Quantum theory of an inhomogeneously broadened two-photon laser

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A quantum statistical analysis of a two-photon laser is presented in a generalization of the one-photon laser. A two-photon loss mechanism is assumed by taking detailed-balance considerations into account. The expressions for the photon statistical distribution exhibit the exact influence of the Doppler parameter and other relaxation parameters. In addition, the photon statistics can be studied as a function of the detuning. Threshold conditions for the laser action are noticed. The peak photon number and half-width of the photon distributions are obtained. It is found that the photon distribution can be written as a ratio of the one-photon laser-photon distribution to the Poisson distribution for photons.

I. INTRODUCTION

In the wake of the success of the quantum theory in explaining one-photon laser phenomenon involving a single photon emission and absorption per atomic transition, the problem of laser action involving stimulated emission of multiple photons per atomic transition has received considerable recent attention.[1-4] Two-photon spectroscopy, however, has been a well-known experimental technique for some time,[5] and two-photon spontaneous emission has been observed more than a decade ago.[6] This has naturally led to speculations about the feasibility of a two-photon laser. The possibility of obtaining high-intensity radiation from a two-photon laser system is very bright as the coupling constant g_2 of the atom-field interaction is proportional to the field intensity in a two-photon laser, while the coupling constant g is proportional to the square root of light intensity in the one-photon laser. Yuen[4] has shown that noise behavior of two-photon coherent states, which are similar to the uncertainty states, may lead to applications that are not available from the one-photon laser. He has briefly discussed a two-photon laser and has shown that when the atomic decay phenomenon is neglected, the two-photon coherent states are the radiation states of an ideal two-photon laser system operating far above threshold in the self-consistent-field approximation.

Though the photon distribution of the two-photon emission process has been well studied,[7-10] none of these studies take pump or loss mechanisms into account. Hence the feasibility of two-photon laser action cannot be firmly predicted on the basis of these theories. McNeil and Walls[1] have formally generalized the Scully-Lamb[11] one-photon laser theory and have derived the photon statistics of the laser fields, assuming the atomic transition frequency to be homogeneously broadened and in resonance with the sum frequency of the photons of the field modes under consideration. As is well known, the relaxation parameters have a considerable influence on the performance of the one-photon gas laser when the Doppler limit is lifted,[12] and are expected to have a stronger influence on the performance of a two-photon laser, which is a higher-order process. The purpose of this paper, therefore, is to present a quantum theory of a two-photon gas laser, which takes atomic motion into account and avoids the Doppler-limit approximation. We present the model for the single-mode (SM) and two-mode (TM) two-photon laser in Sec. II and derive the photon statistics of the radiation fields in Sec. III. The results obtained are discussed in Sec. IV.

II. MODEL

We present here a model for the two-photon laser by generalizing the Riska-Stenholm quantum theory,[13] in order to incorporate the influence of the Doppler parameter. We describe the active atom by its two energy levels $|a\rangle$ and $|b\rangle$, having energies $\hbar\omega_a$ and $\hbar\omega_b$, respectively. The decay constant for the levels $|a\rangle$ and $|b\rangle$ are γ_a and γ_b, respectively. The transition frequency between the levels is

$$\omega = \omega_a - \omega_b . \tag{1}$$

The atoms in the upper state $|a\rangle$ are injected into the cavity at a point z, with the velocity v and at a rate $\lambda_a(z, v)$. The atom injected at the point z_0, at a time t_0 and with velocity v then will be, at a later time t at z, where

$$z = z_0 + v(t - t_0) . \tag{2}$$

This neglects the effect of atomic collisions. We assume a Maxwellian velocity distribution for atoms, and thus $\lambda_a(z,v)$ is given by

$$\lambda_a(z,v)=(r_a/Lu\sqrt{\pi})e^{-v^2/u^2}\,, \tag{3}$$

where u is the speed parameter and L is the length of the cavity. The z dependence of $\lambda_a(z,v)$ is considered negligible. r_a is the total injection rate.

The loss mechanism is treated by introducing atoms in the lower state $|\beta\rangle$ of two broad levels $|\alpha\rangle$ and $|\beta\rangle$ into the cavity at a rate r_β. The decay constants for $|\alpha\rangle$ and $|\beta\rangle$ are γ_α and γ_β respectively. We consider a two-photon loss mechanism by taking detailed balance considerations into account.

