Electron-impact widths and shifts of neutral helium lines in a plasma

J. M. Bassalo
Departamento de Física, Universidade Federal do Pará, Pará, Brazil

M. Cattani
Instituto de Física, Universidade de São Paulo, Caixa Postal 20516, São Paulo, Brazil

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The quantum-mechanical formalism developed by Bassalo and Cattani is used to calculate the broadening and shift of many atomic lines of neutral helium in a plasma. We verify that this theory overestimates the linewidths and that it can be applied satisfactorily only to obtain the line shifts.

I. INTRODUCTION

The width and the shift of spectral lines produced by electronic collisions have been extensively calculated using the semiclassical formalism carried out by many authors.[1] Using a quantum-mechanical formalism,[2-6] Bassalo and Cattani[7-9] have calculated the broadening and shift of a few atomic lines of neutral helium in a plasma[10-13] produced by electronic collisions.

The effect of the ions on the line shape was calculated using the approach developed by Griem *et al.*[14] and Griem.[1,15]

According to Bassalo and Cattani[7,8] the half-half-width $\Delta\nu_e$ and the shift S_e produced by electronic collisions for an isolated and Lorentzian line for a M-degenerate energy states $|\alpha JM\rangle$ are given by

$$\Delta\nu_e = \mathrm{Re}(\overline{H}_{IF})/2\pi, \quad S_e = -\mathrm{Im}(\overline{H}_{IF})/2\pi, \tag{1}$$

where the indices I and F refer to the initial and final states of the line, respectively,

$$\overline{H}_{IF} = Nh^2\left(\frac{\beta}{2\pi m}\right)^{3/2}\int d^3q\, e^{(-\beta q^2/2m)}\langle\vec{q}\,|L_{IF}|\,\vec{q}\rangle\,. \tag{2}$$

N is the density of perturbing electrons, $\beta = 1/K_BT$, K_B is the Boltzmann constant, T is the absolute temperature, m is the reduced mass of the electron and atom, $\vec{q}$ is the relative linear momentum, and L_{IF} for the transition $|\alpha_I J_I\rangle \to |\alpha_F J_F\rangle$ is given by

$$L_{IF} = A_{IF}\sum_{M_I M_F} C_{M_I M_F}[2\pi i\langle\alpha_I J_I M_I|T|\alpha_I J_I M_I\rangle\delta_{M_I M_F} - 2\pi i\langle\alpha_F J_F M_F|T^{+}|\alpha_F J_F M_F\rangle\delta_{M_I M_F}$$
$$- 4\pi^2\langle\alpha_I J_I M_I|T\delta(E_I - H_0)|\alpha_I J_I F_F\rangle\langle\alpha_F J_F M_F|T^{+}|\alpha_F J_F M_F\rangle], \tag{3}$$

where

$$A_{IF} = 3(-1)^{J_I + J_F}/[(2J_I+1)(2J_F+1)]^{1/2}$$

and

$$C_{M_I M_F} = \langle J_F 1 M_F 0\,|J_F M_F\rangle\langle J_I 1 M_I 0\,|J_F M_I\rangle\,.$$

The interaction potential V between the electron and the atom taking the atomic nucleus as the center of the coordinate system, is given by[8]

$$V = Ze^2\exp\left(\frac{-R/l_D}{R}\right) - \sum_a e^2\exp\left(\frac{|\mathrm{R}-\mathrm{r}_a|/l_D}{|\vec{\mathrm{R}}-\vec{\mathrm{r}}_a|}\right)\,. \tag{4}$$

$\vec{\mathrm{R}}$ and $\vec{\mathrm{r}}_a$ are the positions of the incident and of the ath atomic electron, respectively, the Debye length is given by $l_D = [K_BT/4\pi Ne^2(1+z)]^{1/2}$, and z is the ionic charge of the ions in the plasma. It is assumed that the plasma contains only one kind of ion of electric charge ze; the neutrality of the plasma is expressed by the relation $N_I z = N$, where N_I is the ionic density.

