistatic state.

We had assumed that the azimuthal magnetic field H_ϕ remains trapped inside the plasma during its collapse. This implies no decay of the current I. It is easy to show that this assumption is rather good. The decay of the current and hence magnetic field is determined by Eq. (3). From this follows the decay time for the azimuthal magnetic field

$$\tau_H^\phi = \frac{\gamma mc^2}{dE/dt} = \frac{\gamma mcr_b^2}{2e^2\ln\Lambda}\frac{I_A}{I}, \tag{27}$$

or in conjunction with Eq. (19)

$$\tau_H^\phi/\tau_c = (\gamma/\gamma_{\min})^3. \tag{28}$$

and it follows that for $\gamma \gg \gamma_{\min} \simeq 4.5$ the current decay can be neglected.

During its collapse the magnetic energy changes by the change of the self-inductance L of the plasma channel. The self-inductance of the plasma channel with return current conductor radius $R > r_b$ is computed to be

$$L = 2\times 10^{-5} l\ln(R/r_b)\ \mathrm{H}, \tag{29}$$

where l is the length of the plasma column measured in cm. Therefore, if r_b changes from $r_b \simeq 1$ cm down to $r_b \simeq r_0 \simeq 10^{-13}$ cm the self-inductance and with it the magnetic energy increases by the factor $\ln(R/r_0)/\ln(R/r_b)$. If for example $r_b \simeq 1$ cm, $R \simeq 10$ cm this factor is equal to 14. This increase in magnetic energy must go on the expense of the particle kinetic energy. Therefore, the initial value of γ must be actually ~10 times larger than the final value required to satisfy the collapse condition.

Near the surface of the collapsed relativistic electron positron plasma the azimuthal magnetic field can reach enormous values and which are of the order $H_\phi \simeq 0.2I/r_{\min}$. If, for example, $I = I_A$, $\gamma \sim 10^2$ one obtains $H_\phi \sim 2\times 10^{16}$ G.

The magnetic confinement condition for the relativistic beams in the external magnetic field, prior to their coalescence into the plasma, is given by

$$H_0^2/8\pi > E_B^2/8\pi - H_B^2/8\pi, \tag{30}$$

where E_B and H_B are the electric and magnetic self-fields of the beams at the beam surfaces. If the beam radius is $r = r_b$ one finds $E_B = I/r_b c\beta$, $H_B = I/r_b c$, and hence

$$H_0^2 > \frac{I^2}{r_b^2c^2}\left(\frac{1}{\beta^2} - 1\right) \simeq \frac{I^2}{\gamma^2 r_b^2 c^2}. \tag{31}$$

Expressing I in amperes this condition is $H_0 > 0.1I/\gamma r_b$. For $I = I_A = 17$ kA, $r_b = 1$ cm, $\gamma = 10^2$ one finds that $H_0 > 17$ G. The generation of the field for beam confinement thus poses no problem.

The presence of an axial magnetic field introduces another problem that upon first inspection seems to pose an insurmountable difficulty for the proposed collapse towards nuclear dimensions actually to take place. It is to be recalled that the decay time of the azimuthal magnetic field was found to be long compared to the collapse time if $\gamma \gg 4.5$. Therefore, if the same should be the case also for an axial magnetic field trapped within the plasma, it would follow that below a certain plasma radius the repulsive magnetic force from this field will become larger than the attractive magnetic pinch force from the azimuthal field. If at an initial plasma radius $r_b = r_b^0$ the axial field is $H_z(0) = H_0$ it would at a smaller radius, $r_b < r_b^0$, be equal to $H_z = H_0(r_b^0/r_b)^2$. It thus follows that the repulsive magnetic force, which is proportional to H_z^2, increases as r_b^{-4}, in contrast to the pinch force, which is proportional to H_ϕ^2, and which only increases as r_b^{-2}. Therefore, after having reached a plasma radius where $H_z^2 \simeq H_\phi^2$, the collapse should be stopped. Taking the above given example $r_b^0 = 1$ cm, $I = 17$ kA, and $H_0 = 17$ G one finds that this would happen already at $r_b = 5\times 10^{-3}$ cm which would be not a very interesting value. To overcome this difficulty one might consider a plasma configuration satisfying the confinement condition (30), where H_0 is very small inside but not outside the plasma. However, for a collapse down to a radius of $\sim 10^{-13}$ cm this would require that the residual axial magnetic field trapped within the plasma to be less than $\sim 10^{-10}$ G, a completely unrealistic demand. Fortunately though, on closer inspection it turns out that the lifetime of the axial magnetic field is much shorter than for the azimuthal field. The axial magnetic field is connected with azimuthal electron and positron trajectories, in contrast to an azimuthal field which is connected with axial trajectories. But because the scale length for the azimuthal motion is smaller by many orders of magnitude, the axial field can diffuse out the electron-positron plasma very rapidly. Furthermore, since the particles are relativistic the diffusion by radiation under certain conditions can predominate collisional diffusion.

