Compton scattering in the presence of coherent electromagnetic radiation

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The presence of coherent electromagnetic radiation has a profound effect on the Compton scattering of x rays from electronic systems. Here, two electronic systems are considered—bound electrons in atoms and an electron gas. The scattering cross section is expressed in terms of a power series in even derivatives of the double integral of the electronic-momentum distribution function over $\vec{p}_\perp$, the planar momentum vector perpendicular to $\vec{k}$, the momentum transfer to the electron. It is shown that the effect offers an important experimental technique for measurements of details of electronic-momentum distribution in the scattering system.

I. INTRODUCTION

Compton scattering of x rays from atoms, molecules, and condensed matter is a very useful process for obtaining experimental information on the electronic-momentum distribution in the system.[1] Here an x ray scatters at a large-angle off the sample and knocks out an electron in such a manner that the electronic recoil is much larger than the binding energy. Under these conditions, the spectrum of the scattered x-ray photons is related to $g(\vec{p}\cdot\hat{k})$, where g is the double integral of the electronic-momentum distribution function n_p over $\vec{p}_\perp$, the planar momentum vector perpendicular to $\vec{k}$. Here $\vec{k}$ represents the momentum transfer to the scattered electron.

If the system be placed in the cavity of a laser, we find a remarkable modulation in the differential scattering cross section by the electric field of the laser due to absorptions and emissions through multiphoton processes. In this paper, we demonstrate this effect by considering two electronic systems, bound atomic electrons and a free electron gas. In Sec. II, the case of Compton scattering from bound electrons in hydrogenlike atoms in the presence of laser radiation is discussed. Here we assume that the laser field is small enough so that the bound-electron wave function is unaffected by the laser. The calculations are made for large momentum transfer to the atom, i.e., the impulse approximation is used. The two important small parameters for the scattering process are $\alpha=E_b/E_R$, the ratio of the binding energy of the electron to the recoil energy; and $\epsilon^2=e^2E_0^2/2m\omega_0^2E_R$, which essentially represents the ratio of the classical kinetic energy of the electron in the presence of the field to the recoil energy. The cross section for scattering is calculated under the limitations that these two parameters be small, and terms proportional to $\alpha\epsilon$ and of higher order are neglected. The results are expressed in power series in the field as an infinite sum containing even derivatives of the function g. It is shown that the modulation of the cross section by the laser field offers an important experimental technique for measurement of details of the electronic-momentum distribution in the atom.

In Sec. III, the scattering system is taken to be an electron gas. The formulation of Compton scattering from the electron gas in the presence of the laser field is presented and the inherent difficulties present in solving the problem in the presence of the electron-electron interaction are discussed. For illustrative purposes, the electrons are taken to be free and the scattering cross section is obtained as a power series in the field. The results for the cross section are shown to have importance for the measurements of the momentum distribution near the Fermi surface.

Finally, Sec. IV contains a brief discussion of the results obtained in this paper.

II. SCATTERING FROM BOUND ELECTRONS

Let the incoming x-ray beam be characterized by the frequency ω_1 and wave vector $\vec{k}_1$ while the scattering x ray has corresponding quantities ω_2 and $\vec{k}_2$. We assume the atomic system to be almost transparent to the x ray, and thus the scattering cross section is completely characterized by the energy transfer $\omega=\omega_1-\omega_2$ and the momentum transfer $\vec{k}=\vec{k}_1-\vec{k}_2$. When the momentum transfer $\vec{k}$ is much larger than the typical electronic momentum the ejected electrons will suffer large recoil. It is then that Compton scattering measures the momentum distribution of the electrons.

Before considering the effect of the laser radiation, we would summarize the basic idea behind the impulse approximation (IA) usually used[2] in calculating the Compton scattering from bound electrons. Consider for simplicity a one-electron atom. The scattering cross section is then porportional to

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$$d\sigma \propto \sum_f |\langle f|e^{i\vec{k}\cdot\vec{r}}|g\rangle|^2 \delta(E_f - E_b - \omega)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\, e^{i\omega t}\langle g|e^{iHt}e^{-i\vec{k}\cdot\vec{r}}e^{-iHt}e^{i\vec{k}\cdot\vec{r}}|g\rangle\,, \tag{1}$$

where $|g\rangle$ is the ground state of the electron and $|f\rangle$ the final state, with E_b the binding energy and E_f the final-state energy. H, the Hamiltonian operator for the electron, is given by

$$H = \frac{p^2}{2m} + V(r) \equiv H_0 + V(r)\,, \tag{2}$$

where $\vec{p}$ is the electronic momentum and $V(r)$ is the Coulomb potential in which it moves. The Hamiltonian operator e^{iHt} is expanded as