Expanding the T matrix up to the second Born approximation (see comments about this in Ref. 7) and performing the calculations[7-13] we obtain

$$\Delta\nu_e = \frac{Ne^4}{2\sqrt{\pi}}\left(\frac{\beta^3}{2m}\right)^{1/2}\left(\sum_n [W_{In}(\beta;\Delta_{In}) + W_{Fn}(\beta;\Delta_{Fn})] - 2W_{IF}(\beta;\Delta=0)\right) \tag{5}$$

and

$$S_e = -\frac{N}{h}(V_I - V_F) - \frac{Ne^4}{\pi}\left(\frac{\beta^3}{2m}\right)^{1/2}\times\sum_n [S_{In}(\beta;\Delta_{In}) - S_{Fn}(\beta;\Delta_{Fn})]\,, \tag{6}$$

where the intermediate states of the emitting atom are indicated by $|n\rangle = |\alpha_n J_n M_n\rangle$. The functions $W_{Kn}(\Delta; \beta_{Kn})$ and $S_{Kn}(\Delta; \beta_{Kn})$ with $K=I$ or F, are given by

$$W_{Kn}(\Delta; \beta_{Kn}) = \int_{x_{\min}}^{x_{\max}} \frac{x\,dx}{(x^2+a^2)^2} K_{Fn}\left[\left(\frac{8m}{\beta}\right)^{1/2} x\right] \times \exp\left[-\left(\frac{\gamma_{Kn}}{x} - x\right)^2\right] \tag{7}$$

and

$$S_{Kn}(\Delta; \beta_{Kn}) = \int_0^{\infty} \frac{x\,dx}{(x^2+a^2)^2} F_{Kn}\left[\left(\frac{8m}{\beta}\right)^{1/2} x\right] \times D\left(\frac{\gamma_{Kn}}{x} - x\right), \tag{8}$$

where $\gamma_{Kn} = \frac{1}{4}\beta\Delta_{Kn}$, $\Delta_{Kn} = E_K - E_n$ is the energy difference between the states $|K\rangle \equiv |\alpha_K J_K M_K\rangle$ and $|n\rangle \equiv |\alpha_n J_n M_n\rangle$; $a = (\hbar/l_D)(\beta/8m)^{1/2}$ and $D(\gamma_{Kn}/x - x)$ is the Dawson's integral (Abramowitz and Segun):

$$D\left(\frac{\gamma_{Kn}}{x-x}\right) = D(\xi) = \exp(-\xi^2)\int_0^{\xi} \exp(t^2)\,dt,$$

with $\xi = (\gamma_{Kn}/x - x)$; $x_{\min} = (\beta/8m)^{1/2}(\Delta q)_{\min}$ and $x_{\max} = (\beta/8m)^{1/2}(\Delta q)_{\max}$ where $(\Delta q)_{\min}$ and $(\Delta q)_{\max}$ are, respectively, the minimum and maximum values for the momentum exchange in the electron-atom collision compatible with the energy conservation $q^2 - \bar{q}^2 = 2m\Delta_{Kn}$ assumed in the linewidth calculation.[2-6] In preceding papers[6-13] we have put, inadvertiantly, $(\Delta q)_{\min} = 0$ and $(\Delta q)_{\max} = \infty$ (see comments in Sec. II).

The function $W_{IF}(\beta; \Delta=0)$ is defined by

$$W_{IF}(\beta; \Delta=0) = \int_{\min}^{\max} \frac{x\,dx}{(x^2+a^2)^2} F_{IF}\left[\left(\frac{8m}{\beta}\right)^{1/2} x\right] e^{-x^2}. \tag{9}$$

In the first-order term $S^{(1)}$ for the shift, which is given by $S_e^{(1)} = -N(V_I - V_F)/h$, the function V_K is written

$$V_K = A_{IF} \sum_{M_K} C_{M_K M_K} \int d^3R\, \langle \alpha_K J_K M_K | V | \alpha_K J_K M_K \rangle, \tag{10}$$

where V is given by Eq. (4).

In (7) and (8) the form factors $F_{Kn}[(8m/\beta)^{1/2}x]$ are given by, taking into account that $\Delta q = (8m/\beta)^{1/2}x$,

$$F_{Kn}(\Delta q) = A_{IF} \sum_{M_I M_F} C_{M_I M_F}\delta_{M_I M_F}\left[Z\delta_{Kn} - \langle \alpha_K J_K M_K \left| \sum_a \exp\left(i\frac{\Delta q z_a}{\hbar}\right) \right| \alpha_n J_n M_n \rangle\right]^2, \tag{11}$$

where z_a is the z component of the vector $\vec{r}_a$.

