Molecular-ion formation by $\alpha\mu^-$ and $\alpha\pi^-$ atoms in liquid helium*

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The experimentally determined intensities of x rays due to muonic and pionic atoms formed in gaseous and liquid helium are analyzed. The analysis is based largely, though not entirely, on some estimates of Auger rates. A discussion of Stark transitions, some estimates of the time required to slow a recoiling mesonic atom after it ejects an electron from a nearby atom in liquid helium, and a brief discussion of kaonic x-ray intensities are also employed. It is concluded that an $\alpha\mu^-$ or an $\alpha\pi^-$ atom in liquid helium probably becomes temporarily bound to a helium atom during its cascade to the $1s$ state. More specifically, it is argued that the relatively intense muonic $K\gamma$ line in gas, the relatively faint muonic and pionic $K\gamma$ lines in liquid, and the anomalously high $K\beta/K\alpha$ intensity ratio for pions in liquid indicate that molecular ions are probably formed in liquid but not in gas by $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n = 4$.

I. INTRODUCTION

The purpose of this paper is to show that molecular-ion formation (MIF) probably occurs *during* the atomic cascade that takes place after a negative muon or pion is stopped in liquid helium. The molecular ion that we have in mind is composed of an $\alpha\mu^-$ or an $\alpha\pi^-$ atom and a neutral helium atom. The $\alpha\mu^-$ or $\alpha\pi^-$ atom, which has a net positive charge, is composed of an α particle and a negative muon or pion. We shall devote particular attention to the possibility that MIF occurs when the mesonic atom is in a state with principal quantum number $n=4$.

The possibility that MIF might occur in liquid helium during the cascade appears to have been first suggested by Placci *et al.*[1] These authors measured the relative yields of x rays emitted by $\alpha\mu^-$ atoms in gaseous helium and found that the total relative intensity of the $K\gamma$ line and higher lines of the K series is significantly higher than in liquid. They noted that this difference in relative intensity might be accounted for if molecular ions are formed in liquid but not in gas. Presumably, for some p states with $n\geq 4$, the Auger rate would be relatively small unless MIF occurs, in which case it would be increased to such an extent that it becomes at least comparable to the radiative rate.

Some of the calculations that we shall describe in this paper require some knowledge of the structure of the molecular ion. The molecular ion is similar to the HeH^+ ion,[2] the essential difference being that the helium atom is bound to a small, positively charged mesonic atom instead of a proton. Except for differences due to its larger mass, the mesonic atom behaves chemically like a proton. We shall require a more or less detailed knowledge of the electronic structure of the molecular ion, but we shall not need to know very much about its rotational and vibrational motion. To the extent that the Born-Oppenheimer approximation is valid, the electron wave function and the interatomic potential for the molecular ion should be essentially the same as for the HeH^+ ion.

Some approximate Auger rates are computed in Sec. II. The discussion in Sec. II is devoted primarily to the intensities of the $K\gamma$ lines due to $\alpha\mu^-$ and $\alpha\pi^-$ atoms in liquid and gaseous helium. The results obtained in Sec. II support the conjecture of Placci *et al.*[1] that MIF occurs in liquid during the cascade. Section III is concerned with the implications of the anomalously high experimental value of the $K\beta/K\alpha$ intensity ratio for $\alpha\pi^-$ atoms in liquid helium.[3-5] It is argued that this result, too, indicates that MIF probably occurs in the $n=4$ level. Furthermore, it is suggested that irreversible Stark transitions (IST) are likely to play a role in any explanation of this anomalously high ratio. It is shown in Sec. IV that an $\alpha\mu^-$ or $\alpha\pi^-$ atom, after acquiring a relatively large recoil kinetic energy when it undergoes Auger deexcitation, is very likely to be slowed sufficiently rapidly in liquid helium to permit the occurrence of IST. It is argued in Sec. V that data on the αK^- atom[6] are compatible with the occurrence of IST and MIF in $\alpha\mu^-$ and $\alpha\pi^-$ atoms in liquid helium. Finally, some further experimental work is suggested in Sec. VI.

II. AUGER RATES

In this section we shall estimate mesonic Auger rates for a molecular ion of the type described in Sec. I. Rates will also be estimated for the external Auger transitions that can occur when a positively charged mesonic atom collides with an ordinary atom in liquid helium. The particular

Auger transitions that will be considered are restricted to dipole transitions in which the changes in the meson quantum numbers are $\Delta n = \Delta l = -1$. We shall compare the computed Auger rates with the rates for radiative transitions, and we shall conclude that the measured intensities of the muonic and pionic $K\gamma$ lines relative to the $K\alpha$ and $K\beta$ lines indicate that molecular ions are probably formed in liquid helium by $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n = 4$. The notation that will be employed was chosen so as to coincide as much as practicable with the notation employed in some earlier work on a related problem.[7]

A. Wave functions

It is convenient to refer to the α particle to which the meson is not bound as α_a and the one to which it is bound as α_b. The initial and final meson wave functions are given fairly accurately by hydrogenic functions for a negative meson bound to a free α particle. These wave functions will be denoted by

$$\phi_i(\vec{r}_\mu) = R_{n,l}(r_\mu) Y_{l,m}(\hat{r}_\mu), \tag{2.1a}$$

$$\phi_f(\vec{r}_\mu) = R_{n-1,l-1}(r_\mu) Y_{l-1,m'}(\hat{r}_\mu), \tag{2.1b}$$

where $\vec{r}_\mu$ is the position of the meson with respect to α_b. In each of the instances considered in the present paper, the mean orbital radius of the meson is small compared to the internuclear separation.

The unperturbed initial and final electron wave functions are assumed to be eigenfunctions of the Hamiltonian for two electrons moving in the combined field of an α particle and a proton. The nuclei are spaced a distance R apart. In the calculation described below, the exact electron wave functions are replaced by approximate functions. The initial electron wave function is assumed to be given approximately by

$$\Psi_i = A_1 \psi_a(1)\psi_a(2) + A_2[\psi_a(1)\psi_b(2) + \psi_a(2)\psi_b(1)], \tag{2.2}$$

where ψ_a and ψ_b are 1s hydrogenic wave functions centered, respectively, on α_a and α_b. The effective nuclear charges for ψ_a and ψ_b will be denoted by Z_a and Z_b. The amplitudes A_1 and A_2 and the effective charges Z_a and Z_b depend on R. An approximate wave function of this form was employed many years ago by Coulson and Duncanson[8] in an investigation of the HeH^+ molecular ion. They found the equilibrium internuclear separation for this ion to be $R_0 = 1.43a_0$, where a_0 is the hydrogen Bohr radius. The amplitudes and the effective nuclear charges for $R = R_0$ were found to be $A_1 = 0.655$, $A_2 = 0.293$, $Z_a = 1.93$, and $Z_b = 1.46$. For our purposes, it is convenient to make the simplifying assumption that

$$\left.\begin{array}{l} A_1 = 0.655, \quad A_2 = 0.293 \\ Z_a = 1.93, \quad Z_b = 1.46 \end{array}\right\} a_0 < R < 1.6a_0, \tag{2.3a}$$

$$\left.\begin{array}{l} A_1 = 1.00, \quad A_2 = 0.0 \\ Z_a = 1.69, \end{array}\right\} 1.6a_0 < R. \tag{2.3b}$$

We also assume that internuclear separations less than $1.0a_0$ are not possible because the molecular potential, which is discussed in somewhat more detail in Sec. III B 2, becomes highly repulsive for these values of R.

The final electron wave function is assumed to be given with sufficient accuracy by

$$\Psi_f = (1/\sqrt{2})\{\psi_c(1)\psi_{\vec{k}}^-(2) + \psi_c(2)\psi_{\vec{k}}^-(1)\}, \tag{2.4}$$

where ψ_c is a 1s hydrogenic function centered on α_a, and $\psi_{\vec{k}}^-$ is a distorted plane wave describing an ejected electron with wave vector $\vec{k}$. The effective nuclear charge for ψ_c will be denoted by Z_c. In principle, the function $\psi_{\vec{k}}^-$ is distorted by α_a, by the net positive charge of the atom composed of the meson and α_b, and by the remaining electron. In practice, we shall approximate it with an ordinary Coulomb distorted plane wave, which we shall write in the form

$$\psi_{\vec{k}}^-(\vec{r}_e) = \frac{4\pi}{kr_e} \sum_{L,M} i^L e^{i\sigma_L} F_L(kr_e) Y_{L,M}^*(\hat{k}) Y_{L,M}(\hat{r}_e), \tag{2.5}$$

where σ_L is a Coulomb phase shift and F_L is a regular Coulomb wave function. Depending on whether the overlap with the initial electron wave function involves ψ_a or ψ_b, it will be assumed that $\psi_{\vec{k}}^-$ is centered on α_a or α_b and has an effective nuclear charge denoted by Z_{ka} or Z_{kb}. It seems likely that a large error is not introduced by any of these approximations because, as discussed in somewhat more detail below, the computed Auger rates for the particular states considered here are not very sensitive to reasonable variations in the assumed values of Z_c, Z_{ka}, and Z_{kb}.

B. Computation of rates

The perturbing interaction can be written

$$H' = H_1' + H_2', \tag{2.6a}$$

where

$$H_1' = \frac{e^2}{r_{1\mu}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{a\mu}} + \frac{e^2}{R}, \tag{2.6b}$$

$$H_2' = \frac{e^2}{r_{2\mu}} - \frac{e^2}{r_{2b}} - \frac{e^2}{r_{a\mu}} + \frac{e^2}{R}. \tag{2.6c}$$

The subscripts 1 and 2, a and b, and μ refer, respectively, to the electrons, to α_a and α_b, and to the meson. Only the terms $e^2/r_{1\mu}$ and $e^2/r_{2\mu}$ can be responsible for Auger transitions. If, in a calculation of dipole Auger rates, the electrons and the meson are all described in a coordinate system centered on α_b, the interactions H_1' and H_2' can be replaced by expressions of the form

$$H_{bb}' = e^2 \frac{4\pi}{3} \frac{r_\mu}{r_e^2} \sum_p Y_{1,p}(\hat{r}_\mu) Y_{1,p}^*(\hat{r}_e). \tag{2.7}$$

However, in some instances it is more convenient to describe the electrons in a coordinate system centered on α_a while describing the meson in a system centered on α_b, in which case H_1' and H_2' can be replaced by approximate expressions of the form[9]

$$H_{ab}' \simeq \frac{e^2}{R^3} r_e r_\mu \frac{4\pi}{3}\left(\sum_p Y_{1,p}(\hat{r}_\mu) Y_{1,p}^*(\hat{r}_e) - 3Y_{1,0}(\hat{r}_\mu) Y_{1,0}(\hat{r}_e)\right). \tag{2.8}$$

The preceding expression is fairly accurate if the internuclear separation R is large compared to the mean orbital radius of the initial electron.

