PHYSICAL REVIEW A VOLUME 18, NUMBER 5 NOVEMBER 1978

Temperature effects in photodetection

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(Received 6 March 1978)

We present a theoretical study of photodetection with coupling of the photosensitive atoms to a reservoir. We investigate the effects of nonzero detector temperature, where the initial thermal-equilibrium occupation of the atomic levels and the thermalization of the radiation field interfere with the measurement of the photon statistics. We derive formulas exhibiting the temperature dependence of the field dynamics, the relationship between field and atomic moments, and the photocounting probability. We present examples illustrating how the coherence properties of the incoming field are affected by the detection process in the presence of thermal noise.

I. INTRODUCTION

Widespread interest in developing photoelectric detection techniques has led to numerous theoretical as well as experimental investigations of the relationship between the photocounting data and the statistics of the incident radiation.[1-6] In particular, the quantum treatment of the detection of electromagentic fields, initiated by Glauber[1] and Kelley and Kleiner,[7] has been largely developed during the last decade.[3,4,8-13] One interesting aspect that has so far received little comment is the influence of the "detector temperature": How does the initial equilibrium occupation of the detector levels and the corresponding thermalization of the radiation field interfere with the measurement of photon statistics? How does the initial state of the detector affect the field attenuation[8-10] and the relationship between field and photocurrent statistics? These problems are studied in the present paper.

Conventional perturbation theory[1-3,7,11] neglects virtual transitions and requires a reservoirlike behavior of the field since the field attenuation is not adequately described. More powerful procedures such as the Heisenberg equation of motion[8,9] and the master equation[10] fully account for the field attenuation, but require the atomic system to act as a reservoir. Temperature effects can hardly be studied in the framework of these theories. We therefore apply the open-system detector scheme introduced in our previous paper[13] (referred to as Paper I in the following), where not only the field attenuation, but also the relaxation of the photosensitive atoms is adequately accounted for. Besides incorporating the previous theories, the model of Paper I is well suited for developing a theory of photodetection for any "detector temperature" fixed by the initial state of the detector characterized by the number of excited atoms.

The theory of photodetection presented in Paper I was based on the assumption that the photosensitive atoms are initially in their ground state. This choice is equivalent to fixing the detector temperature at zero. In this paper, nonzero detector temperature is introduced as a parameter which measures the number of excited atoms in the initial state of the detector. We have in mind here an effective temperature describing not only thermal effects, but also any source of atomic excitation. Being maintained by interaction of the photosensitive atoms with a reservoir, the initial number of excited atoms corresponds to the "intrinsic dark current" of the detector. The fluctuation of this number constitutes the "intrinsic noise." Since zero detector temperature cannot be achieved experimentally, it is of interest to know how this intrinsic noise interferes with the detection process.

We refer to Paper I as for the description of our detection scheme and a more comprehensive discussion of the different detector models and list of references. Our main objective here is to establish the relations between atomic and field moments, as such relations are known[13] to determine the transfer of the statistical state of the incoming field to the detector signal. The derivation of the relationship in question is technically involved if we use the master-equation technique of Paper I. In the present publication we rather follow a different approach based on Langevin equations. We apply the Langevin approach to the monomode pointlike Dicke model[13] with atoms undergoing relaxation due to interaction with external reservoirs. The resulting equations correspond to the well-known[14] Langevin equations describing the monomode laser without field losses. The equations for the average of atomic and field variables as derived from the Langevin equations agree with those obtained from the corresponding master equation. This link between the two descriptions is well-known from the laser literature.[15] Our model turns out to be formally identical to that of a laser working under the unusual condition of very small amplification. Therefore we do not reproduce the well-known description of the field here, but rather exploit the relation between atomic and field variables.

In Sec. II we derive the relations between atomic and field moments. These relations are basic to the derivation of the photocounting probability presented in Sec. III. Specific applications of the counting probability are discussed in Sec. IV.

II. LANGEVIN EQUATIONS OF MOTION

A. Field equation

We consider the monomode pointlike Dicke model at resonance with the atoms coupled to external reservoirs at temperature $T \neq 0$. In Paper I we discussed why this model is an appropriate description of a photodector.