In the case of a two-level system, the lasing levels should have the same parity under the usual dipole approximation for the two-photon transition to take place. However, if one considers the contributions of higher-order moments (e.g., the quadrupole moments), two-photon transitions can take place between the states of opposite parity. But, as pointed out by Pantell and Puthoff,[14] two-photon transition between states of opposite parity is at least six orders of magnitude weaker than the two-photon transition between states of the same parity, because of the necessity to consider the contribution of electric quadrupole moment in the former case. In this respect the states of same parity may be preferred for the two-photon transition in a two-level system.

The unperturbed Hamiltonian $\hbar H_0$ for the two-photon laser is

$$H_0=(a_1^\dagger a_1+\tfrac{1}{2})\nu_1+(a_2^\dagger a_2+\tfrac{1}{2})\nu_2+(\sigma^+\sigma\omega_a+\sigma\sigma^+\omega_b)\,, \tag{4}$$

where a_1 ($a_1^\dagger$) and a_2 ($a_2^\dagger$) are the photon annihilation (creation) operators for the radiation fields of frequency ν_1 and ν_2, respectively. The atomic operators σ and σ^+ are defined by

$$\sigma^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}\ \text{and}\ \sigma=\begin{pmatrix}0&0\\1&0\end{pmatrix}, \tag{5}$$

which obey the relation $\sigma^+\sigma+\sigma\sigma^+=1$. For the two-mode two-photon laser, where two photons are emitted to two different modes, the Hamiltonian $\hbar H_2'$ for the interaction between the active atoms and the radiation fields may be written

$$H_2'=g_2(\sigma^+a_1a_2+\sigma a_1^\dagger a_2^\dagger)\sin k_1z\sin k_2z\,. \tag{6}$$

g_2 is the coupling constant given by

$$g_2=\mu_{12}^{(2)}E_1E_2/\hbar^2\,, \tag{7}$$

$\mu_{12}^{(2)}$ is the matrix element for two-photon transition, and E_1 and E_2 have the dimension of electric field. For a single-mode two-photon laser, where two photons are emitted into the same mode, the interaction Hamiltonian $\hbar H_1'$ is

$$H_1'=g_2(\sigma^+aa+\sigma a^\dagger a^\dagger)\sin kz\sin kz\,, \tag{8}$$

where the coupling constant g_2 is now given by

$$g_2=\mu_{12}^{(2)}E^2/\hbar^2\,. \tag{9}$$

We describe the composite atom-field system as usual by the density matrix ρ which obeys the equation of motion

$$i\dot\rho=[H,\rho]\,, \tag{10}$$

where H is the total Hamiltonian of the system. For the TM case, the equations of motion for the elements of ρ are

$$\begin{aligned}\dot\rho(a,n_1,n_2;\ a,n_1,n_2)&\\ =-\gamma_a\rho(a,n_1,n_2;\ a,n_1,n_2)&\\ -i[H_2'(t)\rho(b,n_1+1,n_2+1;\ a,n_1,n_2)&\\ -\rho(a,n_1,n_2;\ n,n_1+1,n_2+1)H_2'(t)]\,,&\end{aligned} \tag{11a}$$

$$\begin{aligned}\dot\rho(a,n_1,n_2;\ b,n_1+1,n_2+1)&\\ =-(\gamma_{ab}+i\Delta_2)\rho(a,n_1,n_2;\ b,n_1+1,n_2+1)&\\ -iH_2'(t)[\rho(b,n_1+1,n_2+1;\ b,n_1+1,n_2+1)&\\ -\rho(a,n_1,n_2;\ a,n_1,n_2)]\,,&\end{aligned} \tag{11b}$$

and

$$\begin{aligned}\dot\rho(b,n_1+1,n_2+1;\ b,n_1+1,n_2+1)&\\ =-\gamma_b\rho(b,n_1+1,n_2+1;\ b,n_1+1,n_2+1)&\\ -i[H_2'(t)\rho(a,n_1,n_2;\ b,n_1+1,n_2+1)&\\ -\rho(b,n_1+1,n_2+1;\ a,n_1,n_2)H_2'(t)]\,,&\end{aligned} \tag{11c}$$