Finally, in (9) $F_{IF}(8m/\beta)^{1/2}x$ is written

$$F_{IF}(\Delta q) = A_{IF} \sum_{M_I M_F} C_{M_I M_F}\delta_{M_I M_F}\left[Z - \langle \alpha_I J_I M_I \left| \sum_a \exp\left(i\frac{\Delta q z_a}{\hbar}\right) \right| \alpha_I J_I M_I \rangle\right] \times \left[Z - \langle \alpha_F J_F M_F \left| \sum_a \exp\left(i\frac{\Delta g z_a}{\hbar}\right) \right| \alpha_F J_F M_F \rangle\right]. \tag{12}$$

II. CALCULATION OF $\Delta\nu$ AND S FOR HELIUM LINES

Let us indicate by $|n^a l\rangle$ and $|n'^a l'\rangle$, where $a=1$ for the *para*helium and $a=3$ for the *ortho*helium, the initial and final states, respectively, of the analyzed transition. The final state $|n'^a l\rangle$ will be chosen as the lowest energy level and, as it is much less polarizable than the initial state, its contribution to the broadening and shift will be negligible compared with that of the initial state.

The first-order term $S_e^{(1)}$ of the shift is zero because $V_I = V_F = 0$ as one can easily verify taking into account the symmetry of the electronic charge distribution of the neutral helium atom.

So, the half-halfwidth $\Delta\nu_e$ and the shift S_e defined by (5) and (6), respectively, letting

$$y = 2\Delta q a_0/\hbar = (8m/\beta)^{1/2}(2a_0 x/\hbar)$$

become

$$\Delta\nu_e = 16Ne^4\left(\frac{\beta m}{2\pi}\right)^{1/2}\left(\frac{a_0}{\hbar}\right)^2 \sum_{l'} \int_{y_{\min}}^{y_{\max}} \frac{y\,dy}{(y^2+\delta^2)^2} F_{n^a l, n^a l'}(y) \exp\left[-\left(\frac{\xi\Delta_{n^a l, n^a l'}}{y} - ny\right)^2\right], \tag{13}$$

and

$$S_e = -\frac{32}{\sqrt{\pi}} Ne^4\left(\frac{\beta m}{2\pi}\right)^{1/2}\left(\frac{a_0}{\hbar}\right)^2 \int_0^{\infty} \frac{y\,dy}{(y^2+\delta^2)^2} \sum_{l'} F_{n^a l, n^a l'}(y) D\left(\frac{\xi\Delta_{n^a l, n^a l'}}{y} - \eta y\right), \tag{14}$$

where $\delta = 2a_0/l_D$, a_0 is the Bohr radius, $\xi = (2m\beta)^{1/2}a_0/\hbar$, and $\eta = (\beta/2m)^{1/2}\hbar/4a_0$. The energy differences $\Delta_{n^a l, n^a l'}$ between the states $|n^a l\rangle$ and $|n^a l'\rangle$ are given by Moore.[16]

In the calculations of the form factors we use hydrogenlike wave functions with principal quantum numbers adjusted to give the measured bound-state energies[12] and in the Appendix are presented to relevant form factors $F_{n^a l, n^a l'}$ to the helium line shape.

We verified that, if the screening parameter $\delta = 2a_0/l_D$ is put equal to zero, which means that l_D is infinite, our predictions for $\Delta\nu_e$ and S_e are modified only for a few percent. So, we can say that in our approach the screening effects are not significant.

Up to now we have evaluated the contribution of the electron impacts. To take into account the ions contribution we use the approach developed by Griem[15] and Griem *et al.*[14] (see also Griem[1]). Following these authors the total width and total shift are given by $\Delta\nu = \Delta\nu_e + \Delta\nu_i$ and $S = S_e + S_i$ where $\Delta\nu_i$ and S_i are estimated in terms of the Stark parameters α and σ.[1,14,15]

In preceding papers[6-13] we have calculated the widths of many neutral helium lines putting $y_{min} = 0$ and $y_{max} = \infty$. Our predictions for $\Delta\nu$ were about 2 times larger than the experimental ones.[17]

Since for these lines we have $K_B T \gg \Delta_{nal, nal'}$, $(\Delta q)_{min}$ and $(\Delta q)_{max}$ can be taken, in a good approximation, as $\Delta_{nal, nal'}/\bar{v}$ and $2m\bar{v}$, respectively, where $\bar{v}$ is the average electron speed. So, $y_{min} \simeq \frac{1}{2}\sqrt{\pi}\,\xi\Delta_{nal, nal'}$ and $y_{max} \simeq 4m\bar{v}a_0/\hbar$.

