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Deflection of atoms by a resonant standing electromagnetic wave

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Deflection of an atom due to momentum transfer from a strong resonant standing electromagnetic wave is investigated theoretically in the limit of short atom-field interaction time. The translational and internal motions of the atom are treated quantum mechanically, while the field is treated classically. It is shown that momentum transfer from a standing wave to an atom proceeds at the induced or Rabi rate, rather than the spontaneous rate characteristic of radiation pressure. In a typical case, atomic deflections of order 1° are achieved with 10^6 W/cm^2 field intensity in a time less than the natural lifetime of the excited atom.

I. INTRODUCTION

The use of a resonant electromagnetic wave, or a combination of resonant and static fields, to deflect a beam of neutral atoms has been the subject of renewed interest since the advent of higher-power tunable lasers. A potential application of laser deflection is to problems of laser isotope separation.

Several methods of photodeflection have been proposed,[1-6] some of which have been demonstrated experimentally.[7-11] Most of these methods require an interaction time that is long compared to the natural lifetime of the excited atoms. This makes the practical application of these methods impossible in many cases, because an atom excited by the resonant radiation makes transitions to metastable states, which are not affected by the applied field.[9] Such transitions remove atoms from the interaction cycle, and little or no deflection is produced. The purpose of this paper is to show that in a strong resonant standing wave, significant atomic deflections can occur in a time less than the spontaneous lifetime of the excited atom, and hence the problem of transitions to metastable levels is circumvented by the speed of the process.

When an atomic beam is irradiated by a strong resonant electromagnetic wave, absorption-emission processes proceed at two distinct rates. Photons are absorbed from and emitted into the applied field at the induced rate Ω, and occasionally photons are spontaneously emitted, in random directions, at the spontaneous rate γ. Deflection or scattering of the atomic beam results when momentum is transferred from the field to the atoms, and the rate of momentum transfer depends on the nature of the applied field.

If the applied field consists of a single plane wave, momentum is transferred to the atoms at the spontaneous rate γ. This momentum transfer, i.e., radiation pressure, proceeds at the spontaneous rate, because absorption followed by induced emission into the same field mode involves no net transfer of momentum, while absorption followed by spontaneous emission transfers an average of one quantum of momentum for each spontaneous event (isotropic spontaneous emission does not carry away the momentum acquired by the atom through absorption).

If the applied field is composed of two or more plane waves, an atom can absorb a photon from one of the plane waves, and induced emission can cause that photon to be emitted into a different plane wave, with a resultant transfer of momentum at the induced rate Ω. Since the induced rate may exceed the spontaneous rate by many orders of magnitude in a strong applied field, it is expected that deflection processes operating at the induced rate will be more efficient and more rapid than processes that operate at the spontaneous rate. In the following we shall show that momentum transfer in a standing wave proceeds at the induced rate, and that it is this feature of the interaction that gives rise to the rapid deflection mentioned above.

In the model adopted here, the internal motion of the atom is treated as a two-level system. The center-of-mass motion of the atom is treated quantum mechanically, and the resonant standing wave is treated as a classically prescribed electric field. Analytical solutions of the Schrödinger equation are obtained, in the rotating-wave approximation, on the assumption that the Doppler width associated with initial beam spread and subsequent atomic deflection is small compared to the frequency width associated with the finite time during which the atom interacts with the resonant radiation.

In Sec. II, the theory of deflection of an atom by a resonant standing wave is developed, and the effect of finite divergence in a beam of atoms is briefly discussed. In Sec. III the limit of validity of our assumption concerning Doppler effect is examined, and a numerical example is given to illustrate the magnitude of deflections obtainable.

II. THEORY

The Hamiltonian for an atom in a classically prescribed electromagnetic field, in the dipole approximation, takes the familiar form

$$H = P^2/2M + H_0 - \vec{\mu}\cdot\vec{E}(\vec{R}, t), \tag{1}$$

where $P^2/2M$ is the kinetic energy associated with the center-of-mass momentum $\vec{P}$, H_0 is the Hamiltonian for the internal notion of the unperturbed atom, $\vec{\mu}$ is the dipole moment operator, and $\vec{E}(\vec{R}, t)$ is the electric field evaluated at the center-of-mass position $\vec{R}$.