During the collapse of the plasma cylinder a trapped axial magnetic field induces an azimuthal current whereby the electrons and positrons would assume an additional azimuthal velocity component v_ϕ of equal but opposite magnitude. Let us assume that the plasma collapse proceeds until

$$H_z \simeq H_\phi = 2I/cr_b , \tag{32}$$

where it would be stopped if H_z does not diffuse out of the plasma. At this radius the electrons and positrons would reach their maximum azimuthal velocity. Expressing the current I in Eq. (32) through the previously found relation $\omega^2 = 2eI/\gamma mcr_b^2$, one finds

$$H_z = (m/e)\gamma\omega^2 r_b . \tag{33}$$

The azimuthal velocity for both electrons and positrons is then simply given by the relativistic Larmor formula

$$v_\phi = (eH_z/\gamma mc)r_b = (\omega r_b)^2/c . \tag{34}$$

We first compute the diffusion loss of H_z due to collisions. The characteristic loss time for this process is given by

$$\tau_H^z = \frac{\frac{1}{2}\gamma m v_\phi^2}{dE/dt} . \tag{35}$$

For this we can also write

$$\tau_H^z = \frac{1}{2}\left(\frac{v_\phi}{c}\right)^2 \tau_H^\phi = \frac{1}{2}\left(\frac{\omega r_b}{c}\right)^4 \tau_H^\phi = \frac{2}{\gamma^2}\left(\frac{I}{I_A}\right)^2 \tau_H^\phi , \tag{36}$$

where we have used Eq. (34) and the previously found value $\omega^2 = (2/\gamma)(c/r_b)^2(I/I_A)$.

Next we compute the diffusion by radiation. The rapidly circulating electrons and positrons emit synchrotron radiation at the rate

$$P_H = \tfrac{2}{3}(e^2 v_\phi^4/c^3 r_b^2)\gamma^4 . \tag{37}$$

This formula can be obtained from Eq. (8) by making the substitution $\overline{v_\perp^2} = v_\phi^4/r_b^2$.

Since both the electrons and positrons have equal but opposite azimuthal velocity and at the same time are oppositely charged, the emitted synchrotron radiation is circularly polarized. The diffusion time by radiation losses is given by

$$\tau_H^z(R) = \tfrac{1}{2}\gamma m v_\phi^2/P_H = \tfrac{3}{4} mc^3 r_b^2/e^2 v_\phi^2 \gamma^3 . \tag{38}$$

For this we can also write

$$\tau_H^z(R) = \frac{3}{2}\,\frac{c^2 \ln\Lambda}{v_\phi^4 \gamma^4}\,\frac{I}{I_A}\,\tau_H^\phi . \tag{39}$$

Again using Eq. (34) and the expression for ω^2, and furthermore the definition of $\gamma_{\min}$, we can write for Eq. (39)

$$\tau_H^z(R) = \frac{1}{6}\,\frac{\gamma_{\min}^3}{\gamma^2}\,\frac{I_A}{I}\,\tau_H^\phi . \tag{40}$$

From Eqs. (36) and (40) we can find out when the diffusion loss by radiation predominates the loss by collision. This obviously happens if $\tau_H^z(R) < \tau_H^z$, which implies that

$$I > (\gamma_{\min}/12^{1/3})I_A \simeq 2I_A . \tag{41}$$

From this result we conclude that in our first example, where $I = I_A$, collisional diffusion still predominates but in the second example, with $I = 10I_A$, diffusion by radiation is far more important.