For a fixed value of R, the Auger rate is given by

$$\gamma(R) = \frac{2\pi}{\hbar} \frac{m_e \hbar k}{(2\pi\hbar)^3} (2l+1)^{-1} \sum_{m,m'} \int |M|^2 d\Omega(\hat{k}), \tag{2.9}$$

where m_e is the mass of the electron. The integration is over all directions of the wave vector $\vec{k}$. The matrix element in Eq. (2.9) is given approximately by

$$M \simeq M_1 + M_2 + M_3, \tag{2.10a}$$

where

$$M_1 = \sqrt{2}(A_1 S_a + A_2 S_b) \times \int \psi_{\vec{k}}^*(\vec{r}_e) \phi_f^*(\vec{r}_\mu) H_{ab}' \psi_a(\vec{r}_e) \phi_i(\vec{r}_\mu) d\tau_e d\tau_\mu, \tag{2.10b}$$

$$M_2 = \sqrt{2} A_2 S_a \times \int \psi_{\vec{k}}^*(\vec{r}_e) \phi_f^*(\vec{r}_\mu) H_{bb}' \psi_b(\vec{r}_e) \phi_i(\vec{r}_\mu) d\tau_e d\tau_\mu, \tag{2.10c}$$

$$M_3 = \sqrt{2} A_2 S_{ka} \times \int \psi_c^*(\vec{r}_e) \phi_f^*(\vec{r}_\mu) H_{bb}' \psi_b(\vec{r}_e) \phi_i(\vec{r}_\mu) d\tau_e d\tau_\mu \tag{2.10d}$$

and

$$S_a = \int \psi_c^*(\vec{r}_e) \psi_a(\vec{r}_e) d\tau_e, \tag{2.10e}$$

$$S_b = \int \psi_c^*(\vec{r}_e) \psi_b(\vec{r}_e) d\tau_e, \tag{2.10f}$$

$$S_{ka} = \int \psi_{\vec{k}}^*(\vec{r}_e) \psi_a(\vec{r}_e) d\tau_e. \tag{2.10g}$$

The evaluation of S_a is trivial. The two-center overlap integral S_b can be evaluated analytically.[10] The angular part of S_{ka} can be evaluated analytically, and its radial part can be evaluated numerically.

It can be shown that

$$\gamma(R) = \gamma^{(1)}(R) + \gamma^{(2)}(R) + \gamma^{(3)}(R), \tag{2.11}$$

where $\gamma^{(1)}$, $\gamma^{(2)}$, and $\gamma^{(3)}$ are, respectively, the rates obtained by assuming that $M = M_1$, $M = M_2$, and $M = M_3$. Each of the interference terms $2\,\mathrm{Re}(M_1 M_2^*)$, $2\,\mathrm{Re}(M_1 M_3^*)$, and $2\,\mathrm{Re}(M_2 M_3^*)$ in the expression for $|M|^2$ gives a vanishing net contribution to γ. Each of the terms in Eq. (2.11) is proportional to the square of the radial dipole matrix element

$$a_0 \frac{m_e}{2m_\mu} \langle n-1, l-1 | r | n, l\rangle = \int_0^\infty R_{n-1,l-1} R_{n,l} r_\mu^3 \, dr_\mu, \tag{2.12}$$

where m_μ is the reduced mass of the meson and an α particle. The preceding equation can be regarded as the definition of the dimensionless matrix element $\langle n-1, l-1 | r | n, l\rangle$ that appears in one of the equations given below. The three terms in Eq. (2.11) are given by[11]

$$\gamma^{(1)} = 2(A_1 S_a + A_2 S_b)^2 (R/a_0)^{-6} Z_a^3 B I_a^2, \tag{2.13a}$$

$$\gamma^{(2)} = (A_2 S_a)^2 Z_b^3 B I_b^2, \tag{2.13b}$$

$$\gamma^{(3)} = (A_2 K_2'/S_b)^2 Z_a^3 B I_{ka}^2, \tag{2.13c}$$

where

$$B = \frac{4}{3}\left(\frac{m_e}{m_\mu}\right)^2 \frac{m_e c^2}{\hbar}\left(\frac{e^2}{\hbar c}\right)^2 \frac{2l}{2l+1} \times \langle n-1, l-1 | r | n, l\rangle^2 (a_0 k), \tag{2.13d}$$

$$I_a = \int_0^\infty \left(\frac{r_e}{a_0}\right)^3 \frac{F_1(kr_e)}{kr_e} \exp\left(-\frac{Z_a r_e}{a_0}\right) \frac{dr_e}{a_0}, \tag{2.13e}$$

$$I_b = \int_0^\infty \frac{F_1(kr_e)}{kr_e} \exp\left(-\frac{Z_b r_e}{a_0}\right) \frac{dr_e}{a_0}, \tag{2.13f}$$

$$I_{ka} = \int_0^\infty \left(\frac{r_e}{a_0}\right)^2 \frac{F_0(kr_e)}{kr_e} \exp\left(-\frac{Z_a r_e}{a_0}\right) \frac{dr_e}{a_0}, \tag{2.13g}$$

$$K_2' = S_b \int \psi_c^*(\vec{r}_e) \psi_b(\vec{r}_e) \frac{1}{r_e^2} \cos\theta_e \, d\tau_e. \tag{2.13h}$$

As mentioned previously, the nuclear charges for the Coulomb wave functions in the expressions for I_a, I_b, and I_{ka} are, respectively, Z_{ka}, Z_{kb}, and Z_{ka}. In the expression for K_2', the origin of the electron coordinate system is α_b, and α_a is situated on the positive z axis. An explicit expression for the integral K_2', which can be evaluated analytically, is given elsewhere.[12]

The Auger rate for a molecular ion should be given approximately by $\gamma(R_0)$, where R_0 is the equilibrium internuclear separation. We shall assume that $R_0 = 1.43a_0$, which is the value determined in Ref. 8. If a molecular ion has not been formed, and if the motion of the mesonic atom relative to a nearby helium atom can be treated classically, the collisional Auger rate should be given approximately by

$$\Gamma_{\rm Aug} = 4\pi N \int_{a_0}^{\infty} \gamma(R) R^2 dR, \tag{2.14}$$

where N is the number density of helium atoms. Arguments are given in Ref. 7 to support the adequacy of Eq. (2.14).

It will henceforth be assumed that

$$\gamma(R_0) \simeq \gamma^{(2)}(R_0), \tag{2.15}$$

because, as discussed in more detail below, the terms $\gamma^{(1)}(R_0)$ and $\gamma^{(3)}(R_0)$ are found to be relatively small for the low lying states considered in the present paper.[13] Table I list some values of $\gamma^{(2)}(R_0)$ for molecular ions formed with muons, pions, and kaons. Kaon rates have been included because some experimental work on the αK^- atom,[6] which will be discussed in Sec. V, appears to be relevant to some additional material on the cascade in $\alpha\mu^-$ and $\alpha\pi^-$ atoms that will be presented in Sec. III. Table I also lists approximate values of the collisional Auger rate in liquid helium obtained using the relation

$$\Gamma_{\rm Aug}^{(2)} = 4\pi N \int_{a_0}^{\infty} \gamma^{(2)}(R) R^2 dR, \tag{2.16}$$

with $N = 1.9 \times 10^{22}$ cm^{-3}. In computing $\gamma^{(2)}(R_0)$ and $\Gamma_{\rm Aug}^{(2)}$, the amplitude A_2 and the effective nuclear charges Z_a and Z_b were assumed to be given by Eq. (2.3). It was also assumed that $Z_c = 2.0$ and $Z_{kb} = 1.46$. Table I also lists values of the total radiative rate $\gamma_{\rm rad}$.

C. Discussion of approximations

Before attempting to relate these computed Auger rates to the experimental results, it seems appropriate to discuss some of the approximations that have been made. The rates $\gamma^{(2)}$ and $\Gamma_{\rm Aug}^{(2)}$ can be interpreted as being due to the ejection of an electron that is initially in the atomic orbital ψ_b. For a molecular ion, the best values of the amplitude and effective charge for this orbital are probably $A_2 = 0.293$ and $Z_b = 1.46$, as determined in Ref. 8. But for a collision between a positively charged mesonic atom and a neutral helium atom, the assumption that A_2 and Z_b may be adequately approximated by Eq. (2.3) is merely a guess which, we believe, cannot be given any further justification without an excessive amount of calculation. In the case of the ejected electron, assuming that the effective charge is $Z_{kb} = 1.46$, as we have done, is a way of roughly taking into account the distorting effect of the singly ionized helium atom that, in the final state, is situated a distance R from the positively charged mesonic atom on which $\psi_{\vec{k}}$ is assumed to be centered. However, in the region of largest overlap between $\psi_{\vec{k}}$ and ψ_b, the most appropriate value of Z_{kb} may very well be less than the particular value that we have chosen. For the states considered in Table I, it was found that employing an effective charge for the final electron within the range $1.0 \leq Z_{kb} < 1.46$, while keeping $Z_b = 1.46$, causes the computed values of $\gamma^{(2)}$ and $\Gamma_{\rm Aug}^{(2)}$ to be smaller, but by only ~5% at most. For a given value of Z_{kb} within this range, the magnitude of the decrease becomes smaller as the energy of the ejected electron becomes larger.

For given values of R, Z_c, Z_{ka}, and Z_{kb}, the ratios $\gamma^{(1)}/\gamma^{(2)}$ and $\gamma^{(3)}/\gamma^{(2)}$ depend only on the wave number k of the ejected electron. Values of the neglected terms $\gamma^{(1)}(R_0)$ and $\gamma^{(3)}(R_0)$ were computed for a number of representative transitions spanning the range of values of k relevant to Table I. In each instance it was assumed that the amplitudes A_1 and A_2 and the effective nuclear charges Z_a and Z_b, all of which depend on R, are given by Eq. (2.3). It was also assumed that $Z_c = 2.0$. In each instance it was found, for $1.0 \leq Z_{ka} \leq 1.93$, that

$$\gamma^{(1)}(R_0)/\gamma^{(2)}(R_0) \lesssim 4 \times 10^{-3},$$

$$\gamma^{(3)}(R_0)/\gamma^{(2)}(R_0) \lesssim 2 \times 10^{-4},$$

where $\gamma^{(2)}(R_0)$ is the rate given in Table I. These ratios decrease as k increases. Therefore, to the extent that the initial electron wave function can be adequately represented by the function Ψ_i specified in Eqs. (2.2) and (2.3), $\gamma^{(2)}(R_0)$ is probably a fairly accurate approximation to the $\Delta n = \Delta l = -1$ Auger rate for a molecular ion with relatively low n.