The reservoirs are responsible for the atomic relaxation and for the stochastic forces occuring in the Langevin equations of motion,[15-17] viz.,

$$\dot{a} = -i\lambda V^{-1/2}\sum_i s_i^- \equiv -i\lambda V^{-1/2} S^-, \tag{2.1}$$

$$\dot{s}_i^- = -\gamma s_i^- + 2i\lambda V^{-1/2} s_{3i} a + f_i^-, \tag{2.2}$$

$$\dot{s}_{3i} = -2\gamma(s_{3i} - \tfrac{1}{2}\sigma_0) + i\lambda V^{-1/2}(a^+ s_i^- - s_i^+ a) + f_{3i}, \tag{2.3}$$

with $a^\pm$ denoting e.m. field operators and $s_i^\pm$ and s_{3i} atomic operators referring to atom i. The dipolar coupling constant λ does not depend on the volume V. By $\xi^{(i)} \equiv \{f_i^+, f_i^-, f_{3i}\}$ we denote the stochastic forces on the ith atom. The description of a deadtime effect, namely, the deadtime related to the extraction of excited atoms, is included in the atomic relaxation constant γ.

We make the usual hypotheses, namely, (i) the forces on different atoms are uncorrelated, i.e., each atom has its own reservoir and the various reservoirs, though identical, are independent; (ii) the forces on a single atom are Gaussian δ-correlated random functions with

$$\langle \xi_\mu^{(i)} \rangle_R = 0, \quad \langle \xi_\mu^{(i)}(t)\xi_\nu^{(i)}(t')\rangle = 2D_{\mu\nu}\delta(t-t'), \tag{2.4}$$

$\langle \cdots \rangle_R$ denoting the average over the reservoir state; and (iii) the values of the constants $D_{\mu\nu}$ in Eq. (2.4) can be determined by applying the fluctuation-dissipation theorem to the equations of motion without atom-field coupling terms: the properties of the reservoir are not affected by the λ-coupling.

In particular, we have

$$\langle f_i^\pm(t) f_i^\pm(t')\rangle_R = 0, \tag{2.5}$$

$$\langle f_i^\pm(t) f_i^\mp(t')\rangle_R = \gamma(1 \pm \sigma_0)\delta(t-t'), \tag{2.6}$$

$$\langle f_{3i}(t) f_{3i}(t')\rangle_R = \gamma(1-\sigma_0^2)\delta(t-t'). \tag{2.7}$$

Here $\sigma_0 \equiv -\tanh(\frac{1}{2}\beta\epsilon)$ is the expectation value of the population difference operator s_{3i} and ϵ is the energy spacing between the two levels of each atom and $\beta = (k_B T)^{-1}$. In contrast to laser theory no field losses are assumed here.

By a well-known procedure,[15] namely, the linearization of the atomic Langevin equation (2.3) around the stationary solution, from Eqs. (2.1)–(2.3), we can derive a closed-form equation of motion for the field variables. This equation exhibits the damping and the fluctuations of the field produced by the coupling to the atoms, viz.,

$$\dot{a}_t = -\kappa a_t + g_t, \tag{2.8}$$

where

$$\kappa = \kappa_0(-\sigma_0) \equiv \lambda^2\gamma^{-1}(N/V)(-\sigma_0), \tag{2.9}$$

and

$$g_t = -i\lambda V^{-1/2}\sum_i \int_0^t e^{-\gamma(t-t')} f_i^-(t')\,dt'. \tag{2.10}$$

Equation (2.8) is obtained introducing the same approximations as in I, namely,

$$\lambda(N/V)^{1/2} \ll \gamma \tag{2.11}$$

and

$$t \gg \gamma^{-1}. \tag{2.12}$$

With S_{3t} substituted by its stationary value $\frac{1}{2}N\sigma_0$, Eq. (2.8) is thus equivalent to a linearization of the equation of motion for the field operators a_t, a_t^+.

The equation of motion for the moments of the field reads

$$\frac{d}{dt}\langle a^{+m}a^m\rangle_t = -2\kappa m\langle a^{+m}a^m\rangle_t + 2\kappa m^2\bar{n}\langle a^{+m-1}a^{m-1}\rangle_t \tag{2.13}$$

where $\bar{n}$ is the thermal equilibrium expectation value for a^+a and $\langle\cdots\rangle$ denotes the average over the states of the total system (atoms, e.m. field, and reservoirs). For zero temperature Eq. (2.13) reduces to the equation of motion (26) in I.