The inclusion of an axial magnetic field now leads to the following modified sequence of events. After the formation of the electron-positron plasma the collapse first proceeds by the time constant τ_c until the plasma radius becomes ~10^{-3} cm, where $H_z \simeq H_\phi$. From there on the collapse proceeds to continue down to the radius determined by the uncertainty relation, but only if the two following conditions are met: (i) The diffusion time for the axial magnetic field must be short in comparison to the diffusion time by the azimuthal field; (ii) the time for the diffusion dominated collapse must be short in comparison to the time for electron-positron annihilation. In the diffusion-dominated region below ~10^{-3} cm the time scale for the collapse is the diffusion time τ_H^z or $\tau_H^z(R)$, whichever is shortest. In general, the diffusion time can become larger than the time τ_c. The diffusion-dominated collapse therefore will be, in general, slower.

The first condition implies that

$$\tau_H^z \ll \tau_H^\phi , \tag{42}$$

or

$$I \ll (\gamma/\sqrt{2})I_A , \tag{43}$$

for collision dominated diffusion, and

$$\tau_H^z(R) \ll \tau_H^\phi , \tag{44}$$

or

$$I \gg \tfrac{3}{8} I_A/\gamma^2 , \tag{45}$$

for radiation-dominated diffusion. Let us check if these inequalities are satisfied for our two numerical examples given above. In the first case it was assumed that $I = I_A$, $\gamma = 10^2$. It follows that $I = I_A \ll 70I_A$ and inequality (43) is well satisfied. In the second case it was assumed that $I = 10I_A$ and $\gamma = 2\times 10^3$. One verifies that inequality (45) is here even better satisfied.

The second condition implies that

$$\tau_d \gg \tau_H^z\,, \tag{46}$$

or

$$\frac{16}{9}\,\frac{\gamma_{\min}^3\gamma^3}{\ln(2\gamma^2)}\left(\frac{I_A}{I}\right)^2 \gg 1\,, \tag{47}$$

and

$$\tau_d \gg \tau_H^z(R)\,, \tag{48}$$

or

$$\frac{64}{3}\,\frac{\gamma}{\ln(2\gamma^2)}\,\frac{I}{I_A} \gg 1\,. \tag{49}$$

Once again one easily verifies that for both examples given above the inequalities (47) and (49) are well satisfied.

We thus have arrived at the important conclusion that even the presence of a trapped axial magnetic field is unable to stop the collapse of the electron-positron plasma down to nuclear densities. Since below a radius of ~10^{-3} cm the collapse is determined by the diffusion velocity of the axial magnetic field, the collapse time is increased. This increase is given by the factor

$$\frac{\tau_H^z}{\tau_c} = 2\,\frac{\gamma}{\gamma_{\min}^3}\left(\frac{I}{I_A}\right)^2, \tag{50}$$

respectively

$$\tau_H^z(R)/\tau_c = \tfrac{1}{6}\gamma I_A/I\,. \tag{51}$$

For our two examples we find $\tau_H^z/\tau_c = 2.2$ and $\tau_H^z(R)/\tau_c = 33$. It therefore follows that the diffusion process can substantially slow down the collapse. However, since without a trapped axial magnetic field the collapse time decreases with r_b^{-2}, it would be of the order ~10^{-6} sec at a radius of ~10^{-3} cm. This time would thus be increased by about one order of magnitude. In the last stages of the collapse where r_b becomes very small the collapse time would in any case become extremely small, or alternatively, the collapse velocity would become always very large.