The contribution to the collisional Auger rate due to the term $\gamma^{(1)}(R)$ was also found to be relatively small. This contribution is

$$\Gamma_{\rm Aug}^{(1)} = 4\pi N \int_{a_0}^{\infty} \gamma^{(1)}(R) R^2 dR. \tag{2.17}$$

Calculations were performed for a number of representative instances spanning the range of values of k relevant to Table I. As in the calculation of $\gamma^{(1)}(R_0)$, the amplitudes A_1 and A_2 and the effective charges Z_a and Z_b were assumed to be given by Eq. (2.3). It was also assumed that $Z_c = 2.0$ and $1.0 \leq Z_{ka} \leq 1.93$. It was found in each instance that $\Gamma_{\rm Aug}^{(1)}/\Gamma_{\rm Aug}^{(2)} \lesssim 0.01$. This ratio decreases as the energy of the ejected electron increases. We con-

TABLE I. Auger and radiative rates for $\alpha\mu^-$, $\alpha\pi^-$, and αK^- atoms. The estimated Auger rate for an atom bound in a molecular ion is $\gamma^{(2)}(R_0)$, where $R_0 = 1.43a_0$ is the assumed value of the equilibrium internuclear separation; the estimated collisional Auger rate for a free atom in liquid helium is $\Gamma^{(2)}_{Aug}$; and the total radiative rate is γ_{rad}.

Atom	n	l	$\gamma^{(2)}(R_0)$ (10^{11} sec^{-1})	$\Gamma^{(2)}_{Aug}$ (10^{11} sec^{-1})	γ_{rad} (10^{11} sec^{-1})
$\alpha\mu^-$	5	4	96	3.5	0.14
		3	58	2.1	0.23
		2	33	1.2	0.46
		1	17	0.61	1.4
	4	3	27	0.99	0.44
		2	14	0.51	0.89
		1	6.0	0.22	2.6
	3	2	4.1	0.15	2.1
		1	1.4	0.052	6.1
	2	1	0.14	0.0050	20
$\alpha\pi^-$	5	4	54	2.0	0.18
		3	33	1.2	0.30
		2	19	0.69	0.61
		1	9.3	0.34	1.8
	4	3	15	0.54	0.58
		2	7.6	0.28	1.2
		1	3.3	0.12	3.4
	3	2	2.2	0.081	2.7
		1	0.77	0.028	8.0
	2	1	0.071	0.0026	26
αK^-	6	5	11	0.41	0.23
		4	7.5	0.27	0.34
		3	4.9	0.18	0.57
		2	3.0	0.11	1.2
		1	1.6	0.060	3.4
	5	4	4.0	0.15	0.58
		3	2.4	0.088	0.97
		2	1.4	0.051	2.0
		1	0.69	0.025	5.7
	4	3	1.0	0.037	1.9
		2	0.51	0.019	3.8
		1	0.22	0.0081	11
	3	2	0.14	0.0050	8.8
		1	0.047	0.0017	26

clude that, for the relatively low-lying states being considered in the present paper, $\Gamma^{(1)}_{Aug}$ may be neglected. Although we have performed no further calculations involving $\gamma^{(3)}(R)$, we assume that a similar conclusion is justified concerning the term

$$\Gamma^{(3)}_{Aug} = 4\pi N \int_{a_0}^{\infty} \gamma^{(3)}(R) R^2 dR. \tag{2.18}$$

The semiclassical expression specified by Eq. (2.14) for the collisional Auger rate was used in Ref. 7 for states with large n in order to avoid the large overestimates that might result if the Born approximation is employed. As it happens, the rate obtained using the Born approximation is not very different from that obtained using Eq. (2.16) for each of the relatively low-lying states considered in the present paper. We have computed approximate values of $\Gamma^{n-1,l-1}_{n,l}$, which is the rate for $\Delta n = \Delta l = -1$ collisional Auger transitions obtained using the Born approximation. A detailed description of this method of calculating Auger rates, which was first employed in this type of problem by Russell and Shaw,[14] has been given by Leon and Bethe.[15] For each of the states considered in Table I, it was found that the ratio of the Born rate to the semiclassical rate lies in the interval

$$1.1 \le \Gamma^{n-1,l-1}_{n,l} / \Gamma^{(2)}_{Aug} \le 2.3.$$

This ratio increases as n increases. The difference between the values of $\Gamma^{n-1,l-1}_{n,l}$ and $\Gamma^{(2)}_{Aug}$ does not appear to be large enough to affect significantly any conclusion regarding the relative importance of collisional Auger transitions and radiative transitions or the relative importance of collisional Auger transitions and molecular-ion formation. We should also add that in view of the rather different ways in which $\Gamma^{n-1,l-1}_{n,l}$ and $\Gamma^{(2)}_{Aug}$ are computed, the agreement between these two rates is surprisingly good.

D. Comparison with experiment

Our estimated Auger rates are in accord with the conjecture of Placci *et al.*[1] that MIF occurs during the cascade in liquid helium. As we have already mentioned, this conjecture was prompted by the fact that the total relative intensity of the muonic $K\gamma$ line and higher lines of the K series is significantly higher in gas than in liquid. Of course, it could be argued that this difference in intensity can be accounted for by the collisional Auger rates, which should be much smaller in gas than in liquid. But if our estimated collisional Auger rate for $4p$ muons is at all reliable, this possibility is ruled out. Compared to the total radiative rate for $4p$ muons, the collisional Auger rate $\Gamma^{(2)}_{Aug}$ in liquid is nearly 12 times smaller. But the molecular rate $\gamma^{(2)}$ for $4p$ muons is more than twice as large as the radiative rate. Therefore, it seems reasonable to conclude that molecular ions are probably formed in liquid by $\alpha\mu^-$ atoms in the $4p$ state. The comparatively high intensity in gas of the $K\gamma$ line and higher lines of the K series can be accounted for by the fact that MIF, which should require a ternary collision, is far less likely to occur. We note that if an $\alpha\mu^-$ atom in the $4p$ state is likely to undergo MIF in liquid, then so should an $\alpha\mu^-$ in any state with $n = 4$ and

$l \geq 2$ because none of these other states has an estimated atomic deexcitation rate, $\Gamma_{Aug}^{(2)} + \gamma_{rad}$, larger than that of the $4p$ state. For the same reason, an $\alpha\pi^-$ atom with $n = 4$ and $l \geq 2$ should also undergo MIF in liquid, provided nuclear absorption caused by Stark mixing is unimportant in the $n = 4$ level. It is also probable that MIF is important for $4p$ pions because the estimated total rate for atomic deexcitation or nuclear absorption[16] is only about twice as large as the estimated atomic deexcitation rate for $4p$ muons.

The difference between the relative intensities of the muonic $K\gamma$ line in liquid and gas is not the only experimental indication that MIF occurs during the cascade in liquid. Even if x-ray data for muons in gas are not taken into account, x-ray data for muons and pions in liquid can still be interpreted as indicating that MIF occurs in the $n = 4$ level. A discussion of some of the implications of the data for liquid will be given in Sec. III; but for the time being, we merely cite an analysis by Backenstoss *et al.*,[5] who employed the collisional Auger rates defined by Eq. (2.14) and found that their x-ray data, especially their muon data, can be fitted fairly well if each value of Γ_{Aug} is multiplied by a factor of 57. The need for this factor of 57 is due to the relatively small observed yields of the $K\gamma$ and $K\delta$ lines. The molecular rates in Table I are uniformly larger than the collisional rates by a factor of about 28. Since the approximations made in computing $\gamma^{(2)}$, particularly the use of the initial electron wave function Ψ_i, could have resulted in that rate being too low by a factor of 2, we believe that the data analysis reported in Ref. 5, together with our molecular Auger rates, can also be interpreted as indicating that MIF occurs in liquid helium. But even if MIF is assumed to occur, this data analysis, which specifically excludes the possibility of any sort of Stark mixing in low-lying principal levels,[17] does not give an entirely satisfactory explanation of the muonic and pionic x-ray data because it does not account for an anomaly connected with the $K\beta/K\alpha$ intensity ratios, particularly the pion ratio. This anomaly, which was recognized in Ref. 5, will be discussed in Sec. III, where various kinds of Stark transitions will be considered.

III. $K\beta/K\alpha$ INTENSITY RATIOS

A. Experiments

This section is concerned with an anomalous feature of the K-series x rays due to $\alpha\mu^-$ and $\alpha\pi^-$ atoms in liquid helium. The ratio

$$\rho = I_{K\beta}/I_{K\alpha}, \tag{3.1}$$

where $I_{K\alpha}$ and $I_{K\beta}$ are the intensities of the $K\alpha$ and $K\beta$ lines, has been found to be much higher for pions than muons. X-ray yields for these atoms have been determined by three groups, and all three experiments are in essential agreement with one another. The first experiment was performed by Wetmore *et al.*[3] Their results were later confirmed by Berezin *et al.*[4] and, more recently, by Backenstoss *et al.*[5] We shall quote only the results reported in Ref. 5, which is the experiment in which the $K\beta$ line was most clearly resolved from the higher lines of the K series.

Backenstoss *et al.*[5] have found the $K\beta/K\alpha$ intensity ratios for muons and pions to be $\rho_\mu = 0.54$ and $\rho_\pi = 1.24$, respectively. If radiation is assumed to be the only important process for excited states with $n \leq 3$, a ratio $\rho > 0.44$ indicates a $3p$ population higher than statistical. Of course, nuclear absorption, not radiation, is the dominant process for low-lying pion s states, and it has been found to be competitive with radiation for pion p states.[4,5] But it is difficult to account for the high value of ρ_π in terms of the pion-nucleus interaction *alone* because, as is easily shown by a straightforward calculation, the rate of nuclear absorption for p-state pions relative to the rates of both $np \to 1s$ and total x-ray emission slowly *increases* as n increases. Furthermore, if all s-state pions are assumed to undergo nuclear absorption, and if p-state nuclear absorption is assumed to be negligible, a statistical distribution of pions among the $3p$ and $3d$ states leads to the intensity ratio $\rho_\pi = 0.53$, which is still much less than the experimental value.

We believe that Stark transitions,[18] particularly irreversible transitions to states with lower orbital angular momentum, are partly responsible for the experimental values of ρ. A largely qualitative discussion of various kinds of Stark transitions is given in Sec. III B. But because Stark rates are difficult to estimate reliably, we shall not attempt in this paper to give a quantitative explanation of the $K\beta/K\alpha$ intensity ratios. However, we shall argue that the anomalously high value of ρ_π indicates that MIF probably occurs in the $n = 4$ level, no matter what is the correct explanation of the intensity ratios.

The anomalously high experimental value of ρ_π indicates that the $3p$ state of the $\alpha\pi^-$ somehow acquires a relatively high population. A pion can reach the $3p$ state only by a transition from a d state. This should be either the $3d$ state or the $4d$ state because Auger ejection, which we believe to be the dominant means of deexcitation for all d states with $n \geq 5$, is generally accompanied by the smallest possible change in n, while x-ray emission, which is the only other means of deexcitation, is usually accompanied by the largest.