We shall calculate the motion of an atom that starts out moving in the positive z direction, enters a region of resonant radiation at $z = 0$, and exits the interaction region at $z = L$. The electric field in the interaction region is taken to be a standing wave of the form

$$\vec{E}(x, t) = 2(8\pi I/c)^{1/2}\hat{\epsilon}\cos kx \cos\omega t, \tag{2}$$

which is equivalent to two plane waves, each of intensity I, counterpropagating along the x axis. The polarization vector $\hat{\epsilon}$ is a unit vector transverse to the x direction.

For an electric field of this form, only the x coordinate of the center of mass appears in the Hamiltonian. It follows that motion of the atom in the y and z directions is unaffected by the field, and only motion in the x direction is of interest. Elimination of the inessential degrees of freedom yields the Hamiltonian

$$H = P_x^2/2M + H_0 - \vec{\mu}\cdot\vec{E}(x, t) \tag{3}$$

for atomic motion in the x direction. As the atom moves along the z axis, the interaction term in Eq. (3) is switched on as the atom enters the interaction region, and is switched off as it leaves this region.

Upon exiting the interaction region, the atom has a certain probability density $W(p)$ for momentum p in the x direction. This momentum density determines the probability density for displacement x as $z\to\infty$, $P(x) = (p_z/z)W(p_z x/z)$. If the deflections are small ($x/z \ll 1$, $\theta \approx x/z$), the probability density for deflection θ is

$$P_\theta(\theta) = p_z W(p_z\theta), \tag{4}$$

where p_z is the z component of atomic momentum.

To obtain the transverse momentum density $W(p)$, we solve the Schrödinger equation in the momentum representation. We start by writing down the general equations of motion, and then simplify these by using the two-level-atom and rotating-wave approximations. The unperturbed Hamiltonian $H' = P_x^2/2M + H_0$ has eigenvectors $|n, p\rangle = |n\rangle|p\rangle$ and eigenvalues $\epsilon_n(p) = p^2/2M + E_n$, where $|n\rangle$ and E_n are the eigenvectors and eigenvalues of H_0, and $|p\rangle$ is the eigenvector of P_x with eigenvalue p. An arbitrary state vector is expanded as

$$|\psi\rangle = \sum_n \int dp\, \phi_n(p)|n, p\rangle, \tag{5}$$

where $\phi_n(p)$ is the amplitude for momentum p and internal energy E_n. Upon substituting this expansion into the Schrödinger equation, $i\hbar\partial|\psi\rangle/\partial t = H|\psi\rangle$, and using the orthonormality of the basis states $|n, p\rangle$, we obtain the equations of motion

$$i\hbar\frac{\partial\phi_n(p)}{\partial t} = \sum_m \int dp'\, \langle n, p|H|m, p'\rangle\phi_m(p'). \tag{6}$$

Evaluation of the matrix elements $\langle n, p|H|m, p'\rangle$, using Eqs. (2) and (3), is straightforward. The explicit equations of motion are

$$i\hbar\frac{\partial\phi_n(p)}{\partial t} = \epsilon_n(p)\phi_n(p) - \cos\omega t\sum_m g_{nm}[\phi_m(p-\hbar k) + \phi_m(p+\hbar k)], \tag{7}$$

where $g_{nm} = (8\pi I/c)^{1/2}\langle n|\vec{\mu}\cdot\hat{\epsilon}|m\rangle$. Equation (7) shows that a change of the excitation state of the atom is accompanied by a transfer of momentum $\pm\hbar k$. Note that $g_{nn} = 0$ if the atomic levels are nondegenerate. The transformation

$$\phi_n(p) = C_n(p)\exp[-i\epsilon_n(p)t/\hbar], \tag{8}$$

with $\epsilon_n(p) = p^2/2M + E_n$, puts Eq. (7) in the form

$$i\hbar\frac{\partial C_n(p)}{\partial t} = -\cos\omega t\sum_m g_{nm}\{C_m(p-\hbar k)\exp[-i(\omega_{mn}+\delta-\omega p/Mc)t] + C_m(p+\hbar k)\exp[-i(\omega_{mn}+\delta+\omega p/Mc)t]\}, \tag{9}$$

where $\omega_{nm}=(E_n-E_m)/\hbar$, $\delta=\hbar\omega^2/2Mc^2$, and the amplitudes $C_n(p)$ are now slowly varying functions of time (interaction picture).

