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Bohm-Aharonov experiment without an electron microscope

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A new-type Bohm-Aharonov experiment is proposed. The experiment can be carried out in a large scale without the help of an electron microscope.

Recent theoretical activities on the magnetic monopole and non-Abelian gauge fields[1] have created renewed interest in the famous Bohm-Aharonov experiment,[2] which was first performed by Chambers.[3] This experiment is basically an interference experiment involving two coherent electron beams. Its objective is to observe the phase shift of the electron wave function by altering the amount of magnetic flux passing between two coherent electron beams, even though the beams themselves pass only through magnetic-field-free regions.

So far there are only two types of known electron interferometers which have been proved successful. In the Marton-type interferometer,[4] the electron beam traverses in succession three thin equally spaced crystalline laminae. This is a rough equivalent of an optical interferometer of the Mach-Zehnder type. In the Möllenstedt-Düker interferometer,[5] the coherent electron beams are produced by macroscopic electric fields. This instrument is an electron-optical analog of the Fresnel biprism.

There are certain stringent restrictions known to experimenters working in electron-interference experiments.[6] To obtain reasonable penetration into the diffractor and to avoid perturbations by stray laboratory magnetic fields, it is highly desirable to work at electron energies of the order of 40 keV. At this energy the electron wavelength corresponds to 0.06 Å. The difficulties associated with such short wavelength are numerous. First, the precise alignment required in electron-interference experiments are several orders of magnitude beyond those familiar to light opticians. Secondly, the apparatus slits, size of source, "biprism," etc., must be scaled down by a factor of almost 10^5 from those of light optics. Thirdly, to have a visible interference pattern, the angle between the two interfering electron beams in radians must be less than $10^{-9}/d$, where d in centimeters is the least-resolved distance in the electron microscope.

The successful Bohm-Aharonov experiments were performed with the use of a Möllenstedt-Düker interferometer.[3] In the Chambers experiment, the field was produced by an iron whisker about 1 μm in diameter. Möllenstedt *et al.*[7] have used a miniature solenoid about 15 μm in diameter. While Chambers had observed only the interference fringe shifts, Möllenstedt was able to measure the magnetic-flux period within 14% of the true value. The result obtained by Boersch is slightly better.

Does the Bohm-Aharonov experiment need to be as difficult to perform as the electron-interference experiment seems to imply? Not necessarily so. The purpose of the electron-interference experiment is to obtain an interference pattern. The main objective of the Bohm-Aharonov experiment is to measure the phase shifts of two coherent electron beams. Of course, through the observation of the interference patterns one can deduce directly phase shifts. This, however, is not the only way to deduce phase shifts. It has been shown theoretically that through a crystal-diffraction experiment involving two coherent beams, one can deduce the relative phase shift between beams by measuring the intensity variation of a diffracted beam. Through this method, experiments were successfully performed for measuring phase shifts in coherent x rays[8] and neutron beams.[9] In these experiments the beam energy, crystal, and crystal orientation are properly chosen such that only two diffracted beams are present. The measured intensity variation is owing to the interference between a zeroth-order and a first-order diffracted beam. The theoretical result[10] is more general and is reviewed here.

The incident coherent beams are described by the wave function

$$e^{i\vec{k}_1\cdot\vec{r}} + ae^{i\vec{k}_2\cdot\vec{r}}, \tag{1}$$

where a denotes the relative phase and amplitude of these two waves. The wave vector satisfies the relations

$$\vec{k}_1^2 = \vec{k}_2^2 = \vec{k}^2 \tag{2}$$

and

$$\vec{k}_1 - \vec{k}_2 = \vec{\kappa}\,, \tag{3}$$

where $\vec{k}$ is the wave vector of a diffracted beam and $\vec{\kappa}$ is a reciprocal-lattice vector of the crystal. The experimentally measured intensity is proportional to

$$(1+|a|^2)^{-1}|E|^2|G|^2\,, \tag{4}$$

where

$$E=f(\vec{k}_1;\vec{k})+af(\vec{k}_2;\vec{k}) \tag{5}$$

is the structure factor and G the lattice factor. The function $f(\vec{k}_1;\vec{k})$ or $f(\vec{k}_2;\vec{k})$ is the structure factor of the crystal in the diffraction of a single incident beam. In the Bohm-Aharonov experiment, the phase of the complex factor a depends linearly on the magnetic flux between two coherent interfering electron beams. A change in magnetic flux will cause $\cos^2$ intensity variation of the diffracted beam. The variation visibility depends on the noncoherent components such as crystal distortions and improper alignments. However, the period of the variation purely depends only on the phase of a. By observing the diffraction-intensity variation in the detector, one is relieved from the difficulties associated with the small fringe spacing of the interference pattern. With the above in mind, a Marton-type Bohm-Aharonov experiment is proposed.