with

$$\rho(b,n_1+1,n_2+1;\ a,n_1,n_2)=\rho^*(a,n_1n_2;\ b,n_1+1,n_2+1)\,, \tag{11d}$$

where n_1 and n_2 are the photon numbers of the fields of frequencies ν_1 and ν_2, respectively, $\Delta_2=\omega-\nu_1-\nu_2$ and $\gamma_{ab}=\frac{1}{2}(\gamma_a+\gamma_b)$. For the SM case, we have

$$\dot{\rho}(a,n;\ a,n) = -\gamma_a\rho(a,n;\ a,n) - i[H_1'(t)\rho(b,n+2;\ a,n) - \rho(a,n;\ b,n+2)H_1'(t)], \quad (12a)$$

$$\dot{\rho}(a,n;\ b,n+2) = -(\gamma_{ab}+i\Delta_1)\rho(a,n;\ b,n+2) - iH_1'(t)[\rho(b,n+2;\ b,n+2) - \rho(a,n;\ a,n)], \quad (12b)$$

and

$$\dot{\rho}(b,n+2;\ b,n+2) = -\gamma_b\rho(b,n+2;\ b,n+2) - i[H_1'(t)\rho(a,n;\ b,n+2) - \rho(b,n+2;\ a,n)H_1'(t)], \quad (12c)$$

with

$$\rho(b,n+2;\ a,n) = \rho^*(a,n;\ b,n+2), \quad (12d)$$

where $\Delta_1 = \omega - 2\nu$, and n is the photon number of the radiation field of frequency ν. We follow the iteration procedure in the Riska-Stenholm theory[13] to obtain expressions for the matrix elements of ρ up to fourth order. The integrations involved in the calculations are solved by using the method of Mohanty and Nayak.[12] The loss mechanism is treated as a linear process, and so the density-matrix elements describing it are calculated up to second order under the assumption that the energy levels $|\alpha\rangle$ and $|\beta\rangle$ of the loss atoms are broad compared to the Doppler and detuning parameters. The equation of motion for the density matrix describing the radiation fields only are finally obtained by tracing over atomic variables.

III. PHOTON STATISTICS

As the atoms are in the upper state, as when they are introduced into the cavity, we have in the zeroth order, for the SM case,

$$\rho^{(0)}(a,n;\ a,n;\ t) = \rho(n,n;\ t_0)\exp[-\gamma_a(t-t_0)], \quad (13a)$$

$$\rho^{(0)}(b,n+2;\ b,n+2;\ t) = 0. \quad (13b)$$

We have, for the TM case,

$$\rho^{(0)}(a,n_1,n_2;\ a,n_1,n_2;\ t) = \rho(n_1,n_2;\ n_1,n_2;\ t)\exp[-\gamma_a(t-t_0)], \quad (13c)$$

$$\rho^{(0)}(b,n_1+1,n_2+1;\ b,n_1+1,n_2+1;\ t) = 0. \quad (13d)$$

A. Two-mode case

As the moving atoms interact with the two modes of the radiation at fields with the wave vectors k_1 and k_2 in a single atomic transition, we find, under reasonable approximation,[13] that the transition frequency ω is Doppler shifted by a magnitude $(k_1+k_2)v$. Following the method described in Sec. II, we find the steady-state solution of the equation of motion for the density matrix describing the radiation fields to be only

$$\rho(n_1,n_2;\ n_1,n_2) = \frac{A_2}{C}U(x_2,y_2)\left(1 - \frac{n_1n_2}{32}\frac{B_2}{A_2}F(x_2,y_2)\right) \times \rho(n_1-1,n_2-1;\ n_1-1,n_2-1), \quad (14)$$

where A_2, B_2, and C are, respectively, linear, nonlinear, and loss parameters given by

$$A_2 = \frac{g_2^2\gamma_a}{4\gamma_a}\frac{\sqrt{\pi}}{(k_1+k_2)u}, \quad B_2 = \frac{4g_2^2}{\gamma_a\gamma_b}A_2, \quad \text{and } C = \frac{g_2^2\gamma_\beta}{4\gamma_\beta\gamma_{\alpha\beta}}, \quad (15)$$