$$e^{iHt} = e^{iH_0t}e^{iVt}e^{-[H_0,V]t^2/2}\cdots. \tag{3}$$

The higher-order terms involve multiple commutators and are higher order in powers of the time t. An examination of Eq. (1) shows that those times that are of importance in the integration are of order $\omega^{-1} \sim \hbar/E_R$, where E_R is the recoil energy. When E_R is much larger than other characteristic energies associated with the ground state of the electron, i.e., typically, $\alpha \equiv E_b/E_R \ll 1$, one can set $\exp(-\frac{1}{2}[H_0, V]t^2) = 1$, etc. This results in $V(r)$ canceling out in Eq. (1) and we obtain,

$$d\sigma \propto \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\, e^{i\omega t}\,\langle g|e^{iH_0t}e^{-i\vec{k}\cdot\vec{r}}e^{-iH_0t}e^{i\vec{k}\cdot\vec{r}}|g\rangle\,. \tag{1'}$$

The integral is now simple to evaluate and the cross section can be expressed

$$\frac{d^2\sigma}{d\omega\, d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 (\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2 \times \sum_p |\psi_p|^2 \delta(E_{p+k} - E_p - \omega)\,. \tag{4}$$

Here $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are the incoming and outgoing x-ray polarizations, $E_p = p^2/2m$, and ψ_p is the Fourier transform of $|g\rangle$. From comparing Eqs. (1) and (4), it is clear that the IA is the semiclassical approximation for the scattering process. Here, we consider the scattering time $\hbar/E_R$ to be so short in comparison with the electron period that the x ray is scattered from a freelike electron moving with a definite momentum $\vec{p}$ with probability $|\psi_p|^2$, which recoils to a state $\vec{p}+\vec{k}$. The character of the bound electron manifests itself through $|\psi_p|^2$ which is the probability for the electron to find itself in a state $\vec{p}$.

We shall now apply the above theory to the situation in which the Compton scattering takes place in the presence of a circularly polarized laser field characterized by a vector potential $\vec{A}(t)$ taken to be space independent in the dipole approximation. In principle, the ground state $|g\rangle$ of the electron would be modulated by the laser. However if the laser strength is such that its electric field is small compared to the atomic field experienced by the electron, one can effectively neglect the modulation of the electronic-bound state by the laser and simply take the state to be $|g\rangle$. There would be, however, a small constant shift ΔE_b in the ground-state energy E_b of the bound electron. This second-order energy shift of the ground state is obtained as[3,4]

$$\Delta E_b = e^2A_0^2/2mc^2\,, \tag{5}$$

where A_0 is the laser vector-potential amplitude. Thus the energy of the ground state becomes $E_b + \Delta E_b$ while the time-independent wave function for the ground state, taken to be the $1S$ state for a one-electron atom, is taken to be unperturbed by the laser.

On the other hand, the laser field will greatly influence the recoiling electron. It is difficult to obtain in closed form the influence of the laser field on the fast-moving electron to all orders in the field. In what follows, we shall consider the laser field to be small and obtain the results for the scattering cross section as a power series in the field. The appropriate small parameter, introduced before, is the dimensionless field intensity

$$\epsilon^2 = e^2E_0^2/2m\omega_0^2E_R. \tag{6}$$

This parameter is the relevant small-field parameter in other studies involving interaction of laser radiation with electrons, i.e., heating of a gaseous plasma by multiphoton inverse bremsstrahlung[5] and x-ray absorption in atoms in the presence of an intense laser field.[3,4] (In Ref. 3, the effect of the laser on the absorption cross section is, to the lowest order, proportional to $\mathcal{E}^2/T$ which, after removing atomic parameters, reduces to ϵ^2 used above.)

In what follows, we shall calculate the scattering cross section under the limitation that the two small parameters α and ϵ^2 be small, and neglect terms proportional to $\alpha\epsilon$ and other higher-order terms.

In the presence of the laser, the final-state wave functions for the electron in the continuum would be solutions of the time-dependent Schrödinger equation,

$$\left[\frac{1}{2m}\left(\vec{q} - \frac{e}{c}\vec{A}(t)\right)^2 + V(r)\right]\phi = i\frac{\partial\phi}{\partial t}\,. \tag{7}$$

Making a transformation to an oscillating coordinate system,[6] we let

$$\phi=\psi\exp\left(-\frac{ie}{mc}\int_0^t \vec{q}\cdot\vec{A}(\tau)\,d\tau-\frac{ie^2}{2mc^2}\int_0^t A^2\,d\tau\right) \tag{8}$$

and find

$$\left[\frac{q^2}{2m}+V\left(\vec{r}+\frac{e}{mc}\int_0^t \vec{A}\,dt\right)\right]\psi=i\frac{\partial\psi}{\partial t}. \tag{9}$$