If an atom, initially at rest, absorbs a photon of energy $\hbar\omega$ and momentum $\hbar\omega/c$, the internal energy of the atom increases by the amount $\hbar\omega_{mn}$, its kinetic energy increases by the amount $(\hbar\omega/c)^2/2M=\hbar\delta$, and conservation of energy $\hbar\omega=\hbar\omega_{mn}+\hbar\delta$ shows that the resonant frequency of the transition is $\omega=\omega_{mn}+\delta$. Thus δ is a frequency shift associated with recoil of the atom. The quantities $\pm\omega p/Mc$ are Doppler shifts due to motion of the atom in the x direction. If the interaction time is sufficiently short, then both recoil and Doppler shifts can be neglected. The condition that $\omega pt/Mc\ll 1$ has the simple physical meaning that the frequency width associated with finite transit time of the atom across the field (transit time broadening) is large compared to the accumulated Doppler shift. We make this assumption of short interaction time in the following analysis, and discard exponential factors of the form $\exp[-i(\delta\pm\omega p/Mc)t]$ in Eq. (9).

We shall consider the case where the applied field is resonant with only a single atomic transition ($\omega=\omega_{mn}$, $m=+$, $n=-$). Then if we neglect all but the two amplitudes $C_\pm(p)$ involved in the transition (two-level atom approximation),[12] expand $\cos\omega t$ in exponentials, and keep only terms on the right in Eq. (9) that vary slowly with time (rotating-wave approximation),[12] Eqs. (9) reduce to

$$\begin{aligned}\dot{C}_+(p)&=(i\Omega/2)[C_-(p-\hbar k)+C_-(p+\hbar k)]\,,\\ \dot{C}_-(p)&=(i\Omega/2)[C_+(p-\hbar k)+C_+(p+\hbar k)]\end{aligned}\tag{10}$$

where $\Omega=(8\pi I/c\hbar^2)^{1/2}\langle -|\vec{\mu}\cdot\hat{\epsilon}|+\rangle$. The phases of $|\pm\rangle$ are chosen so that Ω is real.

Equations (10) may be solved exactly. The transformation

$$\begin{aligned}D_+(p)&=[C_+(p)+C_-(p)]/2^{1/2}\\ D_-(p)&=[C_+(p)-C_-(p)]/2^{1/2}\end{aligned}\tag{11}$$

decouples Eqs. (10) as

$$\begin{aligned}\dot{D}_+(p)&=(i\Omega/2)[D_+(p-\hbar k)+D_+(p+\hbar k)]\,,\\ \dot{D}_-(p)&=-(i\Omega/2)[D_-(p-\hbar k)+D_-(p+\hbar k)]\,.\end{aligned}\tag{12}$$

Upon substituting the trial solution

$$D_\pm(p)=\exp[isp\pm i\alpha t]\tag{13}$$

into Eqs. (12), we obtain the dispersion relation

$$\alpha(s)=\Omega\cos\hbar ks\tag{14}$$

for waves in momentum space. Physically acceptable solutions are obtained for all real values of s. The general solution of Eqs. (12) is a superposition of waves

$$D_\pm(p,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}a_\pm(s)\exp[isp\pm i\alpha(s)t]\,ds\,.\tag{15}$$

At $t=0$, Eq. (15) reduces to a Fourier transform relation between $a_\pm(s)$ and $D^0_\pm(p)=D_\pm(p,0)$. When the inverse of this transform, namely,

$$a_\pm(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}D^0_\pm(p)\exp(-isp)\,dp\,,\tag{16}$$

is substituted into Eq. (15), we obtain

$$D_\pm(p,t)=\int_{-\infty}^{\infty}G_\pm(p-p',t)D^0_\pm(p')\,dp'\,,\tag{17}$$

where

$$G_\pm(p,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp[\pm i\alpha(s)t+ips]\,ds\,.\tag{18}$$