with $\gamma_{\alpha\beta} = \frac{1}{2}(\gamma_\alpha+\gamma_\beta)$. $F(x_2,y_2)$ is given by

$$F(x_2,y_2) = 1 + \frac{\gamma_{ab}^2}{\Delta_2^2+\gamma_{ab}^2} + \frac{2y_2}{\sqrt{\pi}}\frac{1}{U(x_2,y_2)} + \frac{\gamma_{ab}^2}{\Delta_2^2+\gamma_{ab}^2}\frac{y_2}{x_2}\frac{V(x_2,y_2)}{U(x_2,y_2)}, \quad (16)$$

where $x_2 = \Delta_2/(k_1+k_2)u$ and $y_2 = \gamma_{ab}/(k_1+k_2)u$. U and V are, respectively, the real and imaginary parts of the probability integral

$$W(z_2) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{z_2-t}\,dt = U + iV. \quad (17)$$

for complex argument $z_2 = x_2 + iy_2$, and have been widely tabulated.[15] In this TM case, equal number of photons are created and annihilated in the two modes. Thus we have the condition $n_1 - n_2 = 0$, which is a consequence of detailed-balance condition.[1] The steady-state solution (14) thus becomes, for $n_1 = n_2 = n$,

$$\rho(n_1,n_2;\ n_1,n_2) = \rho(n,n)$$

with

$$\rho(n,n) = \left(\frac{A_2}{C}U(x_2,y_2)\right)^n \bar{N}_2 \prod_{j=0}^{n}\left(1 - \frac{j^2}{32}\frac{B_2}{A_2}F(x_2,y_2)\right), \quad (18)$$

where $\bar{N}_2$ is the normalization constant. Equation (18) describes the photon distributions in both modes of the cavity. The photon number $\bar{n}_{\mathrm{TM}}$, at which the peak of the distribution (18) occurs, can be obtained from the assumption

$$\rho(\bar{n}_{\rm TM}-1,\bar{n}_{\rm TM}-1)\cong\rho(\bar{n}_{\rm TM},\bar{n}_{\rm TM}):$$

$$\bar{n}_{\rm TM}=\left[\frac{A_2}{B_2}\,\frac{32}{F(x_2,y_2)}\left(1-\frac{C}{A_2}\,\frac{1}{U(x_2,y_2)}\right)\right]^{1/2}. \tag{19}$$

We find the half width of the distribution (18) to be

$$K^2_{\rm TM}=\tfrac{1}{2}\,\bar{n}_{\rm TM}/[(A_2/C)U(x_2,y_2)-1] \tag{20}$$

from the condition

$$\rho(\bar{n}_{\rm TM}+K_{\rm TM},\bar{n}_{\rm TM}+K_{\rm TM})=\tfrac{1}{2}\,\rho(\bar{n}_{\rm TM},\bar{n}_{\rm TM}).$$

In Eq. (20) $\bar{n}_{\rm TM}$ is given by Eq. (19).

B. Single-mode case

In this case, the active atoms interact with the radiation field twice in each atomic transition. Thus, under the approximation used for the TM case, there will be a Doppler shift of magnitude $2kv$ in the atomic transition frequency ω. Following the procedure leading to Eq. (14) in the TM case, we find the photon statistics for this case to be

$$\rho(n,n)=\frac{A_1}{C}\,U(x_1,v_1)\left(1-\frac{n(n-1)}{32}\,\frac{B_1}{A_1}\,F(x_1,y_1)\right)\times\rho(n-2,n-2)\,, \tag{21}$$

where

$$A_1=\frac{g_2^2\gamma_a}{8\gamma_a}\,\frac{\sqrt{\pi}}{ku}\,,\quad B_1=\frac{4g_2^2}{\gamma_a\gamma_b}A_1\,, \tag{22}$$

and C has already been defined in Eq. (15). $F(x_1,y_1)$ is

$$F(x_1,y_1)=1+\frac{\gamma^2_{ab}}{\Delta_1^2+\gamma^2_{ab}}+\frac{2y_1}{\sqrt{\pi}}\,\frac{1}{U(x_1,v_1)}+\frac{\gamma^2_{ab}}{\Delta_1^2+\gamma^2_{ab}}\,\frac{y_1}{x_1}\,\frac{V(x_1,y_1)}{U(x_1,y_1)}\,, \tag{23}$$

