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Comments and Addenda

The section Comments and Addenda is for short communications which are not appropriate for regular articles. It includes only the following types of communications: (1) Comments on papers previously published in **The Physical Review** *or* **Physical Review Letters**. *(2) Addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section must be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts follow the same publication schedule as articles in this journal, and page proofs are sent to authors.*

Comments on "Simulation of large magnetic islands: A possible mechanism for a major tokamak disruption"

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In this Comment it is established that the numerical results for the nonlinear evolution of the $m = 2$ tearing mode are affected by the density of the poloidal grid. A coarse grid gives results that are quantitatively and qualitatively incorrect. The important qualitative error is that with a coarse grid the growth of the width of the associated magnetic island is exponential rather than algebraic.

In Ref. 1, it was reported that for a certain safety factor profile and sufficiently large values of the resistivity, the $m=2/n=1$ magnetic island grows exponentially until it saturates at a width of $0.7a$, where a is the minor radius; here m and n are the poloidal and toroidal mode numbers, respectively. It is the purpose of this Comment to clarify this phenomenon and point out that the results in Ref. 1 were severely affected by numerical error associated with the coarse poloidal grid. Specifically, if a fine poloidal grid is employed, the island grows algebraically rather than exponentially. It attains a maximum width of $0.48a$, relaxes, and eventually saturates at $0.37a$

The issue of whether or not the growth is exponential for some profiles is important because if the growth is exponential, then the associated time scale is consistent with the time scale for the major disruption. If, on the other hand, the island grows algebraically with time, then the fast disruption time scale can be explained only by arguing that the disruption occurs when the island touches the limiter.

The safety factor profile employed in Ref. 1 can be written in general as

$$q(r)=q(0)[1+(r/r_0)^{2\lambda}]^{1/\lambda},$$

where $q(r)$, r_0, and λ are arbitrary parameters. For the specific profile studied in Ref. 1, $q(0)=1.378$, $r_0=0.6$, and $\lambda=4$. The value of S (the ratio of the resistive diffusion time and the poloidal magnetohydrodynamic time) was taken to be 10^4 at the $q=2$ surface.

The results that we obtained using our version of the MASS code[2] are summarized in Fig. 1. For a uniform radial grid of 60 points, the magnetic island width W as a function of time obtained with a uniform poloidal grid of eight points per quadrant (employed in Ref. 1) is compared in Fig. 1(a) with the result obtained with a uniform poloidal grid of 24 points per quadrant. The time is normalized to the skin time (τ_R). We observe that with $N=8$, where N denotes the number of poloidal grid points per quadrant, the behavior of the island width for $t \leqslant 0.03\tau_R$ is the same as that reported in Ref. 1 (as expected, since the two codes are essentially identical). However, the remarkable relaxation of the island for $t > 0.03\tau_R$ [as shown in Fig. 1(a)] was not reported in Ref. 1, where it was stated that the saturation width is $0.7a$. A detailed description of the corresponding relaxation of the toroidal current density is presented in Ref. 3. More importantly, however, as the poloidal grid is made finer, the maximum value of the island width decreases by about 30%. Judging from the results for N between 8 and 24, we conclude that an N of 24 is sufficiently large to give reliable results. In addition, for $N=24$ the saturation and peak widths do not differ enormously; the saturation width according to the quasilinear analysis in Ref. 4 is $0.28a$, approximately 25% smaller than the numerical value of $0.37a$, which is 50% smaller than the value of $0.7a$ given in Ref. 1. The radial grid of 60 points has been shown to be

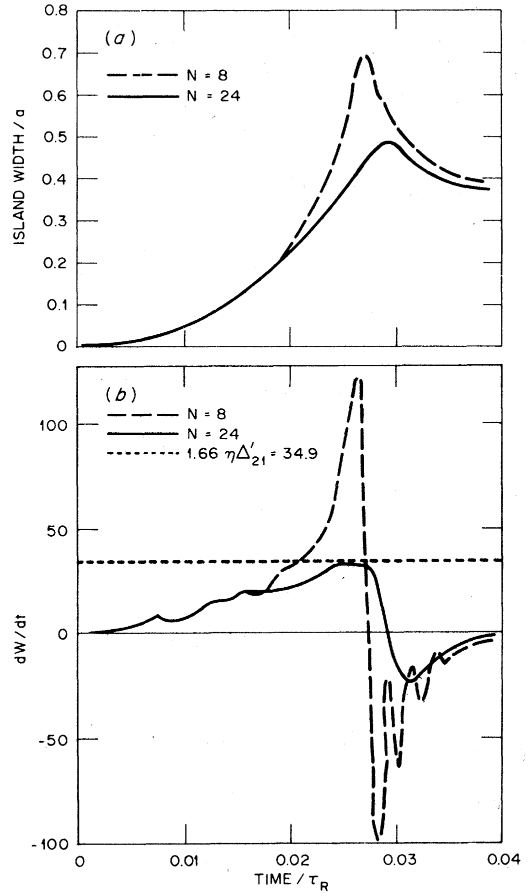

FIG. 1. (a) $m=2/n=1$ island width plotted as a function of time for different poloidal grid densities for $S=10^4$. (b) The rate of change of the island width plotted as a function of time for different poloidal grid densities for $S=10^4$.

sufficiently fine, and similar results have been obtained for smaller values of S.