One notices that, in Eq. (9), the laser field disappears from the electronic coordinates. The appearance of the field in the potential term shows that, in the coordinate system of the electron, the ion appears to be oscillating with the frequency of the field. An exact solution of this equation seems difficult to obtain. However, we limit ourselves to energy eigenvalues much larger than the Coulomb potential, i.e., $\alpha \ll 1$. Hence, the corrections to the Coulomb potential because of the laser field are of order $\alpha\epsilon$ and are neglected. Thus we simply take ψ functions to be the continuum wave functions in the Coulomb field and obtain

$$\phi\simeq\exp\left(-\frac{ie}{mc}\int_0^t \vec{\nabla}\cdot\vec{A}\,d\tau-\frac{ie^2}{2mc^2}\int_0^t A^2\,d\tau+i\vec{q}\cdot\vec{r}-iE_q t\right)\left(\frac{2\pi}{qa}\right)^{1/2}(1-e^{-2\pi/qa})F(i/qa,1,i(qr-\vec{q}\cdot\vec{r}))$$

$$\equiv\exp\left(-\frac{ie}{mc}\int_0^t \vec{\nabla}\cdot\vec{A}\,d\tau-\frac{ie^2}{2mc^2}\int_0^t A^2\,d\tau-iE_q t\right)|\bar{q}\rangle, \tag{10}$$

where F is the confluent hypergeometric function and E_q is the final-state energy. In writing Eq. (10), we have made approximations to the extent that the gradient operator operates only on $e^{i\vec{q}\cdot\vec{r}}$. ∇ operating on F gives terms proportional to $\alpha\epsilon$ which are dropped relative to order ϵ. In the same spirit, we shall take the Hamiltonian H to commute with $\vec{\nabla}\cdot\vec{A}$. Thus, the matrix element of transition is written

$$M_{if}=\int dt\,\langle 1S|\exp[i(E_b+\Delta E_b)t]\exp(i\vec{k}\cdot\vec{r}+i\omega t)\exp\left(-\frac{ie}{mc}\int\vec{\nabla}\cdot\vec{A}\,d\tau-i\frac{e^2A_0^2t}{2mc^2}\right)e^{-iE_qt}|\bar{q}\rangle$$

$$\simeq\int dt\,e^{i\omega t}\langle 1S|e^{iHt}e^{i\vec{k}\cdot\vec{r}}e^{-iHt}\exp\left(-\frac{ie}{mc}\int\vec{\nabla}\cdot\vec{A}\,d\tau\right)|\bar{q}\rangle. \tag{11}$$

Using the impulse approximation discussed before, we write

$$e^{iHt}=e^{iH_0t}e^{iVt} \tag{12}$$

for large recoil energy ($\alpha\ll 1$) and get

$$M_{if}=\int dt\,e^{i\omega t}\langle 1S|e^{iH_0t}e^{i\vec{k}\cdot\vec{r}}e^{-iH_0t}\times\exp\left(-\frac{ie}{mc}\int\vec{\nabla}\cdot\vec{A}\,d\tau\right)|\bar{q}\rangle. \tag{13}$$

Using a complete set of momentum eigenfunctions for the state $|1S\rangle$, we take

$$|1S\rangle=\sum_p \psi_p e^{i\vec{p}\cdot\vec{r}}=\sum_p \psi_p|\vec{p}\rangle,$$

and obtain

$$M_{if}=\int dt\,e^{i\omega t}\sum_p\exp\left(i\frac{p^2}{2m}t\right)\exp\left(-i\frac{(\vec{p}+\vec{k})^2}{2m}t\right)\times\exp\left(-\frac{ie}{mc}\int_0^t(\vec{k}+\vec{p})\cdot\vec{A}\,d\tau\right)\times\psi_p\langle\vec{p}+\vec{k}|\bar{q}\rangle.$$

For large recoil, $\vec{p}$ can be neglected in comparison with $\vec{k}$ in the modulation term. Performing the time integration, with ω_0 as the laser frequency, we get

$$M_{if}=\int dt\,e^{i\omega t}\sum_p\exp\left(i\frac{p^2}{2m}t\right)\exp\left(-i\frac{(\vec{p}+\vec{k})^2}{2m}t\right)\times\exp\left(\frac{ie}{mc\omega_0}\vec{k}\cdot\vec{A}_0\cos\omega_0 t\right)\times\psi_p\langle\vec{p}+\vec{k}|\bar{q}\rangle.$$