with $x_1=\Delta_1/2ku$ and $y_1=\gamma_{ab}/2ku$. In the SM case, two photons from the same mode are involved in each atomic transition. Thus the photon number of the field should remain even. Here also, this result is a consequence of the detailed balance considerations. Assuming the initial photon distribution to be $\rho(2n,2n;\ t_0)$, we get from Eq. (21)

$$\rho(2n,2n)=\left(\frac{A_1}{C}\,U(x_1,y_1)\right)^n\bar{N}_1\times\prod_{j=0}^{n}\left(1-\frac{2j(2j-1)}{32}\,\frac{B_1}{A_1}\,F(x_1,y_1)\right), \tag{24}$$

where $\bar{N}_1$ is the normalization constant. The distribution (24) has a peak at

$$2\bar{n}_{\rm SM}=\frac{1}{2}+\left[\frac{1}{4}+\frac{A_1}{B_1}\,\frac{32}{F(x_1,y_1)}\left(1-\frac{C}{A_1}\,\frac{1}{U(x_1,y_1)}\right)\right]^{1/2}, \tag{25}$$

and its half width is given by

$$(2K_{\rm SM})^2=2\bar{n}_{\rm SM}/[(A_1/C)U(x_1,y_1)-1]\,, \tag{26}$$

under similar conditions as taken in the TM case. In Eq. (26), $2\bar{n}_{\rm SM}$ is given by Eq. (25).

IV. DISCUSSION

It is seen from Eqs. (18) and (24) that the threshold condition for the oscillation to start inside the cavity is

$$A_i/C\geqslant 1/U(x_i,y_i)\,,\quad i=1,2\,. \tag{27}$$

We note that Eq. (27) is of the same form as the threshold condition for one-photon laser.[12] It may be noted that in general v_i for a two-photon laser is relatively small compared to the corresponding parameters for a one-photon laser. This makes the numerical value of $U(x_i,y_i)$ higher compared to that of one-photon laser, and thus reduces the threshold condition compared to a one-photon laser.

In Fig. 1 we compare the photon distribution for the SM as well as TM two-photon lasers with that for a one-photon laser as given by Eq. (7) in Ref. 12. We find that the photon distribution curve for a one-photon laser is considerably broader than that of the two-photon laser. The reason for this nature is not immediately known, though one may suggest that interference effects between pairs of photons, which should be correlated, have something to do with it, and a more detailed analysis of the correlation of two-photon transitions might provide an answer. It is interesting to note in Fig. 1 that the photon statistics curve for the SM two-photon laser is broader than the distribution for the TM two-photon laser. This is due to the fact that in the SM case, two photons are spontaneously emitted to the same mode, which give rise to a higher amplification of spontaneous noise. Figs. 2 and 3 show the photon distribution curves for a SM two-photon laser for various values of Δ_1 and two values of A_1/C. In Fig. 3 one may note that as the values of Δ_1 is increased, the peak of the curve shifts towards a higher photon number, that is, towards higher intensity. As the value of Δ_1 is further increased, the peak photon number is reduced, and thus a dip is clearly indicated at resonance ($\Delta_1=0$). The dip does not occur for smaller values of A_1/C ($\leqslant 1.2$), and in this case the photon distribution resembles the blackbody distribution for $\Delta_1=4\gamma_{ab}$ (Fig. 2). For the TM two-photon laser, Figs. 4 and 5 show that the dip occurs for both $A_2/C=1.2$ and 1.5. In this

