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Potential scattering of charged particles in the field of a low-frequency laser

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The problem of scattering of a charged particle (an electron) by a potential in the presence of a single-mode classical electromagnetic field (a laser) in the dipole approximation is considered. No expansion in the strength of the field is made, but an expansion in the frequency of the field is carried out. This problem has been considered previously by Kroll and Watson, who showed that, if only the first two orders in ω are retained, then the scattering is describable by an on-shell T matrix obtained in the absence of the field. They show this to be an essentially classical result. A different method is used to obtain their result and the next order in ω which yields the first off-shell correction is obtained. Far off-shell contributions are found in this order.

A recent experiment[1] on the scattering of 11-eV electrons by argon atoms in the field of an intense CO_2 laser has yielded the first observation of multiphoton (± three photons) free-free transitions. The laser field intensity is too high to allow a perturbation theory in the electron-laser interaction, but the CO_2 laser photon energy (~1.2 $\times 10^{-3}$ eV) is low enough to allow an expansion in this parameter. The complete problem does not seem to have been considered in the published literature, but the approximation in which the atom is replaced by a structureless potential has been discussed by Kroll and Watson.[2] They expanded in powers of the laser frequency and retained only the first two terms. In that approximation they obtained the result that the T matrix for transfer of l photons could be related to an on-shell T matrix in the absence of the laser for slightly different initial and final momenta. They also show that this is an essentially classical result. It is difficult to see how to generalize their method to higher powers of ω so we shall use a different technique to reproduce their results and also obtain the next-order correction, which yields the first off-shell corrections to the T matrix.

The method used is a Born series in the scattering potential. To that end we start with the Schrödinger equation ($\hbar=1$),

$$\left(\frac{i\partial}{\partial t} - T - V - \frac{e}{m}\vec{p}\cdot\vec{A}(t)\right)\Psi = 0\,, \tag{1}$$

where for a linearly polarized laser

$$\vec{A}(t) = (\vec{E}/\omega)\cos\omega t\,. \tag{2}$$

In the absence of V, solutions to Eq. (1) are

$$\chi_q = \exp\{i[\vec{q}\cdot\vec{r} - \vec{q}\cdot\vec{\alpha}(t) - \epsilon_q t]\}\,, \tag{3}$$

where

$$\epsilon_q = q^2/2m\,,$$
$$\vec{\alpha}(t) = \vec{\alpha}_0 \sin\omega t = (e\vec{E}/m\omega^2)\sin\omega t\,, \tag{4}$$

with $\vec{\alpha}_0$ being the amplitude of a classical particle moving in the vector potential, (2). The transition amplitude for scattering from state χ_q to $\chi_{q'}$, is given by

$$\tau_{q'q} = \langle\chi_{q'}, V\Psi_q^{(+)}\rangle\,, \tag{5}$$

where the brackets indicate both space and time integration. The solution of (1) can be expanded in powers of V such that

$$\tau_{q'q} = \sum_{n=0}^{\infty}\tau_{q'q}^{(n+1)} = \sum_{n=0}^{\infty}\langle\chi_{q'}, V(GV)^n\chi_q\rangle\,, \tag{6}$$

where G is the Green's function in the absence of V:

$$G(\vec{r}t, \vec{r}'t') = -i\Theta(t-t')\int\frac{d^3k}{(2\pi)^3}\chi_k(\vec{r},t)\chi_k^*(\vec{r}',t')\,. \tag{7}$$

This is now substituted into $\tau^{(n)}$. The spatial integrals now may be performed in terms of

$$\tilde{V}(\vec{k}) = \int d^3r e^{i\vec{K}\cdot\vec{r}}V(r)\,, \tag{8}$$

with the result

$$\tau_{q'q}^{(n+1)} = (-i)^n\int\frac{d^3k\cdots d^3k_n}{(2\pi)^{3n}}\tilde{V}(\vec{K}_1)\cdots\tilde{V}(\vec{K}_{n+1}) \times\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{t_1}dt_2\cdots\int_{-\infty}^{t_n}dt_{n+1}\exp\{i[(\epsilon_{q'}-\epsilon_{k_1})t_1 + (\epsilon_{k_1}-\epsilon_{k_2})t_2 + \cdots + (\epsilon_{k_n}-\epsilon_q)t_{n+1}]\} \times\exp\{i[\vec{K}_1\cdot\vec{\alpha}(t_1) + \cdots + \vec{K}_{n+1}\cdot\vec{\alpha}(t_{n+1})]\}\,, \tag{9}$$