Calculating $\Delta\nu_e$ with y_{min} and y_{max} given above we verify that the discrepancy between theory and experiment is not avoided: the new predicted values for $\Delta\nu$ are about 1.7 times larger than the experimental results.

Since the region of Δq plays the principal role in the integral over Δq, we verified that: (i) putting $(\Delta q)_{max} = \infty$ instead of $(\Delta q)_{max} \simeq 2m\bar{v}$, the integral is modified only by a few percent; and (ii) to obtain a good agreement with the experimental results, $(\Delta q)_{min}$ must be substituted by $\theta\Delta_{nal, nal'}/\bar{v}$, where θ is a numerical factor which varies from 5 up to 100, assuming different values for different lines. No criterion was found to justify θ.

We believe that, as pointed out by Griem,[17] the Born approximation is not suited to calculating the widths, which are caused mainly by inelastic collisions. The effect of strong collisions, for which the perturbation theory breaks down, can be better estimated by using the semiclassical approach.[1,14]

As one can see from preceding papers[6-13] our predictions for the shifts are in reasonable agreement with the experimental results. It seems that, in those cases, Born approximation is satisfactory to estimate the shifts, which are due mostly to elastic collisions.

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APPENDIX

We present here the form factors $F_{n^a l, n^a l'}(y)$ that have been calculated assuming that the excited electron is bound to a nucleus with effective charge $Z_{eff} = 1$ (Bethe and Salpeter, 1957) and with a hydrogenlike functions. In this approximation, the form factors have the same form for *ortho*helium and *para*helium. We remember that the form factor is defined by

$$F_{nl, nl'}(y) = |\delta_{l, l'} - \langle nl0|e^{i\Delta q z/\hbar}|nl'0\rangle|^2 .$$

So, we have

$$F_{2s,2s} = [1 - (y^4 - 6y^2 + 8)/8(\tfrac{1}{4}y^2 + 1)^4]^2 ,$$

$$F_{2s,2p} = (\tfrac{9}{4}y^2)(\tfrac{1}{4}y^2 - 1)^2(\tfrac{1}{4}y^2 + 1)^{-8} ,$$

$$F_{2p,2p} = [1 + (\tfrac{5}{4}y^2 - 1)/(\tfrac{1}{4}y^2 + 1)^4]^2 ,$$

$$F_{3s,3s} = [1 - (1230.19y^8 - 11\,664y^6 + 31104y^4 - 21\,504y^2 + 4096)/(\tfrac{9}{4}y^2 + 4)^6]^2 ,$$

$$F_{3s,3p} = \tfrac{128}{3}\,y^2(615.09y^6 - 4374y^4 + 6264y^2 - 2304)^2/(\tfrac{9}{4}y^2 + 4)^{12} ,$$

$$F_{3s,3d} = 32768(22.78y^4 - 162y^2 + 120)^2y^4/(\tfrac{9}{4}y^2 + 4)^{12} ,$$

$$F_{3p,3p} = [1 + (14\,580y^6 - 53\,136y^4 + 41\,472y^2 - 4096)/(\tfrac{9}{4}y^2 + 4)^6]^2 ,$$

$$F_{3p,3d} = \tfrac{256}{3}\,y^2(1822.5y^4 - 4752y^2 + 1152)^2/(\tfrac{9}{4}y^2 + 4)^{12} ,$$

$$F_{3d,3d} = [1 - (22\,032y^4 - 19\,968y^2 + 4096)/(\tfrac{9}{4}y^2 + 4)^6]^2 ,$$

$$F_{4s,\,4s}=[1-(4y^{12}-46y^{10}+166y^{8}-215y^{6}+109y^{4}-19y^{2}+1)/(y^{2}+1)^{8}]^{2}\,,$$

$$F_{4s,\,4p}=45(2y^{10}-18y^{8}+43y^{6}-37y^{4}+11y^{2}-1)^{2}y^{2}/(y^{2}+1)^{16}\,,$$

$$F_{4s,\,4d}=\tfrac{1}{16}(64y^{8}-568y^{6}+992y^{4}-536y^{2}+80)^{2}y^{4}/(y^{2}+1)^{16}\,,$$

$$F_{4s,\,4f}=\tfrac{1}{320}(160y^{6}-1470y^{4}+2240y^{2}-560)^{2}y^{6}/(y^{2}+1)^{16}\,,$$