Using the identity

$$e^{i\lambda\cos\sigma}=\sum_{n=-\infty}^{n=+\infty}i^nJ_n(\lambda)e^{in\sigma}$$

$$M_{if}=\int dt\,e^{i\omega t}\sum_p\exp\left(i\frac{p^2}{2m}t\right)\exp\left(-i\frac{(\vec{p}+\vec{k})^2}{2m}t\right)\times\sum_n i^nJ_n\left(\frac{e\vec{k}\cdot\vec{A}_0}{mc\omega_0\hbar}\right)e^{in\omega_0t}\times\psi_p\langle\vec{p}+\vec{k}|\bar{q}\rangle. \tag{14}$$

The transition probability per unit time is therefore obtained,

$$\frac{1}{T}\sum_q |M_{if}|^2 = 2\pi\hbar \sum_p \sum_n \delta\left(\omega - \frac{k^2}{2m} - \frac{\vec{k}\cdot\vec{p}}{m} + n\omega_0\right) \times J_n^2\left(\frac{e\vec{k}\cdot\vec{A}_0}{mc\omega_0\hbar}\right)|\psi_p|^2,$$

and the differential cross section now reads

$$\frac{d^2\sigma}{d\omega\, d\Omega} = r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2) \times \sum_n \int |\psi_p|^2 d\vec{p}\, \delta\left(\omega - \frac{k^2}{2m} - \frac{\vec{k}\cdot\vec{p}}{m} + n\omega_0\right) \times J_n^2\left(\frac{e\vec{k}\cdot\vec{A}_0}{mc\omega_0\hbar}\right), \tag{15}$$

where $r_0 = e^2/mc^2$ and $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are the incoming and the outgoing x-ray polarizations, respectively.

From Eq. (15), it is clear that only the component of $\vec{A}_0$ parallel to $\vec{k}$ is effective in modulating the outgoing electron. If $\vec{k}$ be taken to be along the z direction, the δ function becomes independent of angular variations, and the equation simplifies to

$$\frac{d^2\sigma}{d\omega\, d\Omega} = r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2 \times \sum_n \int |\psi_p|^2 d\vec{p}\, \delta\left(\omega - \frac{k^2}{2m} - \frac{kp_z}{m} + n\omega_0\right) \times J_n^2(\lambda), \tag{16}$$

where $\lambda = eE_0k/m\omega_0^2\hbar$, with E_0 representing the component of the laser field along z direction. To present the results in terms of dimensionless parameters, we normalize the momenta in terms of the momentum transfer k and the energies in terms of the recoil energy $E_R = k^2/2m$. Letting $\eta = p_z/k$, the double integral of $|\psi_p|^2$ over the planar momentum in the x-y plane is defined as the dimensionless quantity

$$g(\eta) = \frac{1}{k^2}\int p_\perp dp_\perp |\psi_p|^2 . \tag{17}$$

Using the δ function in Eq. (16) to carry out the p_z integration, the cross section becomes

$$\frac{d^2\sigma}{d\omega\, d\Omega} = 2\pi r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2 mk \times \sum_n J_n^2(\lambda) g(\eta)\bigg|_{\eta = (\hbar\omega/E_R - 1)/2 + n\hbar\omega_0/2E_R} . \tag{18}$$

The infinite sum over n is difficult to perform analytically. However, for large recoil energy, $\hbar\omega_0/E_R \ll 1$ and one can expand $g(\eta)$ in a Taylor series around $\eta_0 \equiv \frac{1}{2}(\hbar\omega/E_R - 1)$. For J_n^2 functions, we use a power-series expansion and the cross section can be written in a formal double summation

$$\frac{d^2\sigma}{d\omega\, d\Omega} = 2\pi r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2 mk \times \sum_{l=0}^{\infty} \frac{\epsilon^{2l} 2l!}{2^{2l} l!^2} \sum_{j=0}^{\infty}\left(\frac{\hbar\omega_0}{2E_R}\right)^{2j} \frac{1}{(2j+2l)!} \times \frac{d^{2j+2l}g(\eta)}{d\eta^{2j+2l}}\bigg|_{\eta_0},$$

where $\epsilon^2 = e^2E_0^2/2m\omega_0^2E_R$ is the small parameter representing dimensionless intensity. For $\hbar\omega_0 \ll E_R$, it suffices to keep only the lowest order terms in $\hbar\omega_0$. Therefore the j summation drops out to give

$$\frac{d^2\sigma}{d\omega\, d\Omega} = 2\pi r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2 mk \sum_{l=0}^{\infty} \frac{\epsilon^{2l}}{2^{2l} l!^2} \frac{d^{2l}g(\eta)}{d\eta^{2l}}\bigg|_{\eta=\eta_0} . \tag{19}$$

Thus the differential cross section is expressed as an infinite sum containing even derivatives of the function $g(\eta)$. In the absence of the laser field, only the $l = 0$ term contributes and the expression reduces to the usual Compton-scattering formula.