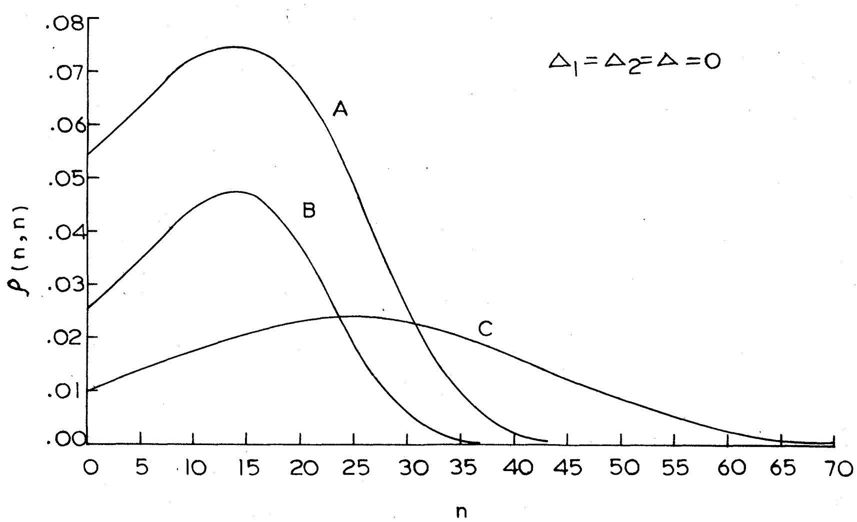

FIG. 1. Photon distribution for (a), SM two-photon laser; (b) TM two-photon laser; (c) one-photon laser; $A_1/C=A_2/C=A/C=1.2$, $B_1/A_1=B_2/A_2=B/A=0.005$, $y_1=y_2=y=0.1$. As defined in Ref. 12, $y=\gamma_{ab}/ku$; (a), (b), and (c) are, respectively, linear, nonlinear, and loss parameters for one-photon laser.

case also, laser oscillation does not take place for $\Delta_2=4\gamma_{ab}$ when $A_2/C=1.2$ (Fig. 4). These observations give useful information regarding the numerical values of the relaxation terms and the linear and nonlinear parameters.

Under the conditions $x_1=x_2$, $y_1=y_2$, $A_1/C=A_2/C$, and $B_1/A_1=B_2/A_2$, we note in Figs. 2–5 that the peaks of the distributions for both the SM and TM case occur at the same photon number. A comparison of the respective peak-photon-number expressions (19) and (25) for the TM and SM cases confirms that the peaks for the two cases occur almost at the same photon number under the abovementioned condition. Figures 2–5 indicate that the photon distribution for the SM case is broader than that for the TM case. This conclusion is verified from a comparison of half widths [Eqs. (20) and (26)] in Fig. 6 for the TM and SM cases. Figure 6 also shows that increase of A_i/C gives rise to a narrower photon distribution curve, which is thus an indication of a higher-ordered state. Equations (20) and (26), as expected, have the same form as that for the half width of the photon distribution for the one-photon laser ob-

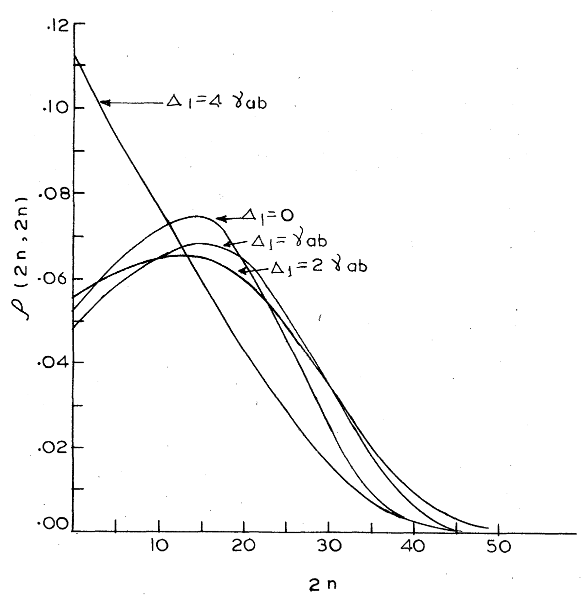

FIG. 2. Photon distribution for SM two-photon laser: $A_1/C=1.2$, $B_1/A_1=0.005$, $y_1=0.1$.