The presence of some axial magnetic field H_z, not exceeding the azimuthal field H_ϕ, may actually be an important advantage since it should help to suppress the $m=0$ pinch instability. The $m=0$ instability is suppressed if $H_z > (1/\sqrt{2})H_\phi$. It is clear that this condition will here be automatically met, in case the $m=0$ instability proceeds in a time shorter than τ_H^z or $\tau_H^z(R)$. In this case the ratio H_z/H_ϕ would locally increase as r_b^{-2} near the neck of the $m=0$ instability and therefore rapidly exceed the critical value $1/\sqrt{2}$ above which stability is restored. As the experience with relativistic electron beam suggests, magnetohydrodynamic instabilities are much less violent in a relativistic plasma, and there is therefore good reason to expect that the next most serious $m=1$ kink instability, as well as all other higher instabilities are of no great concern.

Besides magnetohydrodynamic instabilities one has also to deal with microinstabilities, the most important of which is the electron-positron two-stream instability. However, the high beam temperature combined with the large radiation damping should work strongly against the growth of this fastest rising instability.

We will present here some semiquantitative arguments which seem to be in support of the view why for a highly relativistic electron-positron plasma the two-stream instability is likely to be insignificant. The growth rate of this instability is given by

$$\sigma \simeq \omega = (4\pi n e^2/\gamma m)^{1/2}\,. \tag{52}$$

In Eq. (52) the transverse mass γm enters into the expression for the plasma frequency because the fastest-growing instability propagates in an oblique direction relative to the two counterstreaming beams.

The effect of the two-stream instability results in collective bunching of the beam particles, forming clusters with the dimension of the Debye-length $\lambda_D \simeq c/\omega$. This particle clustering leads to enhanced radiation losses, since it requires to replace the single particle charge e by the collective charge $q = Ne \simeq (c/\omega)^3 ne$ in the loss equation (9). Furthermore, since these losses apply to N particles one computes as the average collective radiation loss per one particle P_e^c:

$$NP_e^c = N^2 P_e\,, \tag{53}$$

or simply

$$P_e^c = NP_e = (c/\omega)^3 n P_e\,. \tag{54}$$

Using this expression one can define the collective damping time τ_D to be

$$\tau_D = E/P_e^c = \tau_c/N\,. \tag{55}$$

From this one obtains that

$$\sigma\tau_D \simeq \omega\tau_D = 12\pi/\gamma^4\,, \tag{56}$$

and it follows that for $\gamma \gg (12\pi)^{1/4} = 2.5$ the radiative damping would ensure no growth of the two-stream instability.

We would like to mention a few possible applications for the proposed device.

(i) The high magnetic field of ~10^{16} G makes it conceivable that in case the collapsed electron-positron plasma makes an impact on some thermonuclear material, this would lead to nuclear reactions by magnetic shielding of the Coulomb barrier.

The proposed scheme may thus present another way towards the release of thermonuclear energy and with almost any thermonuclear material.

(ii) The intense bursts of γ radiation conceivably can also be used to fission heavy elements, such as lead, for the generation of energy, by letting the collapsed plasma hit a target consisting of fissionable material.

(iii) If the circular torus geometry of the collapsed electron-positron plasma is deformed into the shape of a racetrack, the radition emitted from the straight segments will have the character of collimated highly coherent γ-ray beams with a very small diameter. Such beams could be used to make large scale photonuclear reactions at a rate otherwise not possible. The reason for this is that a coherent highly focused γ-ray beam will act like a classical large-amplitude γ wave, and which is much more potent to induce photonuclear reactions that single γ quants.

(iv) In its collapsed state, the electron-positron plasma has the characteristics of an upper laser level where the final state is obtained by complete electron-positron annihilation.

(v) It is conceivable that the intense emitted γ radiation can be used to accelerate protons by radiation pressure to very high energies unattainable with conventional methods of particle acceleration.

(vi) The very high plasma density in the collapsed state combined with its comparatively long lifetime may also open new avenues in high-energy physics, in the study of the electron-positron collisions.

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