In order to demonstrate the usefulness of the result given in Eq. (19), we assume that an experimental method of differentiating with respect to the intensity of the laser beam $I \sim E_0^2$ is available and we can measure

$$\frac{d}{dI}\left(\frac{d^2\sigma}{d\omega\, d\Omega}\right)\bigg|_{I\to 0} .$$

An examination of Eq. (19) shows that the modulation of the differential cross section by the laser field (to the lowest order in ϵ^2) is proportional to the second derivative of $g(\eta)$, which is a more sensitive function, in general, to changes in the frequency shift of the outgoing x-ray photon than the function $g(\eta_0)$ itself. Thus, many small variations in $g(\eta_0)$ which might not be obvious in the $g(\eta_0)$ vs η_0 plot, which is conventionally used, would be observed much more prominently by studying

$$\frac{d}{dI}\left(\frac{d^2\sigma}{d\omega d\Omega}\right)\bigg|_{I\to 0}$$

as a function of η_0.

For the $1S$ hydrogenic state presently being considered,

$$|\psi_p|^2 = \frac{1}{(1 + a^2p^2/\hbar^2)^4} = \frac{1}{[1 + a^2(p_\perp^2 + \eta^2k^2)/\hbar^2]^4} . \tag{20a}$$

Using Eq. (17), one obtains

$$g(\eta) = \frac{\alpha}{6}\frac{1}{(1+\eta^2/\alpha)^3} , \tag{20b}$$

where $\alpha = E_b/E_R = \hbar^2/a^2k^2$. A substitution in Eq. (19) results in

$$\frac{d^2\sigma}{d\omega\, d\Omega} = 2\pi r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)^2\,\frac{m}{6ak^2}\left[1+\frac{1}{4\alpha}\left(\frac{\hbar\omega}{E_R}-1\right)^2\right]^{-3} \times\left(1+\tfrac{3}{2}\epsilon^2\,\frac{\frac{7}{4}(\hbar\omega/E_R-1)^2-\alpha}{[\alpha+\frac{1}{4}(\hbar\omega/E_R-1)^2]^2}\right). \tag{21}$$

Thus, one gets

$$\frac{d}{dI}\left(\frac{d^2\sigma}{d\omega\, d\Omega}\right)\Bigg|_{I\to 0}\sim\frac{d^2g(\eta)}{d\eta^2}\Bigg|_{\eta_0}=\frac{7\eta_0^2/\alpha-1}{(1+\eta_0^2/\alpha)^5}. \tag{22}$$

Fig. 1 shows a plot of $d^2g(\eta)/d\eta^2|_{\eta_0}$ as a function of $\eta_0/\alpha^{1/2}$. For comparison, $g(\eta_0)/\alpha$ is also shown on the same graph. Since both of these functions are symmetric about $\eta_0=0$, only positive values of η_0 are shown. It is clear that small variations in $g(\eta_0)$ from the form given in Eq. (20b) would be amplified many times in $d^2g(\eta)/d\eta^2|_{\eta_0}$, and thus can be observed much more accurately.

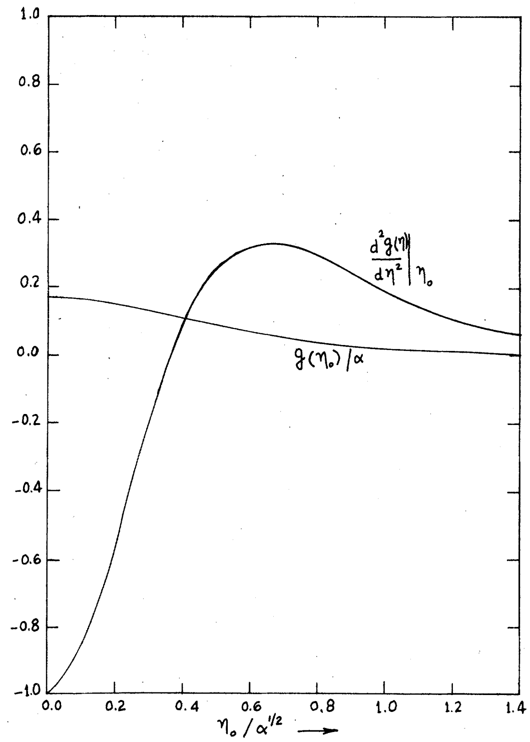

FIG. 1. $d^2g(\eta)/d\eta^2|_{\eta_0}$ and $g(\eta_0)/\alpha$ plotted as a function of $\eta_0/\alpha^{1/2}$ for positive values of η_0 for scattering from bound electrons.