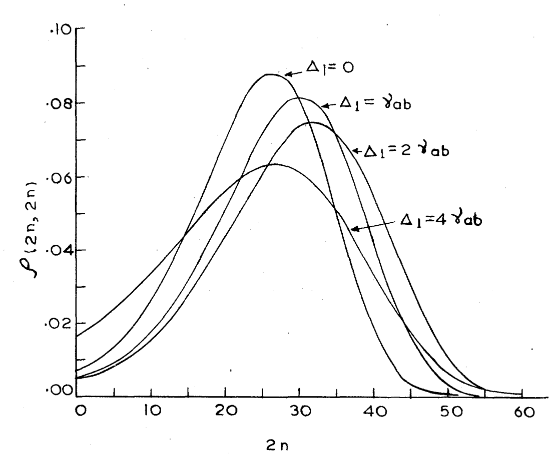

FIG. 3. Photon distribution for SM two-photon laser: $A_1/C=1.5$, $B_1/A_1=0.005$, $y_1=0.1$.

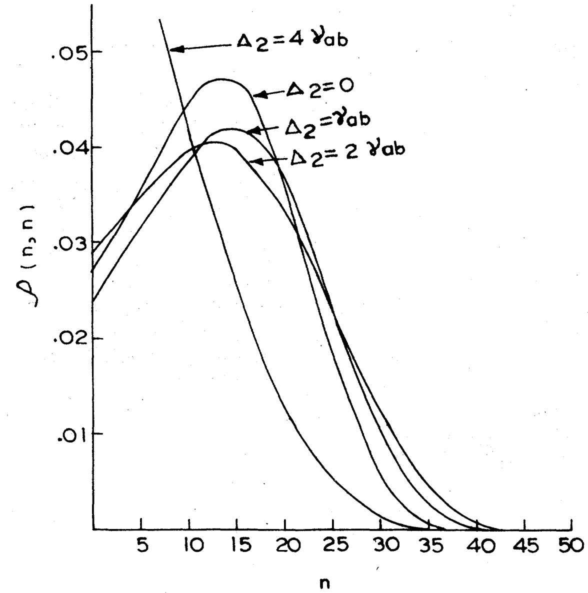

FIG. 4. Photon distribution for TM two-photon laser: $A_2/C=1.2$, $B_2/A_2=0.005$, $y_2=0.1$.

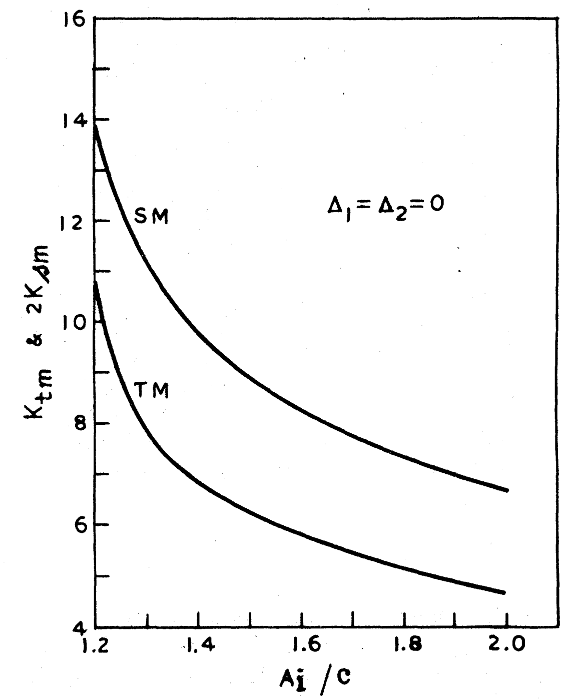

FIG. 6. Variation of half width of photon distribution curve with A_i/C: $B_i/A_i=0.005$, $y_i=0.1$, $i=1,2$.

tained in Ref. 12. A comparison among these equations verifies the conclusion drawn from Fig. 1 about the relative widths of the one- and two-photon lasers.