where

$$\vec{K}_1=\vec{q}'-\vec{k}_1,\ \vec{K}_2=\vec{k}_1-\vec{k}_2,\ldots,\vec{K}_{n+1}=\vec{k}_n-\vec{q}\,. \qquad (10)$$

The time integrals may be performed by an $(n+1)$-fold use of the identity

$$e^{-i\vec{K}_j\cdot\vec{\alpha}(t_j)}=\sum_{l_j=-\infty}^{\infty} J_{l_j}(\vec{K}_j\cdot\vec{\alpha}_0)e^{-il_j\omega t_j}\,, \qquad (11)$$

with the result

$$T_{q'q}^{(n+1)}=2\pi\sum_{l_1\cdots l_{n+1}}\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1)\cdots\tilde{V}(\vec{K}_{n+1})}{(\Delta_1+\omega L_1)\cdots(\Delta_n+\omega L_n)}\times J_{l_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{l_{n+1}}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)\times\delta(\epsilon_{q'}-\epsilon_q-\omega L_0)\,, \qquad (12)$$

where

$$\Delta_j=\epsilon_q^+-\epsilon_{k_j},\ L_s=\sum_{j=s+1}^{n+1}l_j\,. \qquad (13)$$

We may set $L_0=l$ and rewrite this as

$$T_{q'q}^{(n+1)}=-2\pi i\sum_{l=-\infty}^{\infty}\delta(\epsilon_{q'}-\epsilon_q-\omega l)\,T_{q'q}^{(n+1)}(l)\,, \qquad (14)$$

where

$$T_{q'q}(l)=\sum_{n=0}^{\infty}T_{q'q}^{(n+1)}(l) \qquad (15)$$

is identified as the transition matrix for scattering with transfer of l photons from which the cross section is

$$\frac{d\sigma}{d\Omega}(\vec{q}',\vec{q};l)=\frac{q'(l)}{q}\left|\frac{m}{2\pi^2}T_{q'q}(l)\right|^2, \qquad (16)$$

where $q'(l)$ is obtained from the δ function in (14). The expression for $T_{q'q}^{(n)}(l)$ implied by (12) and (14) may be expanded in ω (holding α_0 fixed) with the result

$$T_{q'q}^{(n+1)}(l)=i\sum_{L_1\cdots L_n}\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1)\cdots\tilde{V}(\vec{K}_{n+1})}{\Delta_1\cdots\Delta_n}J_{l-L_1}(\vec{K}_1\cdot\vec{\alpha}_0)J_{L_1-L_2}(\vec{K}_2\cdot\vec{\alpha})\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)\times\left(1-\omega\sum_{s=1}^{n}\frac{L_s}{\Delta_s}+\omega^2\sum_{s=1}^{n}\frac{L_s^2}{\Delta_s^2}+\omega^2\sum_{s>t=1}^{n}\frac{L_sL_t}{\Delta_s\Delta_t}+\cdots\right), \qquad (17)$$

where we have shifted to the capital L's of Eq. (13) as summation variables.

There are three Bessel-function sum rules which we shall need to perform the L sums.

$$\sum_{n=-\infty}^{\infty}J_n(y)J_{N-n}(y')=J_N(y+y')\,, \qquad (18a)$$

$$\sum_{n=-\infty}^{\infty}nJ_n(y)J_{N-n}(y')=\frac{Ny}{y+y'}J_N(y+y')\,, \qquad (18b)$$

$$\sum_{n=-\infty}^{\infty}n^2J_n(y)J_{N-n}(y')=\frac{y}{y+y'}\left(y'J_N'(y+y')+\frac{N^2y}{y+y'}J(y+y')\right). \qquad (18c)$$