The possibility that most $3p$ pions come from the $3d$ state will be discussed later on, in Sec. III C. For the moment, we shall assume that most of them come from the $4d$ state. This assumption, together with the observed value of ρ_π, requires deexcitation from the $4d$ state to proceed largely by Auger effect rather than by radiation, because the rate for $4d \to 2p$ x-ray emission is 3 times greater than the rate for $4d \to 3p$ x-ray emission, which means that a substantial amount of radiative deexcitation from the $4d$ state would result in a $K\beta/K\alpha$ ratio much smaller than the experimental value. It then follows from Table I that MIF should occur in the $n=4$ level if most $3p$ pions come from the $4d$ state, because the molecular Auger rate $\gamma^{(2)}$ for a $4d$ pion is ~6 times larger than the total radiative rate γ_{rad}, whereas the collisional Auger rates $\Gamma^{(2)}_{\text{Aug}}$ is ~4 times smaller than γ_{rad}. This conclusion would hold no matter what causes the pions to go to the $4d$ state in the first place. We note in passing that one possible explanation of a relatively large $4d$ population is the occurrence of irreversible Stark transitions, which are discussed in Sec. III B.

B. Stark transitions

1. *Energy splittings*

Besides depending on an electric dipole matrix element, the probability of a Stark transition can also depend on the energy splitting between the states involved. Table II gives values of the energy splitting

$$\Delta E = E_{l,j} - E_{l-1,j'} . \tag{3.2}$$

This is the splitting between states belonging to the same principal level and differing in orbital angular momentum by one unit. The computation of the binding energy $E_{l,j}$ for a state with $l \geq 1$ included the effects of vacuum polarization and relativity but omitted all other corrections, including those due to strong interactions and finite nuclear size. Corrections to s-state binding energies due to strong interactions and finite nuclear size have been included in the calculation of np-ns splittings. The n dependence of these corrections is n^{-3}. In the case of the $\alpha\mu^-$, finite nuclear size effects were computed using the correction to the $2p$-$2s$ splittings given by Campani.[19] We have assumed that Campani's result is due only to the $2s$ state. In the case of the $\alpha\pi^-$, nuclear size corrections were found using a result computed for the $1s$ state by Backenstoss *et al.*,[5] while the contributions of strong interactions to energy shifts were found using the value determined experimentally for the $1s$ state by the same authors. Table II

TABLE II. Energy splittings and rms Stark matrix elements for transitions of the type $(n,l,j) \to (n,l-1,j')$. The splitting is defined by $\Delta E = E_{l,j} - E_{l-1,j'}$. The matrix element M_r was computed using the field strength $\mathcal{E}_r = 25 \times 10^8$ V/cm.

Atom	Transition	ΔE (eV)	M_r (eV)
$\alpha\mu^-$	$4f_{7/2} \to 4d_{5/2}$	0.009	0.228
	$4f_{5/2} \to 4d_{5/2}$	0.006	0.051
	$4f_{5/2} \to 4d_{3/2}$	0.012	0.191
	$4d_{5/2} \to 4p_{3/2}$	0.052	0.250
	$4d_{3/2} \to 4p_{3/2}$	0.046	0.102
	$4d_{3/2} \to 4p_{1/2}$	0.064	0.228
	$4p_{3/2} \to 4s_{1/2}$	0.183	0.255
	$4p_{1/2} \to 4s_{1/2}$	0.164	0.255
	$3d_{5/2} \to 3p_{3/2}$	0.125	0.121
	$3d_{3/2} \to 3p_{3/2}$	0.110	0.049
	$3d_{3/2} \to 3p_{1/2}$	0.154	0.110
	$3p_{3/2} \to 3s_{1/2}$	0.437	0.140
	$3p_{1/2} \to 3s_{1/2}$	0.394	0.140
	$2p_{3/2} \to 2s_{1/2}$	1.52	0.057
	$2p_{1/2} \to 2s_{1/2}$	1.37	0.057
$\alpha\pi^-$	$5g \to 5f$	0.002	0.218
	$5f \to 5d$	0.012	0.285
	$5d \to 5p$	0.053	0.315
	$5p \to 5s$	$-0.46+0.18i$	0.308
	$4f \to 4d$	0.022	0.151
	$4d \to 4p$	0.105	0.191
	$4p \to 4s$	$-0.89+0.35i$	0.195
	$3d \to 3p$	0.255	0.092
	$3p \to 3s$	$-2.11+0.83i$	0.107
	$2p \to 2s$	$-7.1 \ +2.8i$	0.044
αK^-	$6h \to 6g$	0.024	0.090
	$6g \to 6f$	0.066	0.120
	$6f \to 6d$	0.146	0.137
	$6d \to 6p$	0.278	0.144
	$5g \to 5f$	0.118	0.067
	$5f \to 5d$	0.259	0.088
	$5d \to 5p$	0.485	0.097
	$4f \to 4d$	0.532	0.047
	$4d \to 4p$	0.963	0.059
	$3d \to 3p$	2.331	0.028

also lists values of a matrix element M_r that will be defined later.

Except where a pion s state is involved, most of the splittings listed in Table II are due largely to vacuum polarization, with relativity and finite nuclear size, particularly the latter, being comparatively unimportant. The contributions of vacuum polarization and relativity to ΔE are of the same sign and are such that the state with the lower orbital angular momentum is the more tightly bound if these are the only contributions that must be considered. Finite nuclear size effects for s states are, by themselves, not large enough to reverse this tendency. The np-ns splittings of the $\alpha\pi^-$ can be regarded as exceptional because the nuclear interaction causes the ns state to be less tightly bound than the np state. We note that the energy splittings listed in Table II decrease in magnitude as either n or l increases. We also note that for given values of n and l in Table II, the pion splitting is roughly twice as large in magnitude as the muon splittings, unless s states are involved, in which case the relative difference is even greater.

2. *Types of transitions*

Stark transitions can be classified in several ways. We shall first discuss recoil and collisional transitions. We shall then discuss irreversible transitions.

Recoil transitions are the Stark transitions that may take place immediately after the occurrence of an Auger transition in which an $\alpha\mu^-$ or an $\alpha\pi^-$ ejects an electron from a nearby helium atom. Prior to electron ejection, the mesonic atom and the helium atom may have been bound together in a molecule, or the two may have only been colliding with each other. In either case, the electron moves rapidly away within a time $\lesssim 10^{-17}$ sec, and then the mesonic atom, which was left behind in the relatively intense field of a He^+ ion, begins to recoil.

Later on, in order to simplify a calculation, we shall assume that the electric field responsible for a recoil transition can be adequately represented by a square pulse of magnitude

$$\mathcal{E}_r = 25\times 10^8 \text{ V/cm}. \tag{3.3}$$

This is the strength of the field at a distance $1.43a_0$ from a He^+ ion. This distance is approximately the equilibrium internuclear separation in the HeH^+ molecular ion.[8] Besides energy splittings, Table II also lists values of M_r, which is the *rms* matrix element of the Stark interaction $e\vec{\mathcal{E}}_r \cdot \vec{r}_\mu$ between initial meson states with orbital and total angular momentum l and j and final states with l' and j'. It is defined by

$$M_r^2 = (2j+1)^{-1} \times \sum_{m,m'} |\langle j',l',m'|e\vec{\mathcal{E}}_r\cdot\vec{r}_\mu|j,l,m\rangle|^2. \tag{3.4}$$

Stark transitions are also possible when an $\alpha\mu^-$ or an $\alpha\pi^-$ collides with a neutral helium atom but does not eject an electron. The electric field responsible for such transitions *may* have values much smaller than $\mathcal{E}_r$ because, for most interatomic separations, the nuclear charge is much more effectively shielded in a helium atom than in a He^+ ion. An estimate of this electric field can be easily obtained. Since the relative motion of the two atoms is comparatively quite slow, the electron wave function depends almost adiabatically during the collision on the positions of the nuclei, with the result that a quasimolecular structure is temporarily formed. The electric field responsible for collisional Stark transitions is essentially the same as the electric field governing the motion of the mesonic atom relative to the helium atom. This field can be given in terms of the gradient of the potential between the two atoms. This potential should not be very different from the molecular potential for the ground $^1\Sigma$ state of the HeH^+ ion. The molecular potential for this ion is given approximately by the Morse potential

$$U(R) = U_0(e^{-2(R-R_0)/a} - 2e^{-(R-R_0)/a}), \tag{3.5}$$

where

$$U_0 = 1.85 \text{ eV}, \quad R_0 = 1.44a_0, \quad a = 0.625a_0. \tag{3.6}$$

The numerical values of the parameters U_0, R_0, and a were obtained from some calculations by Michels.[2] Figure 1(a) shows the Morse potential U. Figure 1(b) shows the electric field at the mesonic atom obtained by differentiating U. At $R = a_0$, which is for zero impact parameter approximately the distance of closest approach at the energies we shall consider, the field strength is $\sim 25\times 10^8$ V/cm. Therefore, to the extent that the curve in Fig. 1(b) is accurate, we can regard $\mathcal{E}_r$ as an approximate upper limit to the electric field strength encountered during a collision. But it must be borne in mind that, as pointed out by Mueller *et al.*,[20] the use of an approximate potential to compute the electric field may lead to gross inaccuracies at low values of R.

We have already noted that the binding energies of states with the same n generally increase as l decreases. We believe that the anomalously high value of ρ_π, which indicates an unusually large number of $3p$ pions, is partly due to the occurrence of irreversible Stark transitions[21] (IST) in the $n=3$ or the $n=4$ level. A Stark transition between two nearly degenerate states of a mesonic atom is irreversible if the binding energy of the final state is larger in magnitude than that of the initial state and if the atom emerges from the collision with a laboratory kinetic energy less than a certain value $E_{irr} \simeq 2|\Delta E|$, where ΔE is the splitting. Of course, the preceding statement is correct only if $|\Delta E|$ is much greater than the mean kinetic energy of the helium atoms with which the mesonic atom subsequently collides, but this condition is satisfied in liquid helium by $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n\le 4$. Because the mass difference between an $\alpha\mu^-$ or an $\alpha\pi^-$ and a helium atom is relatively small, IST can occasionally occur even if the initial laboratory kinetic energy

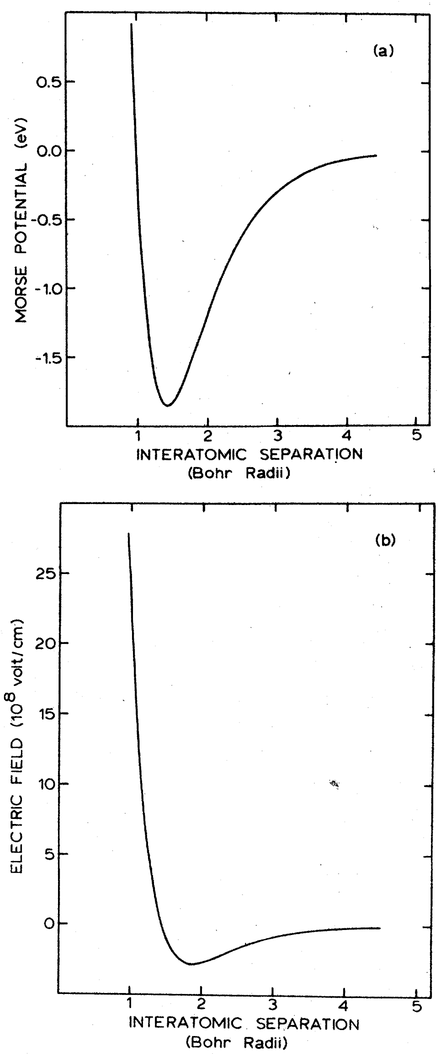

FIG. 1. (a) Morse potential for HeH^+ ion. (b) Electric field obtained by differentiating the Morse potential.

of the mesonic atom is fairly high. It is only necessary that the c.m. scattering angle be sufficiently large. However, IST should become relatively probable only when the mesonic atom has been slowed to an initial laboratory energy not too much greater than E_{irr}. Some calculations that will be presented in Sec. IV indicate that this condition probably becomes fulfilled in the case of certain low-lying states of the $\alpha\mu^-$ and $\alpha\pi^-$ atoms. A transition is sure to be irreversible if the initial laboratory kinetic energy is less than a certain value $E_t \simeq \frac{9}{16} E_{irr}$. In practice, IST should occur only as a result of collisions, not recoils.