III. SCATTERING FROM AN ELECTRON GAS

We now consider the effect of laser modulation on x-ray Compton scattering from an electron gas. If the frequency of the incoming x-ray beam is much higher than the plasma frequency, i.e., $\omega_1 \gg \omega_p$ and $E_F \ll \hbar\omega_1 < mc^2$, where E_F is the Fermi energy, then the differential scattering cross section per unit volume of the plasma is written as[7,8]

$$\frac{d^2\sigma}{d\omega\, d\Omega}=r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)\sum_p\sum_{p'}\frac{1}{T}\int_{-T/2}^{T/2}dt\int_{-T/2}^{T/2}dt'\,\frac{e^{-i\omega(t-t')}}{2\pi}\langle a_p^\dagger(t)a_{p+k}(t)a_{p'+k}^\dagger(t')a_{p'}(t')\rangle, \tag{23}$$

where a_p and $a_p^\dagger$ are, respectively, the annihilation and the creation operators for an electron in a momentum state $\vec{p}$. The symbol $\langle\,\rangle$ represents, for zero temperature, the ground-state expectation value and, for finite temperature, the usual statistical average. For zero temperature, the ground state of the electron gas is completely described by the momentum distribution $n_p=\langle a_p^\dagger a_p\rangle$.

If the gas is assumed noninteracting, n_p is of course unity for $p<p_F$, the Fermi momentum, and equals zero for $p>p_F$. In an interacting electron system, collisions will produce a smearing of the distribution.

For large momentum transfer k, $|p+k|$, $|p'+k| \gg p_F$. This implies that $n_{p+k}=n_{p'+k}=0$. Thus, to a very good approximation, the fast particle behaves like a free particle and, in the presence of the laser field, it will be modulated like a plane wave. Therefore, we can write

$$a_{p+k}(t)\simeq a_{p+k}\exp(-i\epsilon_{p+k}t)\times\sum_n J_n\left(\frac{e}{mc\hbar\omega_0}\vec{A}_0\cdot(\vec{p}+\vec{k})\right)e^{in\omega_0 t}, \tag{24}$$

where $\epsilon_{p+k}=(p+k)^2/2m$.

Using Eq. (24), one gets

$$\langle a_p^\dagger(t)a_{p+k}(t)a_{p'+k}^\dagger(t')a_{p'}(t')\rangle \simeq \langle a_p^\dagger(t)a_{p'}(t')a_{p+k}a_{p'+k}^\dagger\rangle e^{-i\epsilon_{p+k}t}\times e^{i\epsilon_{p'+k}t}\sum_n J_n\left(\frac{e\vec{A}_0\cdot(\vec{p}+\vec{k})}{mc\hbar\omega_0}\right)e^{in\omega_0 t}\times\sum_m J_m\left(\frac{e\vec{A}_0\cdot\vec{p}'+\vec{k}}{mc\hbar\omega_0}\right)e^{-im\omega_0 t'}.$$

The operator $a_{p'+k}^\dagger$ operating on the ground state of the system creates with probability 1 an electron of momentum $p'+k$ (since the state $p'+k$ is unoccupied). The operator a_{p+k} must annihilate with unit probability this fast particle so that the matrix element vanishes unless $p=p'$. Therefore, we obtain,

$$\langle a_p^\dagger(t)a_{p+k}(t)a_{p'+k}^\dagger(t')a_{p'}(t')\rangle = \langle a_p^\dagger(t)a_p(t')\rangle \exp[-i\epsilon_{p+k}(t-t')] \times \sum_{n,m} J_n\left(\frac{e\vec{A}_0\cdot\vec{p}+\vec{k}}{mc\hbar\omega_0}\right) J_m\left(\frac{e\vec{A}_0\cdot\vec{p}+\vec{k}}{mc\hbar\omega_0}\right) \times \exp(in\omega_0 t - im\omega_0 t'). \quad (25)$$

The quantity $\langle a_p^\dagger(t)a_p(t')\rangle$ is difficult to calculate in the presence of both the electron-electron interactions and the laser field. Even in the absence of the laser, evaluation of $\langle a_p^\dagger(t)a_p(t')\rangle$ requires the solution of an integral equation. In the presence of laser radiation, the problem becomes significantly more complicated. Since the laser field is periodic in time, one ends up with infinitely coupled integral equations because any time-dependent function having a Fourier component Ω in the absence of laser field will now be coupled to all frequency components $\Omega+n\omega_0$. That treatment is beyond the scope of the present discussion and will be dealt with in a subsequent publication. Here, for illustrative purposes, we calculate the effect of laser modulation on Compton scattering from the electron gas, taking the electrons to be free. This is certainly correct for systems having weak electron-electron interaction and when the effect of the laser field is much larger than the electron-electron correlation energy.