The first of these can be used n times to give

$$\sum_{L_1\cdots L_n}J_{l-L_1}(\vec{K}_1\cdot\vec{\alpha}_0)J_{L_1-L_2}(\vec{K}_2\cdot\vec{\alpha}_0)\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)=J_l(x)\,, \qquad (19)$$

where $x=(\vec{q}'-\vec{q})\cdot\vec{\alpha}_0$ and where we have used

$$\sum_{j=1}^{n+1}\vec{K}_j=\vec{q}'-\vec{q}\,. \qquad (20)$$

Equations (18a) and (18b) can be used to give

$$\sum_{s=1}^{n}\sum_{L_1\cdots L_n}\frac{L_s}{\Delta_s}J_{l-L_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)=\sum_{s=1}^{n}\frac{lJ_l(x)}{x}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s} \qquad (21)$$

and finally Eqs. (18a) and (18b) can be combined to give

$$\left(\sum_{s=1}^{n}\sum_{L_1\cdots L_n}\frac{L_s^2}{\Delta_s^2}+\sum_{s>t=1}^{n}\sum_{L_1\cdots L_n}\frac{L_sL_t}{\Delta_s\Delta_t}\right)J_{l-L_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)$$
$$=\frac{l^2J_l(x)}{2x^2}\left[\sum_{s=1}^{n}\frac{1}{\Delta_s^2}[(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0)]^2+\left(\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}\right)^2\right]$$
$$+\frac{J_l'(x)}{2x}\left[\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0(\vec{q}'-\vec{k}_s)\cdot\vec{\alpha}_0}{\Delta_s^2}+\left(\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}\right)\left(\sum_{t=1}^{n}\frac{(\vec{q}'-\vec{k}_t)\cdot\vec{\alpha}_0}{\Delta_t}\right)+x\sum_{s>t=1}^{n}\frac{(\vec{k}_s-\vec{k}_t)\cdot\vec{\alpha}_0}{\Delta_s\Delta_t}\right]. \qquad (22)$$

Equations (19), (21), and (22) may be used to simplify the first three orders in ω in Eq. (17):

$$T_{q'q}^{(n+1)}(l)=i\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1)\cdots\tilde{V}(\vec{K}_{n+1})}{\Delta_1\cdots\Delta_n}$$
$$\times\Bigg[J_l(x)-\frac{\omega lJ_l(x)}{x}\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}+\frac{\omega^2l^2J_l(x)}{2x^2}\sum_{s=1}^{n}\left(\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}\right)^2+\frac{\omega^2}{2}\frac{l^2J_l(x)}{x^2}\left(\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}\right)^2$$
$$+\frac{\omega^2}{2}\frac{J_l'(x)}{x}\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0(\vec{q}\,'-\vec{k}_s)\cdot\vec{\alpha}_0}{\Delta_s^2}+\frac{\omega^2}{2}\frac{J_l'(x)}{x}\left(\sum_{s=1}^{n}\frac{(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0}{\Delta_s}\right)\left(\sum_{t=1}^{n}\frac{(\vec{q}\,'-\vec{k}_t)\cdot\vec{\alpha}_0}{\Delta_t}\right)$$
$$+\frac{\omega^2J_l'(x)}{2}\sum_{s>t=1}^{n}\frac{(\vec{k}_s-\vec{k}_t)\cdot\vec{\alpha}_0}{\Delta_s\Delta_t}\Bigg].\tag{23}$$

We can follow the results of Kroll and Watson by noting that we may define

$$D_s=\Delta_s+(\omega l/x)(\vec{k}_s-\vec{q})\cdot\vec{\alpha}_0=\epsilon_{q-\lambda}-\epsilon_{k_s-\lambda}\,,\tag{24}$$

where

$$\vec{\lambda}=(m\omega l/x)\vec{\alpha}_0\,.\tag{25}$$

Then

$$J_l(x)/(D_1\cdots D_n)$$

may be expanded in powers of ω with a result which is identical, up to order ω^2, with the first four terms in Eq. (23). If we now shift the integration variables by $\vec{k}_j'=\vec{k}_j-\vec{\lambda}$ and define