3. Rough estimates

In what follows, we shall obtain some very rough estimates of probabilities for Stark transitions. Although we shall be concerned primarily with recoil transitions, some of the discussion will be applicable also to collisional transitions.

In discussions of Stark mixing it is frequently assumed for the sake of simplicity that classical mechanics can be employed to describe the motion of the mesonic atom relative to the other atom or molecule with which it interacts. We shall make this assumption. In principle, the use of a classical trajectory permits the amount of Stark mixing to be estimated by solving a time-dependent Schrödinger equation, with the mixing being caused by the time-dependent dipole interaction

$$H'(t) = e\vec{\mathcal{E}}(t)\cdot\vec{r}_\mu, \tag{3.7}$$

where $\vec{\mathcal{E}}$ is the electric field due to the other atom and $\vec{r}_\mu$ is the meson coordinate. In practice, the use of a realistic trajectory entails a lengthy and complicated calculation, particularly if there are several states among which mixing can occur. For this reason, we shall make some further simplifications. We shall assume $\vec{\mathcal{E}}$ has a fixed direction and a time dependence that is a square pulse. We shall also assume that Stark mixing occurs between only two states.

We consider a simple system consisting of two states.[22] The time-independent orthornormal wave functions for these states, which are eigenfunctions of a Hamiltonian H_0, are denoted by ψ_1 and ψ_2. These states have real energies E_1 and E_2 that may or may not be equal. The wave function for this system will be denoted by

$$\Psi = B_1(t)\psi_1 e^{-iE_1t/\hbar} + B_2(t)\psi_2 e^{-iE_2t/\hbar}. \tag{3.8}$$

We assume that the amplitudes B_1 and B_2 at $t=0$ are

$$B_1(0)=0, \quad B_2(0)=1, \tag{3.9}$$

and we further assume that the subsequent behavior of the system is determined by the Hamiltonian

$$H = H_0 + H', \tag{3.10}$$

where H', which causes mixing between ψ_1 and ψ_2, is a time-independent interaction with matrix elements

$$\langle\psi_2|H'|\psi_1\rangle = M, \tag{3.11a}$$

$$\langle\psi_1|H'|\psi_1\rangle = \langle\psi_2|H'|\psi_2\rangle = 0. \tag{3.11b}$$

The time-dependent Schrödinger equation

$$-\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = H\Psi \tag{3.12}$$

for this simple system can be solved analytically. The probability of the system being in the state ψ_1 at time t is

$$|B_1(t)|^2 = |M/\hbar|^2\tau^2 \sin^2(t/\tau), \qquad (3.13)$$

where

$$\tau = \hbar[|M|^2 + (\tfrac{1}{2}\Delta E)^2]^{-1/2}, \qquad (3.14)$$

$$\Delta E = E_2 - E_1. \qquad (3.15)$$

If $t \gg \tau$, it may be convenient to speak of the average value of $|B_1(t)|^2$, which is given by

$$\langle |B_1|^2 \rangle = \frac{|M|^2}{2[|M|^2 + (\frac{1}{2}\Delta E)^2]}, \quad t \gg \tau. \qquad (3.16)$$

Depending on the relative values of M and ΔE, the average mixing probability can be quite small or as large as 50%.

Table III lists some values of $\langle |B|_r^2 \rangle$ and t_r/τ_r, where

$$t_r = 2.5 \times 10^{-15} \text{ sec}. \qquad (3.17)$$

The quantity $\langle |B|_r^2 \rangle$ is a very rough estimate of the probability of a recoil transition. It was computed using Eq. (3.16), with M replaced by M_r. Imaginary parts of energy splittings were ignored. The quantity τ_r was computed in a similar fashion using Eq. (3.14). We note that inserting values of the rms matrix element M_r in Eqs. (3.14) and (3.16) is a somewhat arbitrary procedure because these two expressions were obtained using a model with only two states. If the mesonic atom and a He^+ ion initially have zero relative velocity and are separated by a distance $R_0 = 1.44a_0$, the quantity t_r defined in Eq. (3.17) is the length of time required for the recoil to increase the interatomic separation to $\sqrt{2}R_0$, at which point the perturbing electric field has dropped to half its original value.

TABLE III. Rough estimates of Stark mixing due to the sudden application of an electric field of strength $\mathcal{E}_r = 25 \times 10^8$ V/cm.

Atom	Transition	$\langle \lvert B\rvert_r^2 \rangle$	t_r/τ_r
$\alpha\mu^-$	$4f_{7/2} \to 4d_{5/2}$	0.50	0.9
	$4d_{5/2} \to 4p_{3/2}$	0.49	1.0
	$4p_{3/2} \to 4s_{1/2}$	0.44	1.0
	$3d_{5/2} \to 3p_{3/2}$	0.39	0.5
	$3p_{3/2} \to 3s_{1/2}$	0.14	1.0
	$2p_{3/2} \to 2s_{1/2}$	3×10^{-3}	2.9
$\alpha\pi^-$	$5g \to 5f$	0.50	0.8
	$5f \to 5d$	0.50	1.1
	$5d \to 5p$	0.50	1.2
	$5p \to 5s$	0.32	1.5
	$4f \to 4d$	0.50	0.6
	$4d \to 4p$	0.46	0.8
	$4p \to 4s$	0.08	1.8
	$3d \to 3p$	0.17	0.6
	$3p \to 3s$	5×10^{-3}	4.0
	$2p \to 2s$	8×10^{-5}	13.2

It is obvious from Table III that the relative values of ΔE and M_r cause the average mixing probability, as defined by Eq. (3.16), to have a wide range of values for low-lying states. Furthermore, for given values of n and l, the value of $\langle |B|_r^2 \rangle$ tends to be lower for pions than muons, in some instances by a relatively large amount. This should also be true of the rates for collisional Stark transitions.[23] We must emphasize, however, that the transition probabilities in Table III should not be taken at face value, if only because the values of t_r/τ_r indicate that a time-averaged mixing probability may not be entirely appropriate in every instance. But even though the interpretation of the transition probabilities in Table III is subject to some uncertainty because of the various approximations that have been made, it seems reasonable to argue that these probabilities, together with Eq. (3.16) and the relative values of ΔE and M_r in Table II, strongly suggest that recoil Stark transitions with increasingly larger changes in l should become probable as n becomes larger, since the transition matrix elements become larger while the energy splittings become smaller, thereby making possible dipole transitions of increasingly higher order. In particular, the transition probabilities in Table III suggest that recoil transitions with relatively large changes in l might very well occur in all levels with $n \geq 4$. In fact, it even seems entirely likely that, with the axis of quantization parallel to the vector separation between the mesonic atom and the He^+ ion, a recoil would result in a uniform population of all states of a level with $n \geq 4$ that have the same magnetic quantum number. This means that recoils would cause the $4p$ states of the $\alpha\mu^-$ to acquire between 13% and 31% of all $n = 4$ muons.

C. $3d \to 3p$ pion transitions

We now assume that the large $3p$ pion population implied by the high experimental value of ρ_π is due mostly to transitions from the $3d$ state rather than the $4d$ state. Such transitions, if indeed they do occur in the $\alpha\pi^-$, would surely have to be Stark transitions. We shall argue that the occurrence of $3d \to 3p$ Stark transitions in the $\alpha\pi^-$, together with the experimentally determined value of ρ_μ and the observed intensities of the muonic and pionic $K\gamma$ lines, would imply the occurrence of MIF in the $n = 4$ level.

Our arguments will depend, among other things, on two assumptions. One of these assumptions is that most of the muons and pions that ultimately are responsible for either a $K\alpha$ or a $K\beta$ x ray first pass through the $n = 4$ level. We believe that the *real* reason why they pass through the $n=4$ level is that MIF also occurs in the $n=5$ level, thereby causing deexcitation to proceed by molecular Auger effect, which obeys the selection rule $\Delta n=-1$. The reason why we believe this is that it is difficult to imagine MIF not occurring in the $n=5$ level if it occurs in the $n=4$ level, since our estimated values of the atomic deexcitation rate, $\Gamma_{\text{Aug}}^{(2)}+\gamma_{\text{rad}}$, are not very different for these two levels. But if our argument that MIF occurs in the $n=4$ level is to be logical, it is necessary that it not depend crucially on the preceding explanation of why Auger deexcitation occurs in the $n=5$ level, since this would be essentially equivalent to assuming what we are attempting to prove. For this reason we note that the rates in Table I indicate that collisional Auger effect, which also obeys the selection rule $\Delta n=-1$, should be more probable than x-ray emission for muons with $n=5$ and $l\geq 2$ and pions with $n=5$ and $l\geq 3$. In fact, in most of these instances it would appear to be much more probable. We also note that it is found experimentally that the muonic and pionic $K\delta$ lines are very faint in liquid helium,[5] which means that radiative transitions from the $5p$ state are unimportant in both the muon and the pion cascades.

These low $K\delta$ yields, together with the rates in Table I, also indicate either that MIF occurs in the $5p$ state or that the relative number of $n=5$ mesons that undergo deexcitation to a lower principal level from this state is considerably smaller than the statistical weight of the state. (For future reference, we note that if MIF occurs in the $5p$ state of the $\alpha\mu^-$, it should also occur in the $5s$ state because the radiative rate is 15 times smaller in the $5s$ state.) It only remains to establish that radiative transitions from the $5d$ state of the $\alpha\pi^-$ and the $5s$ state of the $\alpha\mu^-$ do not play major roles in the cascades in these atoms.