With the help of the dispersion relation Eq. (14) and the identity

$$e^{\pm iz\cos\theta}=\sum_{n=-\infty}^{\infty}(\pm i)^nJ_n(z)e^{\mp in\theta}\,,$$

the propagators $G_\pm(p,t)$ are readily evaluated as series of Bessel functions

$$G_\pm(p,t)=\sum_{n=-\infty}^{\infty}(\pm i)^nJ_n(\Omega t)\delta(p-n\hbar k)\,,\tag{19}$$

and the general solution Eq. (17) becomes

$$D_\pm(p,t)=\sum_{n=-\infty}^{\infty}(\pm i)^nJ_n(\Omega t)D^0_\pm(p-n\hbar k)\,.\tag{20}$$

Consider the case where the atom is in the lower state and has momentum $p=0$ at $t=0$. In this case, $C^0_+(p)=0$, $C^0_-(p)=[\delta(p)]^{1/2}$, and from Eqs. (11), $D^0_\pm=\pm[\frac{1}{2}\delta(p)]^{1/2}$ (square root of the δ-function is used here so that the probability that the atom is in the ground state, namely, $P^0_-=\int|C^0_-(p)|^2\,dp$, is properly normalized to unity). It follows from (11) and (20) that the momentum probability density $W(p)=|C_+|^2+|C_-|^2=|D_+|^2+|D_-|^2$ has the form

$$W(p,t)=\sum_{n=-\infty}^{\infty}J_n^2(\Omega t)\delta(p-n\hbar k)\,.\tag{21}$$

Equation (21) states that the probability $P_n(t)$ that the atom has acquired momentum $n\hbar k$ ($n=0$, ±1, $\pm2,\dots$) is

$$P_n(t)=J_n^2(\Omega t)\,.\tag{22}$$

In cases of practical interest, Ωt is a large number. For $|n|<\Omega t$, the probability $J_n^2(\Omega t)$ is not a monotonic function of n, but tends to increase with $|n|$ and has maximum near $|n|=\Omega t$. For $|n|>\Omega t$, $J_n^2(\Omega t)$ decreases rapidly to zero as $|n|$ increases. In view of Eq. (4), the maximum deflection is $\theta_{max} \approx \hbar k \Omega t / p_z$. This result shows quite clearly that momentum is transferred to the atom at the induced rate Ω.

The mean magnitude of momentum transferred to the atom,

$$\langle |p| \rangle = \sum_{n=-\infty}^{\infty} \hbar k |n| J_n^2(\Omega t), \tag{23}$$

can be expressed in the closed form[13]

$$\langle |p| \rangle = \hbar k (\Omega t)^2 [J_0^2(\Omega t) + J_1^2(\Omega t)] - \hbar k \Omega t J_0(\Omega t) J_1(\Omega t), \tag{24}$$

and approaches the value $\langle |p| \rangle = 2\hbar k \Omega t/\pi \approx 0.64 \hbar k \Omega t$ as $\Omega t \to \infty$. The rms momentum, at any time t, is given by the simple formula $[\langle p^2 \rangle]^{1/2} = \hbar k \Omega t/\sqrt{2} \approx 0.71 \hbar k \Omega t$.[14] Thus the spread of momentum increases linearly with time.

In the above example, we assumed that the incident atomic beam has sharp momentum $p=0$, i.e., the incident beam is a plane wave, and hence has infinite transverse extension. For a finite collimated beam of width Δx, the initial spread of momentum is $\Delta p \sim \hbar/\Delta x$, and the ratio of this spread to the momentum delivered by a single photon is $\Delta p/\hbar k \sim \lambda/2\pi\Delta x$. It follows that there is a little or no overlap of the terms in Eq. (20) when Δx is large compared to the optical wavelength, and the probability density for momentum becomes

$$W(p,t) = \sum_{n=-\infty}^{\infty} J_n^2(\Omega t) W^0(p - n\hbar k), \tag{25}$$

where $W^0(p)$ is the initial momentum density. The pattern of deflections, in this case, is the same in all essential details as that discussed above.