$$\vec{Q}=\vec{q}-\vec{\lambda},\quad \vec{Q}\,'=\vec{q}\,'-\lambda\,,\tag{26}$$

then (dropping the primes on k_j) we get

$$T_{q'q}^{(n+1)}(l)=i\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1)\cdots\tilde{V}(\vec{K}_{n+1})}{\Delta_1'\cdots\Delta_n'}\Bigg\{J_l(x)-\frac{\omega^2}{2}\frac{J_l'(x)}{x}\left[\sum_{s=1}^{n}\left(\frac{\vec{k}_s\cdot\vec{\alpha}_0}{\Delta_s'}\right)^2+\left(\sum_{s=1}^{n}\frac{\vec{k}_s\cdot\vec{\alpha}_0}{\Delta_s}\right)^2\right]$$
$$-\frac{\omega^2}{2}\frac{J_l'(x)}{x}\vec{Q}\cdot\vec{\alpha}_0\vec{Q}\,'\cdot\vec{\alpha}_0\left[\sum_{s=1}^{n}\left(\frac{1}{\Delta_s'}\right)^2+\left(\sum_{s=1}^{n}\frac{1}{\Delta_s'}\right)^2\right]$$
$$+\frac{\omega^2}{2}\frac{J_l'(x)}{x}\vec{\alpha}_0\cdot(\vec{Q}+\vec{Q})\left[\sum_{s=1}^{n}\frac{\vec{k}_s\cdot\vec{\alpha}_0}{\Delta_s'^2}+\left(\sum_{s=1}^{n}\frac{\vec{k}_s\cdot\vec{\alpha}_0}{\Delta_s'}\right)\left(\sum_{t=1}^{n}\frac{1}{\Delta_t'}\right)\right]$$
$$+\frac{\omega^2J_l'(x)}{2}\sum_{s>t=1}^{n}\frac{(\vec{k}_s-\vec{k}_t)\cdot\vec{\alpha}_0}{\Delta_s'\Delta_t'}\Bigg\}.\tag{27}$$

where the $\vec{K}_j$ are given by (10) with $\vec{q}\to\vec{Q}$ and $\vec{q}\,'\to\vec{Q}\,'$ and Δ_j' is given by (13) with $\vec{q}\to\vec{Q}$. The first term of (27) can be rewritten as

$$J_l(x)\langle\vec{Q}\,'|T^{(n+1)}(\epsilon_Q)|\vec{Q}\rangle\,,$$

where the last factor is the nth term in the Born series for the T matrix for scattering in the absence of the laser. The energy δ function in (14) can be rewritten in terms of the new momenta and it becomes $\delta(\epsilon_{Q'}-\epsilon_Q)$ indicating that the T matrix in the absence of the laser is evaluated on shell in this order. Insertion of this result into (15) yields the result of Kroll and Watson.

The remaining four terms in (27) are the ω^2 correction to this result. They can be reexpressed by use of the identities

$$\frac{\partial}{\partial E}\langle\vec{Q}\,'|T^{(n+1)}(E)|\vec{Q}\rangle=-\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1')\cdots\tilde{V}(\vec{K}_{n+1}')}{\Delta_1'\cdots\Delta_n'}\sum_{s=1}^{n}\frac{1}{\Delta_s'},\tag{28a}$$

$$\frac{\partial}{\partial E^2}\langle\vec{Q}\,'|T^{(n+1)}(E)|\vec{Q}\rangle=\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\frac{\tilde{V}(\vec{K}_1')\cdots\tilde{V}(\vec{K}_{n+1}')}{\Delta_1'\cdots\Delta_n'}\times\left[\sum_{s=1}^{n}\frac{1}{\Delta_s'^2}+\left(\sum_{s=1}^{n}\frac{1}{\Delta_s'}\right)^2\right],\tag{28b}$$

$$[\vec{\alpha}_0\cdot(\vec{\nabla}_Q+\vec{\nabla}_{Q'})]^2\langle\vec{Q}'|T^{(n+1)}(E)|\vec{Q}\rangle = \int\left(\frac{d^3k}{(2\pi)^3}\right)^n \frac{\tilde{V}(\vec{K}_1')\cdots\tilde{V}(\vec{K}_{n+1}')}{\Delta_1'\cdots\Delta_n'} \times\left[\sum_{s=1}^n\left(\frac{\vec{k}_s\cdot\vec{\alpha}_0}{m\Delta_s'}\right)^2+\left(\sum_{s=1}^n\frac{\vec{k}_s\cdot\vec{\alpha}_0}{m\Delta_s'}\right)^2+\frac{\alpha_0^2}{m^2}\sum_{s=1}^n\frac{1}{\Delta_s'}\right], \tag{28c}$$