The output variance of the two-photon laser can be calculated from the equation

$$(\Delta n)^2 = \sum_{n=0}^{\infty} n^2\rho(n,n) - \left(\sum_{n=0}^{\infty} n\rho(n,n)\right)^2 . \tag{28}$$

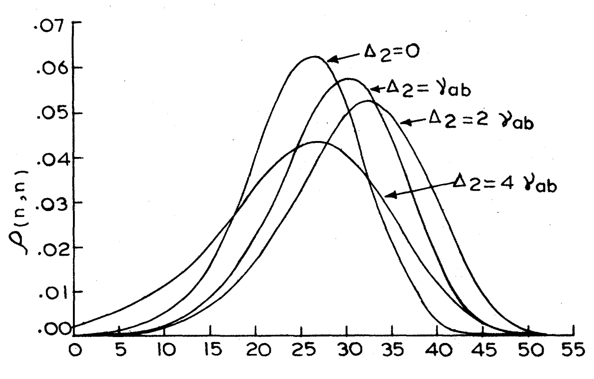

FIG. 5. Photon distribution for TM two-photon laser: $A_2/C=1.5$, $B_2/A_2=0.005$, $y_2=0.1$.

Substituting the photon distribution (18) and (24) in Eq. (28), we obtain the numerical values of the output variance for the TM and SM two-photon lasers, respectively. The results show that the output variance for the SM case is much higher than that for the TM case under the conditions studied. These results are in agreement with our prior observation that the noise in the SM case is expected to be higher than the TM case.

Some interesting inferences can be drawn from the approximate expressions for the two-photon laser. For large n, we can write, following Ref. 16

$$\prod_{j=0}^{n}(1-j^2Q_2) = \exp\sum_{j=0}^{n}\ln(1-j^2Q_2) \approx \exp\left(\int_0^n dj\,\ln(1-j^2Q_2)\right), \tag{29}$$

where $Q_2=(B_2/A_2)F(x_2,y_2)$. On evaluating the integral in Eq. (29), and substituting the result in Eq. (18), we obtain

$$\rho(n,n) = [P_2^n e^{-n}(1-n\sqrt{Q_2})^{n-1/\sqrt{Q_2}}] \times [e^{-n}(1+n\sqrt{Q_2})^{n+1/\sqrt{Q_2}}], \tag{30}$$

where $P_2 = (A_2/C)U(x_2, y_2)$. It may be noted here that the term in the first square brackets on the right-hand side of Eq. (30) represents the one-photon laser photon statistics[12,16] for large n with the peak of the distribution at $(1 - P_2^{-1})Q^{-1/2}$. For large n, the term in the second square brackets on the right-hand side of Eq. (30) can be written

$$e^{-n}(1 + n\sqrt{Q_2})^{n+1/\sqrt{Q_2}} \approx e^{-n}(\sqrt{Q_2})^n n^n . \quad (31)$$

Thus we find that the distribution function for the TM two-photon laser [Eq. (30)] is a ratio of two distribution functions. The numerator represents the distribution functions for one-photon laser, and the denominator represents the Poisson distribution for photons in Stirling's approximation.

Following the same method, the photon statistics (24) for the SM case can be written for large n

$$\rho(2n, 2n) = \{P_1^n e^{-n}(1 - n2\sqrt{Q_1})^{n-1/2\sqrt{Q_1}} \times [e^{-n}(2\sqrt{Q_1})^n n^n]\} (1 - 4n^2 Q_1)^{-1/4} , \quad (32)$$

where $P_1 = (A_1/C)U(x_1, y_1)$ and $Q_1 = (B_1/A_1)F(x_1, y_1)$. In this case also we find that the terms in the curly brackets in Eq. (32) are a ratio of two distribution functions. The numerator of this ratio represents the one-photon laser photon statistics at large n with the peak of the distribution occurring at $(1 - P_1)^{-1}(2\sqrt{Q_1})^{-1}$ and the denominator represents the Poisson distribution for photons in Stirling's approximation. Here the difference from the TM case is the presence of $(1 - 4n^2 Q_1)^{-1/4}$, which increases with n. It follows that this term in Eq. (32) makes the distribution line in the SM case broader than in the TM case.

We have derived the photon statistics of a two-photon laser and shown that the distribution function can be written as the ratio of one-photon laser photon statistics to the Poisson distribution for photons. It seems that this form of the photon statistics makes the two-photon-laser photon distribution line narrower than the one-photon-laser photon distribution line. However, a less restrictive theory that lifts the detailed-balance condition[2] will shed more light on the problem. The results of the present investigation may be helpful in exploring the possibilities of the construction of a two-photon laser which, to the best of our knowledge, is yet to be explored.

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