Though there is no direct experimental evidence regarding x rays from the $5d$ state of the $\alpha\pi^-$, the anomalously high experimental value of ρ_π strongly suggests that such transitions are not of major importance in the pion cascade, since radiative transitions from the $5d$ state are 2.8 times more likely to go to the $2p$ state than the $3p$ state. Furthermore, the total radiative rate for $5d$ pions is slightly less than the estimated rate given in Table I for collisional Auger effect. If radiative transitions from the $5s$ state of the $\alpha\mu^-$ are to play a major role in the muon cascade, not only must this state have a population much greater than statistical, it must also have an effective Auger rate that is not substantially larger than the radiative rate, which would probably be possible only if MIF does not occur. Although we are unable to prove conclusively that either of these conditions is not fulfilled, we believe that the near absence in *both* the $\alpha\mu^-$ and the $\alpha\pi^-$ of radiative transitions from the $5p$ state, which is statistically more favored than the $5s$ state and which has a much higher radiative rate, strongly indicates that x-ray emission from the $5s$ state of the $\alpha\mu^-$ should be unimportant, either because very few muons ever find their way to this state or because MIF occurs, thereby causing deexcitation to proceed by molecular Auger effect. It therefore seems reasonable to assert that our assumption about muons and pions passing through the $n=4$ level is not only correct but also does not depend crucially on MIF occurring in that level.

Our second assumption is that $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n=4$, after undergoing recoils following Auger deexcitation from the $n=5$ level, but before undergoing collisions with nearby neutral helium atoms, have p state relative populations somewhere between 13% and 31%. The rough estimates in Sec. III B, which indicate that $n=4$ recoil mixing is highly probable in both the $\alpha\mu^-$ and the $\alpha\pi^-$, support this assumption.

For the moment, we shall make the additional assumption that $n=4\rightarrow 2$ transitions are unimportant in the $\alpha\mu^-$. Later on, we shall relax this particular requirement. For reasons outlined below, we now assert that if $n=4\rightarrow 2$ transitions do not occur in the $\alpha\mu^-$, and if $3d\rightarrow 3p$ Stark transitions are largely responsible for the high value of ρ_π, the much lower value of ρ_μ cannot easily be understood unless (i) MIF occurs in the $n=4$ level of the $\alpha\mu^-$ or (ii) IST occurs in the $n=3$ level of both the $\alpha\mu^-$ and the $\alpha\pi^-$. (Later, we shall argue further that the occurrence of IST in the $n=3$ level, together with the observed $K\gamma$ intensities, would also imply that MIF occurs in the $n=4$ level.) The relative values of ρ_μ and ρ_π indicate that a much higher proportion of muons than pions passes through the $2p$ state. Since the discussion in Sec. III B indicates that $3d\rightarrow 3p$ transitions should, if anything, be more probable for muons than pions, it would appear that the only transition from the $n=3$ level that could account in a consistent way for the relatively large $2p$ muon population is the $3s\rightarrow 2p$ transition. But the $3s$ state of the $\alpha\mu^-$ can become heavily populated only by Auger transitions from the $4p$ state or by Stark transitions from the $3p$ state. The rates in Table I indicate that $4p\rightarrow 3s$ Auger transitions could not compete with $4p\rightarrow 1s$ radiative transitions unless MIF occurs in the $n=4$ level, which is what we are trying to establish. Therefore, further consideration need be given only to the implications of $3p\rightarrow 3s$ Stark transitions in the $\alpha\mu^-$. Because the $3s$ state has a low statistical weight and a low radiative rate compared to other states of the $n=3$ level, we assert that the only way in which both the 3s state of the $\alpha\mu^-$ and the 3p state of the $\alpha\pi^-$ could, by means of Stark transitions, acquire the populations necessary to account for the experimental values of ρ_μ and ρ_π is by IST.

If IST occurs in the $n=3$ level of both the $\alpha\mu^-$ and the $\alpha\pi^-$, there are reasons why it surely must also occur in both of these atoms in the $n=4$ level. First of all, some calculations that will be presented in Sec. IV indicate that if recoiling mesonic atoms with $n=3$ are slowed quickly enough to velocities at which Stark transitions are likely to be irreversible, the same should be true of atoms with $n=4$. Furthermore, the radiative and collisional Auger rates in Table I and the energy splittings and matrix elements in Table II indicate that if 3p muons and 3d pions are likely to undergo Stark transitions, the same should be true of muons with $n=4$ and $l\geq 1$ and pions with $n=4$ and $l\geq 2$. But if, as we have assumed, between 13% and 31% of all mesonic atoms with $n=4$ are in the 4p state before they begin to undergo collisions with ordinary helium atoms, and if IST is highly probable in the $n=4$ level, it would seem that both the muon and the pion 4p states—or, if not both, then at least one or the other—should then acquire a much higher population. However, the $K\gamma$ line, which accounts for 84% of all radiative transitions from the 4p state, is observed to have a relatively low intensity for both $\alpha\mu^-$ and $\alpha\pi^-$ atoms. The measured $K\gamma/(K\alpha+K\beta+K\gamma)$ intensity ratio is 0.07 for muons and 0.16 for pions.[5] By way of comparison, we note that if mesons are assumed to be distributed statistically in the $n=4$ level, and if only radiation and strong interactions are assumed to be important in the subsequent cascade, this ratio is 0.16 for muons and, depending on the relative rates for nuclear absorption and radiation in p states, between 0.15 and 0.18 for pions. The occurrence of IST in the $n=3$ level would not greatly affect these numbers. And the occurrence of some sort of Auger process in the 3p state and/or the 4d state would only tend to increase these numbers. Therefore, since the 4p collisional Auger rates in Table I are an order of magnitude or more smaller than the radiative rates, the observed $K\gamma$ yields would appear to be inconsistent with the occurrence of IST in the $n=3$ level unless MIF occurs in the 4p state, thereby permitting 4p muons and pions to proceed to the 3s state at rates comparable to or somewhat larger than the $4p\rightarrow 1s$ radiative rates. Finally, it is worth noting that the preceding arguments depend on the assumption, made earlier on, that practically all muons and pions that ultimately contribute to the $K\alpha$ or the $K\beta$ intensities first pass through the $n=4$ level.

We shall now relax our requirement that $n=4\rightarrow 2$ transitions be unimportant in the $\alpha\mu^-$. In other words, we shall consider the possibility that the comparatively high number of muons passing through the 2p state might come largely by radiative transitions from the 4s or the 4d state. If they come from the 4s state, which has a low statistical weight and a low radiative rate compared to other states of the $n=4$ level, the occurrence of IST in the $n=4$ level would be strongly implied because it would be very difficult to account otherwise for a high 4s population. And for reasons outlined above, the occurrence of a substantial amount of IST in the $n=4$ level would in turn strongly imply the occurrence of MIF if the experimentally observed muonic and pionic $K\gamma$ intensities are to be understood. However, the occurrence of MIF in the $n=4$ level would certainly not be implied if the relatively large 2p muon population is due largely to transitions from the 4d state. But the relative values of the radiative and collisional Auger rates in Table I indicate that such transitions would be even more probable for pions than muons, which would make it very awkward to account in a consistent way for the fact that ρ_π is found experimentally to be much higher than ρ_μ. Since Stark mixing, including IST, is most unlikely to be less probable for muons than pions, we can think of no reason why pions should be much less likely than muons to be in the 4d state. It would therefore appear that $4d\rightarrow 2p$ transitions are unlikely to be of crucial importance in the $\alpha\mu^-$.

IV. SLOWING OF MESONIC ATOMS

It was suggested in Sec. III that IST might occur in both the $\alpha\mu^-$ and the $\alpha\pi^-$. As noted in Sec. III B, a necessary condition for IST to be relatively probable is that the laboratory kinetic energy of the mesonic atom not be too much greater than $2|\Delta E|$, where $|\Delta E|$ is the energy difference between the two states involved. However, in most instances the atomic deexcitation process preceding a collisional Stark transition should be some sort of Auger transition that is followed immediately by a recoil during which the mesonic atom acquires a laboratory kinetic energy much greater than $2|\Delta E|$. Therefore, if IST is to be at all likely, the mesonic atom must be moderated to sufficiently low energies before it has much opportunity to undergo radiative deexcitation, collisional Auger deexcitation, or MIF. Rates for radiative and Auger deexcitation were estimated in Sec. II. But we have not attempted to compute the rate at which molecules are formed. Instead, we shall assume somewhat arbitrarily that MIF is unlikely to occur, except at energies rather lower than $2|\Delta E|$. Some brief remarks concerning the MIF rate will be made in Sec. VI. In what follows, we shall assume that for a given initial state, IST *might* be likely to occur only if the mesonic atom is slowed to a sufficiently low energy in a time rather less than

$$\tau_a = (\gamma_{\rm rad} + \Gamma^{(2)}_{\rm Aug})^{-1}, \qquad (4.1)$$

where $\gamma_{\rm rad}$ and $\Gamma^{(2)}_{\rm Aug}$ are the rates for radiation and collisional Auger effect given in Table I. In order to establish conclusively that this condition is fulfilled, it would be necessary to know the differential cross sections for scattering of $\alpha\mu^-$ and $\alpha\pi^-$ atoms by helium atoms at all laboratory energies less than, say, 20 eV. We have not attempted to compute these cross sections. Instead, we shall demonstrate that if these cross sections have what we assert are reasonable values, $\alpha\mu^-$ and $\alpha\pi^-$ atoms should indeed be slowed quickly enough to velocities favorable for IST in the $n=3$ and the $n=4$ levels.

A Monte Carlo method was employed to simulate the moderation of recoiling mesonic atoms in liquid helium. Several very similar calculations were performed. The purpose of each calculation was to determine the time dependence of the relative population of mesonic atoms with laboratory kinetic energies less than a suitably chosen value comparable to one of the $n=3$ or $n=4$ splittings. A large population of mesonic atoms was employed in each calculation, and it was assumed that each of these atoms has the same initial laboratory kinetic energy at time $t=0$. This initial energy will be denoted by E_i. The subsequent slowing of the mesonic atoms was assumed to be caused by purely elastic collisions with helium atoms, which were each assumed to have zero initial laboratory kinetic energy. We note that because Stark transitions may occur during actual collisions and because such transitions result in a change in the energy of relative motion, the assumption that the scattering is purely elastic is not entirely justified, particularly at relatively low energies. Two other simplifying assumptions were also made: The scattering was assumed to be isotropic in the c.m. system; and the total scattering cross section, which will be denoted by σ, was assumed not to depend on the energy of relative motion. These last two assumptions, though not very realistic, provide a convenient starting point for further discussion.