First let us review a Marton-type experiment. In the original Marton interferometer, three crystals were used. The first one is a splitter. This is an amplitude division of two coherent beams. It is well known that interferometers having division of amplitude may be used with an extended source. Hence, there is no need for extreme constancy of source, size, or position, and both the stability and rigidity requirements are greatly relaxed in comparison with the Möllenstedt-Düker interferometer which employs wave-front division of the beam. The remaining two crystals are used as benders of the electron beams. The third one was particularly important in making the angle between the two interfering beams less than 10^{-3} rad. Otherwise these beams could not be accommodated by the aperture of a conventional electron-microscope lens at that time, which was necessary for enlarging the fringe spacing to a size at which the fringes could be resolved by a photographic plate. The small size of the angle had limited the separation of the beams to the order of 1 mm and the choice of the diffracted beams up to the first order. Chambers discarded the idea of a Marton-type experiment, which was based on the stability of the interference fringes in the presence of stray 60-cps laboratory magnetic field. Since his experiment was to observe the interference pattern with and without the magnetic flux, and the main effect of the flux is only to displace the line pattern without changing the interval structure, the stability of the Marton-type experiment would not be a convenient experiment for checking the quantum shift.

In the proposed experiment the functions of the first and second crystals remain the same as before. However, the third crystal is now used as an analyzer for the interfering electron beams. The intensity of a diffraction beam from the third crystal is measured through a detector. The arrangement is similar to that of the x-ray interferometer of Bonse and Hart,[8] and to the neutron interferometer of Rauch, Treimer, and Bonse[9] but without a giant-size crystal. Namely, the crystals are parallel and equally spaced. The identical crystals have the same orientation. For electron transmission, the crystals must be in the form of lamellae. The only remaining difficult part of the proposed experiment is to align these lamellae.

In the proposed experiment, the difficulties associated with the source are relaxed and the difficulties with the fringe spacing are removed. The stability is no longer a problem, but rather is an advantage. There are no restrictions on the angle between the two interference beams. It becomes possible to increase the separation of the beams and to use higher-order diffracted beams. The proposed experiment is basically a two-triple-diffraction experiment. We tend to think that the alignment is relatively easier than the one performed by Marton, Simpson, and Suddeth,[11] and the experiment can be carried out with the crystals spaced at large distance and the interference-beam separation at least 5 mm.

In an original Marton-type interference experiment,[11] the electron gun had a saturated emission of only 300 μA. A present-day field-emission electron gun has stable emission current of 10 A.[12] With the greater sensitivity of the electron detector over the photographic plate, a Bohm-Aharonov experiment of Marton type is relatively easy to perform today.[13]

With the present role played by the Bohm-Aharonov experiment in the foundations of electromagnetism and field theory, a direct, different, and better repetition of the experiment is extremely important.

Note added. The original Marton-type interferometer is based on the Fresnel diffraction of two coherent electron beams while the proposed interferometer of the present paper is based on the Fraunhofer diffraction. The characteristic differences between these two interferometers are discussed in a recent paper [Ming Chiang Li, Z.

Phys. B (to be published)]. In the original Marton-type interferometer, one observes the interference pattern of two coherent beams, which have been diffracted twice by three crystals. In the proposed interferometer, one measures a diffracted-beam intensity from the third crystal, which has been diffracted three times by these crystals. Since there is an intensity lost, however, in the original Marton-type interferometer, one has to perform a differential measurement. But, in the proposed interferometer, one performs the integrated measurement. Hence, the intensity lost does not pose a serious problem.

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[13] Through the Josephson tunneling, R. C. Jaklevic, J. J. Lambe, A. H. Silver, and J. E. Mercereau [Phys. Rev. Lett. 12, 274 (1964)] have observed the effect of a field-free static vector potential on Cooper pairs in superconductor. In the Bohm-Aharonov experiment, one wishes to study the same vector-potential effect on elementary electrons. Hence, the experiment is more fundamental in nature. Besides, the proposed experiment is basically the crystal diffraction of two coherent beams. The success of the proposed experiment implies that the phase of the atomic structure factor can be observed directly (see Ref. 10). The direct phase observation is one of the essential goals in crystallography.