In Fig. 1(b), the quantity dW/dt is plotted as a function of t. The analytic result that $dW/dt = 1.66\,\eta\,\Delta'$ for W larger than the tearing layer width[4,5] is indicated by the dashed line; here η is the resistivity at the $q=2$ surface and Δ' is the discontinuity in the logarithmic derivative of the flux eigenfunction. For $N=8$, the numerical results are substantially different from the analytic results in that dW/dt is much larger than $1.66\,\eta\,\Delta'$ for $W \gtrsim 0.4a$. For $N=24$, however, the numerical results are similar to the analytic results[4,5] and to the numerical results reported in Refs. 6 and 7 in that the growth rate is slow and algebraic rather than exponential. The island growth rate does not become exactly constant because S is small; nevertheless, the island behavior exhibited in Fig. 1(b) for $N=24$ is similar to the behavior for other profiles for which there are no grid problems. (A plot of dW/dt on a logarithmic scale clearly shows that for $N=8$ the island growth is exponential or faster, while for $N=24$, the growth is much less rapid.)

In Ref. 1, it was reported that for $S=10^5$, the island grows linearly with time once it exceeds the tearing layer width until it reaches a width of $0.4a$; the run was not continued because of the large amount of computer time required. For $S=10^5$, a regime where dW/dt is constant, is more clearly demonstrated because the tearing layer width ϵ_T is smaller and thus $W>\epsilon_T$ for a longer period of time. On the basis of Fig. 1, we conclude that no exponential behavior was observed for $S=10^5$ because the code was not run past $W=0.4$, whereas the clear exponential growth due to numerical error occurs mainly for $W \gtrsim 0.4$.

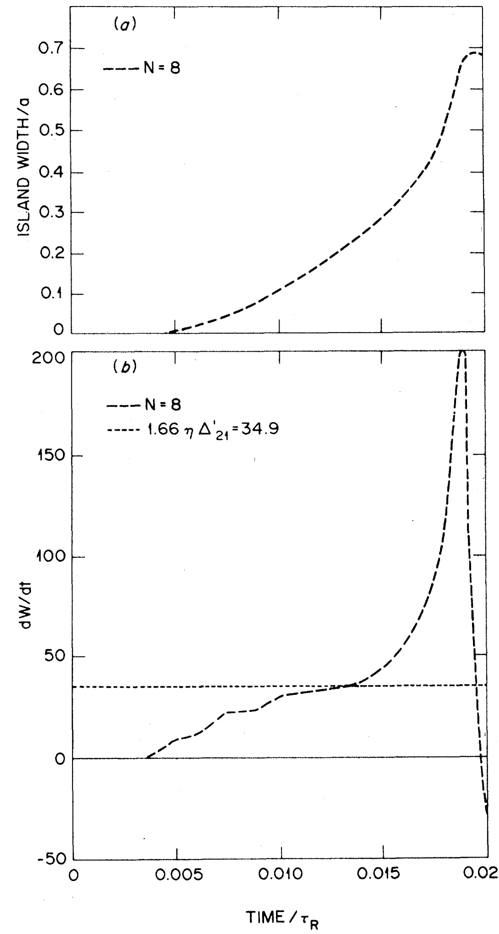

FIG. 2. (a) $m=2/n=1$ island width plotted as a function of time for $N=8$ and $S=10^5$. (b) The rate of change of the island width plotted as a function of time for $N=8$ and $S=10^5$.

In fact, we have run the case $S=10^5$ and found that it does exponentiate for $N=8$ (Fig. 2); $N=24$ was not employed because the computer time required is too large.

Finally, we would like to emphasize that the numerical problems associated with the poloidal grid apparently occur only for cases where the island width becomes very large and thus a large number of poloidal flux harmonics is required. For example, for the safety factor profile specified by $q(0)=1.08$, $r_0=0.5$, and $\lambda=4$, the $m=2$ island exhibits the standard behavior reported in Refs. 6 and 7; i.e., it grows slowly even for $N=8$ once its width exceeds the tearing layer width and then saturates at a width of $0.31a$. Since this case is very similar to the one studied in Ref. 1 and discussed here, apparently the MASS code results for the case studied in Ref. 1 are somewhat pathological.

Consequently, on the basis of the preceding analysis, we conclude that in all cases studied the $m=2$ island width grows slowly (algebraically) from the time it exceeds the tearing layer width until it saturates, irrespective of the value of S. As is obvious from Fig. 1(a) the maximum width of the island, although not as large as reported in Ref. 1, is large enough to hinder confinement. The exponential growth reported in Ref. 1, however, is due to the numerical error associated with the poloidal grid.

ACKNOWLEDGMENT

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