$$F_{4p,\,4p}=[1+(50y^{10}-272y^{8}+425y^{6}-223y^{4}+37y^{2}-1)/(y^{2}+1)^{8}]^{2}\,,$$

$$F_{4p,\,4d}=(\tfrac{1}{5}y^{2})(150y^{8}-648y^{6}+666y^{4}-240y^{2}+12)^{2}/(y^{2}+1)^{16}\,,$$

$$F_{4p,\,4f}=y^{4}(42y^{6}-168y^{4}+114y^{2}-12)^{2}/(y^{2}+1)^{16}\,,$$

$$F_{4d,\,4d}=[1-(102y^{8}-263y^{6}+169y^{4}-25y^{2}+1)/(y^{2}+1)^{8}]^{2}\,,$$

$$F_{4d,\,4f}=(\tfrac{1}{5}y^{2})(161y^{6}-287y^{4}+103y^{2}-9)^{2}/(y^{2}+1)^{16}\,,$$

$$F_{4f,\,4f}=[1+(57y^{6}-39y^{4}+15y^{2}-1)/(y^{2}+1)^{8}]^{2}\,,$$

$$\begin{aligned}F_{5s,\,5s}=[1-(&186\,264\,514.92y^{16}-2\,384\,185\,791y^{14}+10\,162\,353\,510y^{12}\\&-17\,656\,250\,000y^{10}+13\,987\,500\,000y^{8}-5\,104\,000\,000y^{6}\\&+832\,512\,000y^{4}-52\,428\,800y^{2}+1048\,576)/(\tfrac{25}{4}y^{2}+4)^{10}]^{2}\,,\end{aligned}$$

$$\begin{aligned}F_{5s,\,5p}=50y^{2}(&89\,406\,967.16y^{14}-915\,527\,343.7y^{12}+2790\,527\,343y^{10}\\&-3496\,875\,000y^{8}+1941\,000\,000y^{6}-480\,768\,000y^{4}+47\,923\,200y^{2}-1572\,864)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,\end{aligned}$$

$$\begin{aligned}F_{5s,5d}=(\tfrac{1}{35}131\,072y^{4})(&13\,038\,516.04y^{12}-131\,130\,218.75y^{10}+319\,671\,630.86y^{8}-297\,851\,562.5y^{6}+115\,687\,500y^{4}\\&+17\,920\,000y^{2}+896\,000)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,\end{aligned}$$

$$F_{5s,\,5f}=(128\times10^{6}y^{6})(47\,683.72y^{10}-506\,591.8y^{8}+1097\,656.25y^{6}-757\,500y^{4}+192\,640y^{2}-14\,336)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,$$

$$F_{5s,\,5g}=(10^{6}y^{8}/14(732\,421.875y^{8}-8437\,500y^{6}+17\,700\,000y^{4}-9216\,000y^{2}+1032\,192)/(\tfrac{25}{4}y^{2}+4)^{20}\,,$$

$$\begin{aligned}F_{5p,\,5p}+[1+(&2384\,185\,791.01y^{14}-15\,823\,364\,250y^{12}+33\,808\,593\,750y^{10}\\&-29\,543\,750\,000y^{8}+11\,257\,600\,000y^{6}-1789\,440\,000y^{4}+101\,580\,800y^{2}-1048\,576)/(\tfrac{25}{4}y^{2}+4)^{10}]^{2}\,,\end{aligned}$$

$$\begin{aligned}F_{5p,\,5d}=\tfrac{1}{35}y^{2}(&20\,027\,160\,640y^{12}-109\,716\,796\,800y^{10}+175\,546\,875\,000y^{8}\\&-110\,040\,000\,000y^{6}+27\,820\,800\,000y^{4}-2580\,480\,000y^{2}+55\,050\,240)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,\end{aligned}$$

$$\begin{aligned}F_{5p,\,5f}=y^{4}(&2563\,476\,562.5y^{10}-13\,125\,000\,000y^{8}+16\,650\,000\,000y^{6}\\&-1833\,600\,000y^{4}+1105\,920\,000y^{2}-39\,321\,600)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,\end{aligned}$$

$$F_{5p,\,5g}=(10^{4}y^{6}/7)(27\,343\,750y^{8}-140\,000\,000y^{6}+151\,200\,000y^{4}-41\,574\,400y^{2}+2293\,760)^{2}/(\tfrac{25}{4}y^{2}+4)^{20},$$