Thus, as in Eq. (24), we write

$$a_p(t') = a_p e^{-i\epsilon_p t'} \sum_{m'} J_{m'}\left(\frac{e\vec{A}_0\cdot\vec{p}}{mc\hbar\omega_0}\right) e^{im'\omega_0 t'},$$

and obtain

$$\langle a_p^\dagger(t)a_p(t')\rangle = \langle a_p^\dagger a_p\rangle e^{i\epsilon_p(t-t')} \times \sum_{n',m'} J_{m'}\left(\frac{e\vec{A}_0\cdot\vec{p}}{mc\hbar\omega_0}\right) J_{n'}\left(\frac{e\vec{A}_0\cdot\vec{p}}{mc\hbar\omega_0}\right) \times \exp(im'\omega_0 t' - in'\omega_0 t).$$

Substituting in Eq. (25), one gets

$$\langle a_p^\dagger(t)a_{p+k}(t)a_{p'+k}^\dagger(t')a_{p'}(t')\rangle = n_p \exp[i(\epsilon_p-\epsilon_{p+k})(t-t')] \times \sum_{l,s} J_l\left(\frac{e\vec{A}_0\cdot\vec{k}}{mc\hbar\omega_0}\right) J_s\left(\frac{e\vec{A}_0\cdot\vec{k}}{mc\hbar\omega_0}\right) \times \exp(il\omega_0 t - is\omega_0 t'), \quad (26)$$

where we have made use of the identity

$$J_n(x-y) = \sum_{k=-\infty}^{+\infty} J_{n+k}(x)J_k(y).$$

Substitution of Eq. (26) in Eq. (23) followed by time integrations results in

$$\frac{d^2\sigma}{d\omega\, d\Omega} = r_0^2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2)\int d\vec{p}\sum_n \delta\left(\omega+n\omega_0-\frac{k^2}{2m}-\frac{\vec{p}\cdot\vec{k}}{m}\right) \times n_p J_n^2\left(\frac{e\vec{A}_0\cdot\vec{k}}{mc\hbar\omega_0}\right). \quad (27)$$

A comparison of Eq. (27) with Eq. (15) obtained for the case of scattering from bound electrons shows that the scattering cross section for free electrons is obtained from Eq. (15) simply by replacing $|\psi_b|^2$ by n_p, the momentum distribution for the free electron gas. Thus, following the same procedure, one would obtain for the cross section the form given in Eq. (19) with $g(\eta)$ now written

$$g(\eta) \equiv \frac{1}{k^2}\int p_\perp dp_\perp n_p. \quad (28a)$$

Using the Fermi-Dirac distribution function at finite temperature,

$$n_p = \frac{1}{\exp[\beta(p^2/2m - E_F)]+1}, \quad (28b)$$

where $\beta = 1/k_B T$. E_F, the Fermi energy now takes the place of E_b for the bound electrons so that $\alpha \equiv E_F/E_R$. Using Eq. (28b) in Eq. (28a), one gets

$$g(\eta) = \frac{\alpha}{2\beta E_F}\ln\{1+\exp[\beta E_F(1-\eta^2/\alpha)]\}. \quad (29)$$

A substitution of the above expression for $g(\eta)$ in Eq. (19) yields

$$\frac{d^2\sigma}{d\omega d\Omega} = 2\pi r_0^2(\epsilon_1\cdot\epsilon_2)^2 mk\left[\frac{\alpha}{2\beta E_F}\ln\{1+\exp[\beta E_F(1-\eta_0^2/\alpha)]\} + \frac{\epsilon^2}{4}\left(-\frac{1}{1+\exp[-\beta E_F(1-\eta_0^2/\alpha)]} + \frac{2(\eta_0^2/\alpha)\beta E_F\exp[-\beta E_F(1-\eta_0^2/\alpha)]}{\{1+\exp[-\beta E_F(1-\eta_0^2/\alpha)]\}^2}\right) + \cdots\right]. \quad (30)$$

Thus,

$$\frac{d}{dI}\left(\frac{d^2\sigma}{d\omega\, d\Omega}\right)\bigg|_{I\to 0} \sim \frac{d^2 g(\eta)}{d\eta^2}\bigg|_{\eta=\eta_0} = \frac{-1}{1+\exp[-\beta E_F(1-\eta_0^2/\alpha)]} + \frac{2(\eta_0^2/\alpha)\beta E_F \exp[-\beta E_F(1-\eta_0^2/\alpha)]}{\{1+\exp[-\beta E_F(1-\eta_0^2/\alpha)]\}^2}. \tag{31}$$