If the atomic beam diverges with half-angle of, say, $\theta = 10^{-3}$ rad, and has a typical thermal velocity $v_z \sim 5 \times 10^4$ cm/sec, then the initial spread of transverse momentum is not small compared with $\hbar k$, and it is expected that interference due to overlap of the terms in Eq. (20) will affect the probability density $W(p,t)$. It turns out, however, that, due to a rapidly varying phase factor associated with divergence of the beam, the scale of such interference is small compared to $\Delta p = \hbar k$, and therefore is almost certainly unobservable. We do not present this calculation, because the smoothed distribution is the same as Eq. (25).

It is interesting to note that Eq. (21) is formally identical to the equation for Fraunhofer diffraction of a plane wave by a sinusoidal phase grating.[15] In effect, the atomic beam is diffracted by the periodic amplitude $[E(x) \propto \cos kx]$ of the standing wave, and the deflection angles $\theta_n = n\hbar k/p_z$ are precisely what one would expect on the basis of the optical analogy, if the atomic beam is regarded as a wave of wavelength equal to the de Broglie wavelength $\lambda_a = h/p_z$.

III. NUMERICAL EXAMPLE

Our theory is based on the approximation that recoil and Doppler frequency shifts $\delta = \hbar\omega^2/2Mc^2$ and $\pm\omega p/Mc$, respectively, are negligible. Accordingly, exponential factors of the form $\exp[-i(\delta \pm \omega p/Mc)t]$ were replaced by unity in Eq. (9). This approximation is valid when $(\delta + \omega|p|/Mc)t \ll 1$. The recoil shift is half the Doppler shift when $|p| = \hbar k$. Since we are only interested in cases where $|p| \gg \hbar k$, the above condition becomes $\omega|p|t/Mc \ll 1$. Replacing $|p|$ by the rms value $[\langle p^2 \rangle]^{1/2} = \hbar\omega\Omega t/\sqrt{2}\,c$ derived above, we obtain a constraint on the interaction time

$$t < (2^{1/2} Mc^2/\hbar\omega^2\Omega)^{1/2}. \tag{26}$$

The maximum interaction time permitted by Eq. (26), t_{max}, determines the thickness of the interaction region $L = v_z t_{max}$, and the number of absorption-emission processes experienced by the atom, $n = \Omega t_{max}$. If the atoms issue from an oven at temperature T, $p_z \approx (2MkT)^{1/2}$ and $v_z \approx (2kT/M)^{1/2}$ (here k is Boltzmann's constant). The rms deflection is

$$\theta_{rms} = \hbar\omega\Omega t_{max}/2^{1/2} c p_z = [\hbar\Omega/2^{3/2} kT]^{1/2}. \tag{27}$$

Consider a mildly refractory, moderately massive atom with a strong visible absorption. Let

$$T = 1000\ \text{K}, \quad M = 1.6 \times 10^{-22}\ \text{g},$$

$$\mu = \langle - | \vec{\mu} \cdot \hat{\epsilon} | + \rangle = 4\ \text{D}, \quad \omega = 3 \times 10^{15}\ \text{sec}^{-1}.$$

Then, for $I = 2.5 \times 10^6$ W/cm^2 in the interaction region, we calculate

$$\Omega = 5.6 \times 10^{11}\ \text{sec}^{-1}, \quad n = 3500$$

$$t_{max} = 6.2 \times 10^{-9}\ \text{sec}, \quad L = 2.6 \times 10^{-4}\ \text{cm}$$

$$\theta_{rms} = 3.9 \times 10^{-2}\ \text{rad} = 2.2°.$$

Thus a 2° deflection is obtained for our "typical" atom in a field of 2.5×10^6 W/cm^2. The interaction time required for this deflection is less than the natural lifetime of the transition ($\tau \approx 5 \times 10^{-8}$ sec). To achieve the corresponding 2.6-μm beam thickness requires focusing a 2.6-mm diam. laser beam in one dimension by a factor of 10^3. Thus for an unfocused laser intensity of 2.5 kW/cm^2, a total laser power of about 130 W is required.

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