$$\begin{aligned}F_{5d,\,5d}=[1-(&38\,131\,713\,867.19y^{12}-141\,777\,343\,700y^{10}+161\,106\,250\,000y^{8}\\&-68\,368\,000\,000y^{6}+10\,990\,080\,000y^{4}-534\,118\,400y^{2}+7340\,032)/(\tfrac{25}{4}y^{2}+4)^{10}]^{2}\,,\end{aligned}$$

$$\begin{aligned}F_{5d,\,5f}=(2^{20}y^{2}/35)(&26\,869\,773.86y^{10}-77\,972\,412.1y^{8}+64\,082\,031.25y^{6}\\&-18\,100\,000y^{4}+1660\,000y^{2}-46080)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,\end{aligned}$$

$$F_{5d,\,5g}=(2^{24}y^{4}/245)(8010\,864.26y^{8}-20\,141\,601.56y^{6}+11\,859\,375y^{4}-2070\,000y^{2}+96\,000)^{2}/(\tfrac{25}{4}y^{2}+4)^{20}\,,$$

$$\begin{aligned}F_{6s,\,6s}=[1-(&19\,951.54y^{20}-273\,409.998y^{18}+1294\,633.287y^{16}\\&-2674\,850.502y^{14}+2729\,862.949y^{12}-1443\,471.258y^{10}\\&+400\,836.094y^{8}-56\,907.563y^{6}+3877.875y^{4}-108.75y^{2}+1)/(\tfrac{9}{4}y^{2}+1)^{12}]^{2}\,,\end{aligned}$$

$$\begin{aligned}F_{6s,\,6p}=\tfrac{10}{42}y^{2}(&139660.783y^{18}-1551\,786.474y^{16}+5468\,418.903y^{14}\\&-8500\,980.059y^{12}+6495\,620.660y^{10}-2551\,101.328y^{8}\\&+512\,373.094y^{6}-50\,392.125y^{4}+2149.875y^{2}-31.5)^{2}/(\tfrac{9}{4}y^{2}+1)^{24}\,,\end{aligned}$$

$$\begin{aligned}F_{6s,\,6d}=\tfrac{1}{14}y^{4}(&331\,047.781y^{16}-3607\,369.869y^{14}+10\,590\,415.084y^{12}\\&-13\,047\,291.738y^{10}+7667\,143.594y^{8}-2230\,842.516y^{6}\end{aligned}$$

$$+ 314\,745.75y^4 - 19\,561.5y^2 + 420)^2/(\tfrac{9}{4}y^2 + 1)^{24},$$

$$F_{6p,\,6p} = [1 + (258\,631.079y^{18} - 1935\,709.942y^{16} + 4960\,461.989y^{14}$$
$$- 5688\,079.453y^{12} + 3217\,709.18y^{10} - 921\,410.438y^8$$
$$+ 130\,809.938y^6 - 8484.75y^4 + 209.25y^4 + 209.25y^2 - 1)/(\tfrac{9}{4}y^2 + 1)^{12}]^2,$$

$$F_{6p,\,6d} = \tfrac{1}{30}y^2(2069\,048.632y^{16} - 13\,045\,734.782y^{14} + 26\,409\,088.601y^{12}$$
$$- 23\,391\,707.766y^{10} + 9877\,461.152y^8 - 2105\,143.594y^6 + 185\,009.813y^4 - 7506y^2 + 72)^2/(\tfrac{9}{4}y^2 + 1)^{24},$$

$$F_{6d,\,6d} = [1 - (4377\,187.328y^{16} - 19\,849\,243.502y^{14} + 30\,224\,149.919y^{12}$$
$$- 19\,963\,809.519y^{10} + 6197\,966.543y^8 - 894\,232.406y^6 + 55\,333.125y^4 - 1155.75y^2 + 7)/7(\tfrac{9}{4}y^2 + 1)^{12}]^2,$$

$$F_{6d,\,6f} = \tfrac{1}{105}y^2(5922\,076.974y^{14} - 21\,955\,415.805y^{12} + 26\,289\,116.486y^{10}$$
$$- 13\,169\,349.404y^8 + 2942\,711.016y^6 - 284\,051.813y^4 + 10\,303.875y^2 - 121.5)^2/(\tfrac{9}{4}y^2 + 1)^{24}.$$

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