Fig. 2 shows a plot of $d^2g(\eta)/d\eta^2|_{\eta_0}$ as a function of $\eta_0/\alpha^{1/2}$, as given by Eq. (31). For comparison, $g(\eta_0)/\alpha$ given by Eq. (29) is also shown on the same graph. Again, only positive values of η_0 are shown since the curves are symmetrical about $\eta_0 = 0$. The calculations were made for a temperature $T = 300\,°\text{K}$ and $E_F = 2.12$ eV (corresponding to Fermi energy for potassium). One notices that owing to finite temperature, the jump in $g(\eta_0)$ at the Fermi surface has almost smoothed out. However, this jump manifests itself as a δ-function-type discontinuity in $d^2g(\eta)/d\eta^2|_{\eta_0}$ curve and would be noticeable experimentally, even at high temperatures. Thus the effect discussed here suggests an invaluable technique for accurate measurements of the Fermi surface.

It should be noted that the laser light might not penetrate as deep as x rays in metals because of the conduction electrons. The mean free path of optical photons in metals is typically of the order of 500 Å while the effective travel length in and out for 8-keV x rays in solids having atoms with

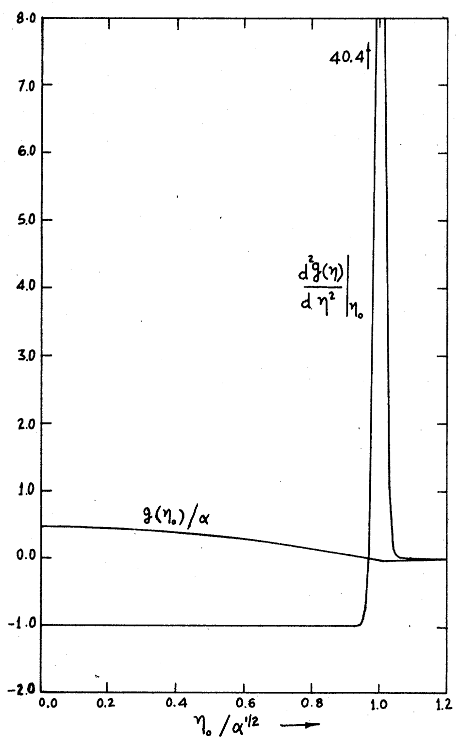

FIG. 2. $d^2g(\eta)/d\eta^2|_{\eta_0}$ and $g(\eta)/\alpha$ plotted as a function of $\eta_0/\alpha^{1/2}$ for positive values of η_0 for scattering from a free-electron gas at $T = 300\,°\text{K}$ and $E_F = 2.12$ eV (corresponding to potassium).

$Z \sim 20$ is estimated to be around 10 000 Å. Thus, for collinear geometry, the range of interaction of x rays and laser radiation for multiphoton process is reduced by an order of magnitude.

IV. DISCUSSION

In the previous sections, we obtained expressions for differential cross sections for Compton scattering of x rays from bound- and free-electron systems. The results were obtained as infinite series in powers of the laser electric field. The relevant small field parameter is given by $\epsilon^2 = e^2E_0^2/2m\omega_0^2E_R$.

In order to neglect the modulation of the ground-state wave function by the laser field, for the bound-electron case considered in Sec. II, one requires that the laser field be much smaller than the atomic field E_{atomic}. A characteristic field $E_0^c = (2m\omega_0^2E_R/e^2)^{1/2}$ is the field value for which $\epsilon^2 = 1$. It is easy to see that the condition $E_0^c \ll E_{\text{atomic}}$ would be valid for a wide variety of situations. For example, consider a Nd laser ($\omega_0 = 1.8\times 10^{15}$ sec^{-1}). Taking a value $E_R = 1$ keV, one obtains $E_0^c \sim 10^9$ V/cm, which can be much smaller than typical atomic fields. In reality, much smaller laser fields than E_0^c would be sufficient to observe the effect suggested here.

The other small parameter in the problem is α which equals E_b/E_R for the bound-electron case and E_F/E_R for the free electrons. Since for x-ray Compton scattering E_R is typically of the order of 1 keV, it is clear that $\alpha \ll 1$ for the situations usually encountered.

The results plotted in Figs. 1 and 2 are completely general with regard to the laser and the x-ray parameters. For both of the scattering systems discussed here, the graphs clearly demonstrate that many small variations in the electronic momentum distribution that barely affect $g(\eta_0)$ would be amplified many times in $d^2g(\eta)/d\eta^2|_{\eta_0}$.

In conclusion, we have shown in this paper that the presence of coherent electromagnetic radiation has a profound effect on the Compton scattering of x rays from bound electrons and a free-electron gas. The effect offers an important experimental technique for measuring details of the electronic momentum distribution and the Fermi surface.

ACKNOWLEDGMENT

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