$$\vec{\alpha}_0\cdot(\vec{\nabla}_Q+\vec{\nabla}_{Q'})\frac{\partial}{\partial E}\langle\vec{Q}'|T^{(n+1)}(E)|\vec{Q}\rangle = \int\left(\frac{d^3k}{(2\pi)^3}\right)^n \frac{\tilde{V}(\vec{K}_1')\cdots\tilde{V}(\vec{K}_{n+1}')}{\Delta_1'\cdots\Delta_n'} \times\left[-\frac{1}{m}\sum_{s=1}^n\frac{\vec{k}_s\cdot\vec{\alpha}_0}{\Delta_s'^2}-\frac{1}{m}\left(\sum_{s=1}^n\frac{\vec{\alpha}_0\cdot\vec{k}_s}{\Delta_s'}\right)\left(\sum_{t=1}^n\frac{1}{\Delta_t'}\right)\right]. \tag{28d}$$

The last term of (27) is, perhaps, the most interesting. It can be rewritten as

$$\langle\vec{Q}'|\delta T^{(n+1)}(E_Q)|\vec{Q}\rangle=\frac{i\omega^2}{2}J_l'(x)\int\frac{d^3k_1d^3k_2}{(2\pi)^6}\sum_{s>t=1}^{n+1}\left(\frac{\langle\vec{Q}'|T^{(t)}(E)|\vec{k}\rangle}{\Delta_{k_1}^2(E)}[\langle\vec{k}_1|T^{(s-t)}(E)|\vec{k}_2\rangle\vec{\alpha}_0\cdot\vec{k}_2 -\vec{\alpha}_0\cdot\vec{k}_1\langle\vec{k}_1|T^{(s-t)}(E)|\vec{k}_2\rangle]\frac{\langle\vec{k}_2|T^{(n-s+1)}(E)|\vec{Q}\rangle}{\Delta_{k_2}^2(E)}\right)\Bigg|_{E=\epsilon_Q}, \tag{29}$$

or in operator notation

$$\delta T^{(n+1)}(E)=\frac{i\omega}{2}J_l'(x)\sum_{s>t=1}^{n+1}T^{(t)}(E)\frac{1}{\Delta^2(E)} \times[T^{(s-t)}(E),\omega\vec{\alpha}_0\cdot\vec{p}]\frac{1}{\Delta^2(E)}T^{(n-s+1)}(E). \tag{30}$$

If we define a new operator

$$X^{(n+1)}(E)=V\frac{1}{\Delta(E)-\omega\vec{\alpha}_0\cdot\vec{p}}V\cdots\frac{1}{\Delta(E)-\omega\vec{\alpha}_0\cdot\vec{p}}V \tag{31}$$

containing $n+1$ factors of V, then it can be expanded in powers of ω up to first order with the result

$$X^{(n+1)}(E)=T^{(n+1)}(E)+\sum_{j=1}^{n+1}T^{(j)}(E)\frac{\omega\vec{\alpha}_0\cdot\vec{p}}{\Delta^2(E)}T^{(n-j+1)}(E). \tag{32}$$

The last term in (32) is the form occurring in (30) which can then be written

$$\delta T^{(n+1)}(E)=\frac{i\omega J_l'(x)}{2}\sum_{s=1}^{n+1}\left(T^{(s)}(E)\frac{1}{\Delta^2(E)}X^{(n+1-s)}(E) -X^{(s)}(E)\frac{1}{\Delta^2(E)}T^{(n+1-s)}(E)\right). \tag{33}$$