Although our simulation of the slowing of a mesonic atom did not take into account the change in the c.m. kinetic energy that must accompany a Stark transition, such a change should be taken into account in determining whether or not a particular Stark transition occurring at a given energy is likely to be irreversible. If the relatively small mass difference between the mesonic atom and a helium atom is neglected, the following two remarks hold for collisional Stark transitions to a state with lower energy. (i) If the inelastic scattering associated with a Stark transition is isotropic in the c.m. system, there is at least a 50% probability that the transition will be irreversible if the laboratory kinetic energy of the mesonic atom before the collision satisfies the condition $E \le 3|\Delta E|$. (ii) If the laboratory kinetic energy of the mesonic atom before the collision satisfies the condition $E \le \frac{9}{8}|\Delta E|$, there is a 100% probability that the transition will be irreversible, even if the inelastic scattering in the c.m. system is not isotropic. Accordingly, the Monte Carlo calculations determined, for given values of E_i, σ, and $|\Delta E|$, the time dependence of the relative populations of mesonic atoms satisfying the conditions

$$E \le 3|\Delta E|, \qquad (4.2a)$$

$$E \le \tfrac{9}{8}|\Delta E|. \qquad (4.2b)$$

Some numerical results are shown in Fig. 2 for $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n=3$ or $n=4$. Each curve is a best fit to the results of a Monte Carlo calculation. Each of the curves is associated with a particular Stark transition and is labeled by the initial state. In each instance the final state, which is lower in energy than the initial state, has orbital and total angular momenta l' and j' that are related to those of the initial state by $l'=l-1$ and $j'=j-1$. Each curve shows the time dependence of the relative population of mesonic atoms with laboratory kinetic energy $E \le 3|\Delta E|$. The unit of time for each curve is the value of τ_a for the initial state. In each instance the scattering cross section is assumed to be $\sigma = \pi a_0^2$, and the laboratory kinetic energy of each mesonic atom at time $t=0$ is assumed to be $E_i = 0.7$ Ry $= 9.5$ eV. This value of E_i is approximately the recoil energy acquired by the mesonic atom if, at the moment the recoil begins, the internuclear separation is equal to the equilibrium separation in the HeH^+ molecular ion. All of the curves in Fig. 2 have a rather similar appearance, which indicates that most atoms with $n=4$ should be slowed quickly enough if the same is true of atoms with $n=3$, and vice versa. Furthermore, if the asssumptions that have been made about the differential scattering cross section were realistic, these results would indicate that the mesonic atoms should indeed be slowed quickly enough to permit the occurrence of IST.

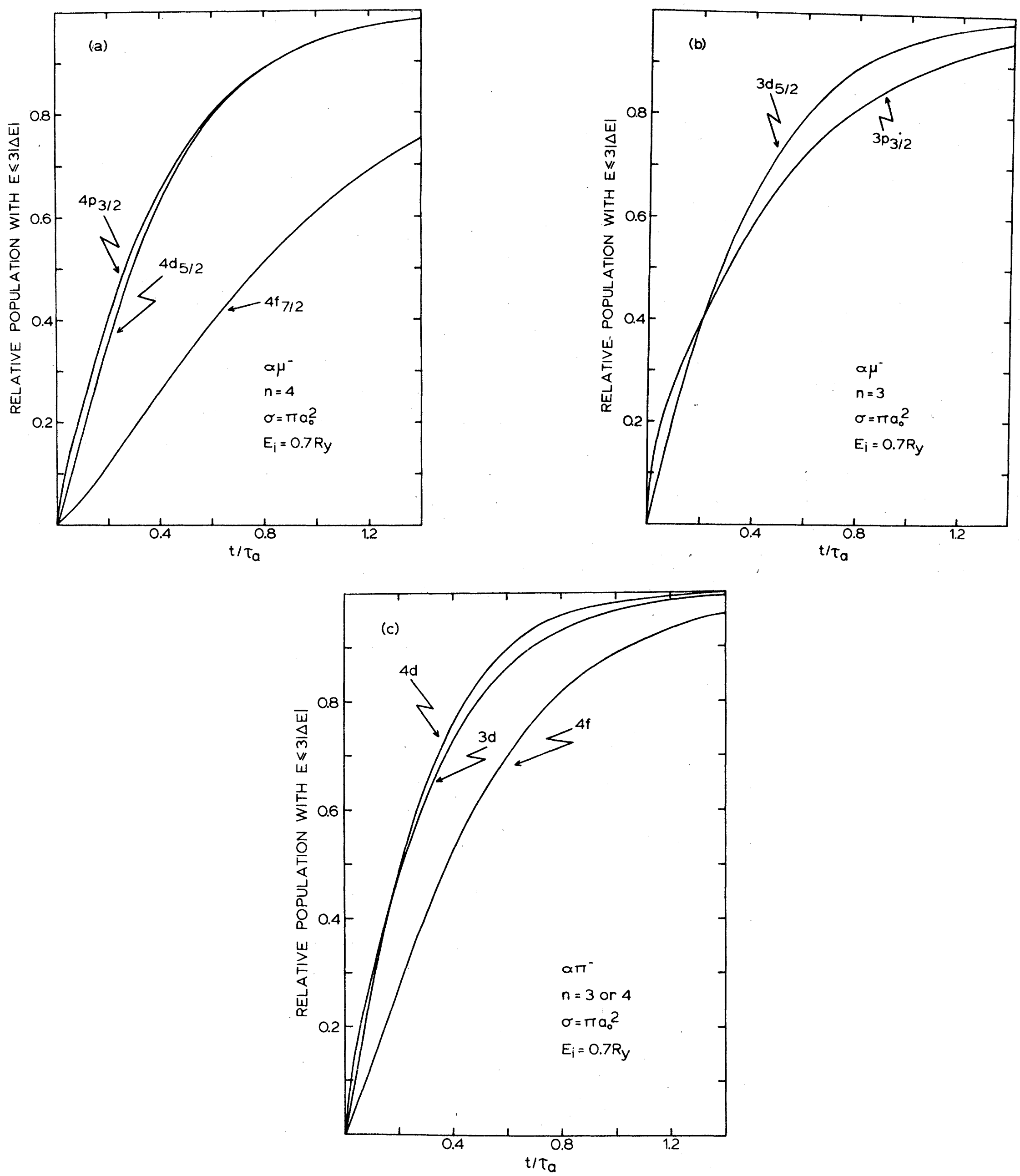

FIG. 2. Slowing of $\alpha\mu^-$ and $\alpha\pi^-$ atoms with $n=3$ or $n=4$ to energies $\leq 3|\Delta E|$.

The extent to which a particular curve in Fig. 2 would change if the assumed value of E_i is varied should depend on the values of $|\Delta E|$ and τ_a. Among the transitions considered in Fig. 2, there is a tendency for the value of τ_a to decrease with increasing $|\Delta E|$, which accounts for the generally similar appearance of all the curves, despite the rather wide range of values of $|\Delta E|$. The $4f\to 4d$ transition of the $\alpha\pi^-$ has nearly the smallest value of $|\Delta E|$, while the $3d\to 3p$ transition of the $\alpha\pi^-$ has nearly the largest. The extent to which our results depend on the assumed value of E_i was investigated for these two transitions. As before, it was assumed in each instance that $\sigma=\pi a_0^2$. It was found for both transitions that assuming that $E_i=0.5$ Ry or $E_i=1.0$ Ry results in a curve that scarcely differs from the one obtained assuming that $E_i=0.7$ Ry. We therefore conclude that none of the curves in Fig. 2 is likely to be changed significantly by reasonable variations in

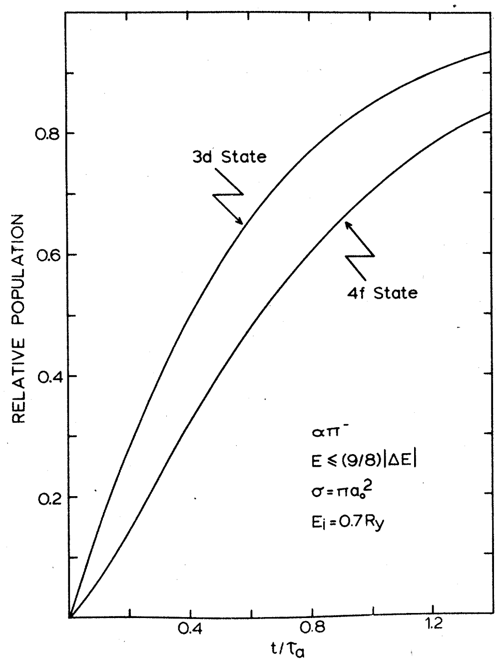

FIG. 3. Slowing of $4f$ and $3d$ $\alpha\pi^-$ atoms to energies $\leq \frac{9}{8}|\Delta E|$.

the assumed value of E_i.

The curves in Fig. 2 show only relative populations with laboratory energy $E \leq 3|\Delta E|$. Figure 3 shows relative populations with $E \leq \frac{9}{8}|\Delta E|$. The curves in Fig. 3 pertain to the $4f \rightarrow 4d$ and $3d \rightarrow 3p$ transitions of the $\alpha\pi^-$. As in Fig. 2, these results were obtained assuming that $\sigma = \pi a_0^2$ and $E_i = 0.7$ Ry. These results indicate that the additional amount of time required to slow the mesonic atom from an energy at which a Stark transition is only fairly likely to be irreversible to an energy at which it is certain to be irreversible is generally rather smaller than τ_a.

It only remains to discuss the assumptions, made in obtaining each of the curves in Fig. 2, that the scattering is isotropic in the c.m. system and that the total scattering cross section is independent of energy and given by $\sigma = \pi a_0^2$. We first note that if the scattering is independent of angle and energy, the time scale for the slowing of the mesonic atoms varies inversely with σ. We further note that any increase in the assumed value of the differential cross section at any angle and at any energy would tend to decrease the estimate of the time required to slow an atom to a very low energy. Therefore, since the results given in Fig. 2 indicate that the mesonic atoms are slowed quickly enough if $\sigma = \pi a_0^2$, it would suffice to show that the actual differential cross section is at least as large as $(\frac{1}{4}a_0^2)$ sr at all angles and at all energies below, say, 20 eV. Because we have not calculated the cross sections for scattering of $\alpha\mu^-$ and $\alpha\pi^-$ atoms by helium atoms, we are unable to state conclusively that this condition is satisfied. But there are some calculations of the scattering of protons and Li^+ ions by helium atoms that would seem to indicate that this condition is satisfied.[24,25] These calculations, though concerned largely with scattering at fairly small angles, suggest that $(\frac{1}{4}a_0^2)$ sr is probably a conservative underestimate of the actual differential cross section for scattering of a positively charged mesonic atom with $E < 20$ eV. For this reason, we believe there is little reason to pursue the matter further.

V. KAONIC X RAYS

Any interpretation of muonic and pionic x-ray data must be compatible with kaonic x-ray data. If IST and MIF occur in $\alpha\mu^-$ and $\alpha\pi^-$ atoms, it seems possible that these effects might also occur in the αK^-. The most thorough experimental investigation of kaonic x rays in liquid helium is that of Wiegand and Pehl.[6] In agreement with the p-state nuclear absorption rates derived by Mazur *et al.*[26] from scattering data, no K-series lines were observed in this experiment. The absolute intensities of the $L\alpha$, $L\beta$, and $L\gamma$ lines were found to be 0.092, 0.052, and 0.024, respectively, with a relative error ~20% in each instance. Absolute intensities as such are probably not relevant to the present discussion because the possibility of nuclear absorption in high s states due to Stark mixing cannot be ruled out.[7] Consequently, we shall only concern ourselves with the $L\beta/L\alpha$ and $L\gamma/L\alpha$ intensity ratios. We shall argue that these intensity ratios are entirely compatible with the occurence of IST and MIF in the αK^-.