The sum over n may now be performed:

$$\delta T=\sum_{n=0}^\infty\delta T^{(n+1)} =\frac{i\omega J_l'(x)}{2}\left(T(E)\frac{1}{\Delta^2(E)}X(E)-X(E)\frac{1}{\Delta^2(E)}T(E)\right), \tag{34}$$

where $X(E)$ is the sum over all n on $X^{(n)}(E)$. It can be written

$$X(E)=V+V\frac{1}{\Delta(E)-\omega\vec{\alpha}_0\cdot\vec{p}-V}V \tag{35}$$

and can be shown to satisfy the integral equation

$$X(E)=V+V\frac{1}{\Delta(E)-\omega\vec{\alpha}_0\cdot\vec{p}}X(E). \tag{36}$$

Similarly T, which is the sum over all n on $T^{(n)}$ satisfies

$$T(E)=V+V\frac{1}{\Delta(E)}T(E). \tag{37}$$

A straightforward comparison of these two yields

$$\langle k'|X(E)|\vec{k}\rangle=\langle\vec{k}'+m\omega\vec{\alpha}_0|T(E+\tfrac{1}{2}m\omega^2\alpha_0^2)|\vec{k}+m\omega\vec{\alpha}_0\rangle. \tag{38}$$

Then if the right-hand side of this equation is expanded in powers of ω, keeping terms up to the linear ones, it can be substituted back into the matrix form of (34) with the result

$$\langle\vec{Q}'|\delta T|Q\rangle=\frac{i\omega^2J_l'(x)}{2}\int\frac{d^3k}{(2\pi)^3}\left(\frac{1}{(\epsilon_Q^+-\epsilon_k)^2}\right)\Big(\langle\vec{Q}'|T(E)|\vec{k}\rangle m\alpha_0\cdot(\vec{\nabla}_k+\vec{\nabla}_Q)\langle\vec{k}|T(E)|\vec{Q}\rangle -[m\vec{\alpha}_0\cdot(\vec{\nabla}_{Q'}+\vec{\nabla}_k)\langle\vec{Q}'|T(E)|\vec{k}\rangle]\langle\vec{k}|T(E)|\vec{Q}\rangle\Big)_{E=\epsilon_Q}. \tag{39}$$

The results may now be assembled to give

$$T_{q'q}(l)=J_l(x)\langle\vec{Q}'|T(\epsilon_Q)|\vec{Q}\rangle-\frac{\omega^2}{2x}J_l'(x)\Big(\vec{Q}\cdot\vec{\alpha}_0\vec{Q}'\cdot\vec{\alpha}_0\frac{\partial^2}{\partial E^2}+m\vec{\alpha}_0\cdot(\vec{Q}+\vec{Q}')\vec{\alpha}_0\cdot(\vec{\nabla}_Q+\vec{\nabla}_{Q'})\frac{\partial}{\partial E}+[m\vec{\alpha}_0\cdot(\vec{\nabla}_Q+\vec{\nabla}_{Q'})]^2$$
$$+m\alpha_0^2\frac{\partial}{\partial E}\Big)\langle\vec{Q}'|T(E)|\vec{Q}\rangle\Big|_{E=\epsilon_Q}+\langle\vec{Q}'|\delta T(\epsilon_Q)|\vec{Q}\rangle\,. \quad (40)$$

The first term is the Kroll and Watson result extended to second order. The next bracket is a collection of second-order corrections which contain the T matrix in the absence of the field evaluated slightly off shell. Finally, the last term is also a second-order correction but it depends upon this T matrix far off shell because of the integral over all k which occurs in (39).

In conclusion, it has been shown that the T matrix for the scattering of an electrons with transfer of l photons by a potential in the presence of a low-frequency laser is describable, in lowest order, by an on-shell T matrix describing the scattering in the absence of the laser, but that the ω^2 corrections are describable in terms of slightly off-shell T matrix terms plus a term that depends upon the T matrix far off shell. However, one should be extremely cautious when applying these results to scattering of an electron by an atom in the presence of a laser field. An atom has internal degrees of freedom, and even when the laser photon energy is much smaller than the excitation energy of the atom, the atom may not act just as a potential. For example, the electron can excite the atom and the energy of a single photon may be enough to couple pairs of excited states. The coupling may even be resonant with the excited $(e+A)$ complex and consequently strong. This will be discussed in a future publication.

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