The experimental intensity ratios are given in Table IV, where they are denoted by $I_{L\beta}/I_{L\alpha}$ and $I_{L\gamma}/I_{L\alpha}$. Table IV also lists calculated intensity ratios obtained assuming that the kaons first have a statistical distribution in some level with principal quantum number n_i and then, in the subsequent cascade, are subject only to radiative transitions and nuclear absorption. Nuclear absorption was assumed to take place only in s and p states. Radiative transitions originating in s states were, of course, neglected. The n dependence of the p-state nuclear absorption rate is $n^{-3}(1-n^{-2})$. It was assumed that the $2p$ nuclear rate is 5×10^{14} sec^{-1}, which is not too much less than the two possible values determined in Ref. 26. The $2p$ nuclear rate can be varied by a factor of 10 without causing the calculated intensity ratios to change significantly.

The kaon data are incompatible with a statistical

TABLE IV. Intensities of the kaonic $L\beta$ and $L\gamma$ lines relative to that of the $L\alpha$ line.

Method of determination		$I_{L\beta}/I_{L\alpha}$	$I_{L\gamma}/I_{L\alpha}$
Experiment [a]		0.57	0.26
Calculation [b]	n_i		
	5	0.14	0.24
	6	0.12	0.04
	7	0.10	0.03

[a] C. E. Wiegand and R. H. Pehl, Phys. Rev. Lett. 27, 1410 (1971).

[b] The kaons are assumed to be initally distributed statistically among the states with principal quantum number n_i. Only radiation and nuclear absorption occur in the subsequent cascade. Only nuclear absorption is possible in s states. The $2p$ nuclear rate is $5\times 10^{14}\ \text{sec}^{-1}$.

initial distribution followed by a cascade in which only radiation and nuclear absorption are possible. The relatively large experimental value of the $L\beta/L\alpha$ intensity ratio calls to mind the relatively large value of the pionic $K\beta/K\alpha$ ratio. Just as the pion ratio indicates that the $3p$ state of the $\alpha\pi^-$ has acquired a relatively high population, the kaon ratio suggests that there is some mechanism causing the $4d$ state of the αK^- to acquire a relatively high population. The $L\beta/L\alpha$ ratio is therefore entirely compatible with the occurrence of IST in the αK^-. We also note that there should be little effect on this intensity ratio if αK^- atoms undergo MIF, because the molecular Auger rate in Table I for the $4d$ state is ~7 times smaller than the radiative rate.

If IST somehow causes the $4d$ state of the αK^- to acquire a large population, it seems reasonable to suppose that it might also cause the $5d$ state to acquire a large population, possibly even larger than that of the $4d$ state. But the experimental $L\gamma/L\alpha$ ratio is not nearly as large as the $L\beta/L\alpha$ ratio, even though the $5d\rightarrow 2p$ transition accounts for almost as large a fraction (66%) of the radiative transitions from the $5d$ state as does the $4d\rightarrow 2p$ transition (75%) from the $4d$ state. This could be accounted for by the deexcitation rates in Table I, which indicate that the $5d\rightarrow 2p$ radiative transition, unlike the $4d\rightarrow 2p$ transition, may face serious competition from a molecular Auger transition. We therefore conclude that the $L\gamma/L\alpha$ ratio is entirely compatible with the occurrence of both IST and MIF in the αK^-.

VI. DISCUSSION

Although we have concluded that MIF probably occurs in the $n=4$ level, we have had little to say thus far about its rate. In order to obtain a fairly reliable estimate of a rate for the reaction

$$\text{He}+\text{He}+\text{mesonic atom}\rightarrow\text{He}+\text{molecular ion},$$

it appears necessary to consider a large number of reacting atoms initially in a state of thermal equilibrium.[27] We have not attempted such a calculation because we are unconvinced that the mesonic atom has already been slowed to thermal velocities when this reaction takes place in liquid helium. However, we note that when MIF occurs in liquid in the $n=4$ level, it should occur at a rate greater than, say, $3\times 10^{11}\ \text{sec}^{-1}$ because the deexcitation rates in Table I indicate that radiation or collisional Auger effect would otherwise occur instead. But we further note that this approximate lower limit need apply only to atoms that have been slowed to velocities that are relatively low, though not necessarily thermal. In other words, it is entirely conceivable that the MIF rate might be substantially smaller at higher energies. An MIF rate that decreases with increasing kinetic energy of the mesonic atom would be entirely compatible with some estimates of the temperature dependence of atomic recombination rates.[27]

Our inability to arrive at a single, well established explanation of the $K\beta/K\alpha$ intensity ratios is due to our inability to obtain accurate estimates of the rates for Stark transitions and MIF. But whatever these rates are, they should depend on the density of the helium. In liquid, the density dependence of either of these rates might be complicated because of many-body effects; but in a dilute gas, and for given values of the temperature of the helium and the kinetic energy of the mesonic atom, the rate for a Stark transition should vary linearly with the density because only binary collisions should be important, while the rate for MIF should vary quadratically because ternary collisions should be required. In view of this, it would be desirable to have measurements of the pionic $K\beta/K\alpha$ intensity ratio in gaseous helium. X-ray measurments have already been made for muons in gas,[1] but such an experiment has not, to our knowledge, been performed with pions. A determination of the pressure dependence of the pion ratio would be especially interesting, particularly if very high pressures could be employed, as they have been in some muonic x-ray experiments.[28,29] However, even a single measurement at a relatively low pressure would be useful because it is difficult to believe that MIF in the $n=4$ level could compete with radiative transitions in gas at, say, 8 atm, where the density is ~100 times smaller than in liquid. Furthermore, $3d \to 3p$ Stark transitions should also be less probable at such a pressure. Therefore, if the arguments in Sec. III are correct, it seems most unlikely that the pionic $K\beta/K\alpha$ intensity ratio would be nearly as high in a relatively low-pressure gas as it is in liquid.

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[2]See, for example, H. H. Michels, J. Chem. Phys. 44, 3834 (1966), and references therein.

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[4]S. Berezin, G. Burleson, D. Eartly, A. Roberts, and T. O. White, Phys. Lett. 30B, 27 (1969); Phys. Rev. A 2, 1630 (1970).

[5]G. Backenstoss, J. Egger, T. von Egidy, R. Hagelberg, C. J. Herrlander, H. Koch, H. P. Povel, A. Schwitter, and L. Tauscher, Nucl. Phys. A232, 519 (1974).

[6]C. E. Wiegand and R. H. Pehl, Phys. Rev. Lett. 27, 1410 (1971).

[7]J. E. Russell, Proc. Phys. Soc.Lond. 85, 245 (1965).

[8]C. A. Coulson and W. E. Duncanson, Proc. R. Soc. Lond. A 165, 90 (1938).

[9]See, for example, L. I. Schiff, *Quantum Mechanics*, 3rd ed. (McGraw-Hill, New York, 1968), p. 260.

[10]See, for example, C. C. J. Roothaan, J. Chem. Phys. 19, 1445 (1951).

[11]These results differ slightly from those given in Ref. 7, where $\gamma^{(1)}$, but not $\gamma^{(2)}$, was calculated assuming that $A_1=1.0$, $A_2=0.0$, and $Z_a=1.69$ for all $R>1.0a_0$. Also, it was tacitly assumed in Ref. 7 that $S_a=1.0$, which is an excellent approximation if $Z_a=1.93$ and $Z_c=2.0$ or if $Z_a=1.69$ and $Z_c=2.0$. The term $\gamma^{(3)}$, which is shown in the present paper to be quite unimportant, was not considered in Ref. 7.

[12]J. E. Russell, Phys. Rev. 188, 187 (1969).

[13]This is in contrast to the results obtained in Ref. 7. In that paper, where much more highly excited mesonic states were considered, it was found in some instances that $\gamma(R_0) \simeq \gamma^{(1)}(R_0)$.

[14]J. E. Russell and G. L. Shaw, Phys. Rev. Lett. 4, 369 (1960).

[15]M. Leon and H. A. Bethe, Phys. Rev. 127, 636 (1962); see also D. N. Michael, *ibid.* 158, 1343 (1967).

[16]According to an analysis by Backenstoss *et al.* (Ref. 5) of x-ray data obtained in liquid helium, the nuclear absorption rate for $2p$ pions is $\Gamma_{2p}=(1.1\pm0.5)\times10^{12}$ sec^{-1}. This implies that the nuclear rate for $4p$ pions is $\Gamma_{4p}=(1.7\pm0.8)\times10^{11}$ sec^{-1}. However, it was emphasized in Ref. 5 that this rate should be employed with a certain amount of caution because the data analysis from which it was derived does not fully account for all of the experimental results. Nevertheless, we shall employ the value of Γ_{2p} obtained in Ref. 5, because even if that result is in error by a factor of 2, none of our arguments should be seriously affected.

[17]The reason given in Ref. 5 for neglecting Stark transitions in the data analysis is that including them would cause the muonic $K\alpha$, $K\gamma$, and $K\delta$ lines to be too intense and the $K\beta$ line to be too faint. However, preliminary calculations that did take into account Stark transitions apparently did not take into account recoil transitions, which are discussed in Sec. III of the present paper. Also, a rather low upper limit to a strength factor for Stark transitions was obtained in Ref. 5 by assuming that all pions decaying during the cascade are in the $n=16$ level of the positively charged $\alpha\pi^-$ atom and have $l>13$. We note that it may also be possible to account for the decaying pions by assuming that they are trapped in orbits of the neutral $\alpha\pi^-e^-$ atom with high n and l: J. G. Fetkovich, B. R. Riley, and I-T. Wang, Phys. Lett. 35B, 178 (1971).

[18]T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Lett. 3, 61 (1959).

[19]E. Campani, Lett. Nuovo Cimento 4, 982 (1970).

[20]R. O. Mueller, V. W. Hughes, H. Rosenthal, and C. S. Wu, Phys. Rev. A 11, 1175 (1975).

[21]The possibility of irreversible Stark transitions is mentioned briefly in Ref. 5.

[22]This paragrah is really an amplification of a remark made by Day, Snow, and Sucher (Ref. 18), who noted that Stark mixing between two states will be inhibited if the matrix element of the perturbing interaction is small compared to the energy splitting involved.

[23]To the extent that the curve in Fig. 1(b) is accurate, the value of $\langle|B|_r^2\rangle$ can be regarded also as a rough upper limit to the amount of Stark mixing in a collision, though in many instances the actual amount of collisional mixing can reasonably be expected to be much smaller, if only because of adiabatic effects. Adiabatic effects should not be of importance in recoil transitions because the onset of the perturbation occurs in a time $\lesssim 10^{-17}$ sec, which is short compared to $\hbar/|\Delta E|$ in each of the instances considered in Table III.

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