Introducing Glauber's coherent-state representation, the solution of Eq. (2.13) takes the form

$$\langle a^{+m}a^m\rangle_t = m!\bar{n}_t^m \int d^2\alpha\, W_0(\alpha) L_m\left(\frac{-|\alpha_t|^2}{\bar{n}_t}\right), \qquad (2.14)$$

where $W_0(\alpha) \equiv W_{t=0}(\alpha)$ is the statistical field distribution at time $t=0$ and L_m denotes the Laguerre polynomial of order m. In Eq. (2.14) $\bar{n}_t = \bar{n}(1-e^{(-2\kappa t)})$ and $|\alpha_t|^2 = |\alpha|^2 e^{(-2\kappa t)}$. For $t \gg \kappa^{-1}$, Eq. (2.14) reduces to the thermal equilibrium value, i.e.,

$$\langle a^{+m}a^m\rangle_t \xrightarrow[t\gg\kappa^{-1}]{} m!\bar{n}^m .$$

Thus κ^{-1} measures the thermalization time for the field. We recall that the temperature enters Eqs. (2.13) and (2.14) not only through $\bar{n}$, but also through the attenuation constant $\kappa \propto -\sigma_0$. We also recall that the temperature entering $\bar{n}$ and κ is fixed by the reservoirs, which are not necessarily thermal baths.

B. Connection between atomic and field dynamics

In order to relate the photon and the photoelecron statistics, let us consider the moments of the atomic variables

$$\langle (S^+)^\nu (S^-)^\nu\rangle_t \equiv \nu! \sum_{j_1\neq j_2\neq\cdots\neq j_\nu} \langle s_{j1}^+ s_{j2}^+ \cdots s_{j\nu}^+ s_{j\nu}^- \cdots s_{j1}^-\rangle_t . \qquad (2.15)$$

For technical convenience we evaluate (2.15) using the equation for $s_i^\pm$ in its integral form

$$\begin{aligned} s_{i,t}^\pm &= h_{i,t}^\pm \mp 2i\lambda V^{-1/2}\int_0^t e^{-\gamma(t-t')} s_{3i,t'} a_{t'}^\pm\, dt' \\ &\equiv {}^0s_i^\pm + {}^\lambda s_i^\pm . \end{aligned} \qquad (2.16)$$

Separating $s_i^\pm$ into a free and an interacting part as indicated in (2.16), for the first moment $\langle S^+S'^-\rangle_t = \sum_i \langle s_i^+ s_i^-\rangle_t$ we obtain

$$\begin{aligned} \langle s_i^+ s_i^-\rangle_t = \langle h_i^+ h_i^-\rangle_t &+ 2i\lambda V^{-1/2}\int_0^t e^{-\gamma(t-t')}\langle s_{i,t}^+ s_{3i,t'} a_{t'}\rangle - 2i\lambda V^{-1/2}\int_0^t e^{-\gamma(t-t')}\langle a_{t'}^+ s_{3i,t'} s_{i,t}^-\rangle \\ &- 4\lambda^2 V^{-1}\int_0^t e^{-\gamma(t-t')}\int_0^t e^{-\gamma(t-t'')}\langle a_{t'}^+ s_{3i,t'} s_{3i,t''} a_{t''}\rangle . \end{aligned} \qquad (2.17)$$

We have now to determine the form of the correlation functions $\langle s_{i,t}^+ s_{3i,t'} a_{t'}\rangle$, $\langle a_{t'}^+ s_{3i,t'} s_{i,t}^-\rangle$, and $\langle a_{t'}^+ s_{3i,t'} s_{3i,t''} a_{t''}\rangle$. For the latter expression, in agreement with the linearization leading Eq. (2.8) we obtain

$$\langle a_{t'}^+ s_{3i,t'} s_{3i,t''} a_{t''}\rangle = \tfrac{1}{4}\sigma_0^2 \langle a_{t'}^+ a_{t''}\rangle . \qquad (2.18)$$

In order to evaluate $\langle s_{i,t}^+ s_{3i,t'} a_{t'}\rangle$ or, equivalently, $\langle a_{t'}^+ s_{3i,t'} s_{i,t}^-\rangle$ we must take into account the following two conditions. (i) For $t=t'$ we must satisfy the relations

$$\langle s_i^+ s_{3i} a\rangle_t = -\tfrac{1}{2}\langle s_i^+ a\rangle_t ,$$

$$\langle a^+ s_{3i} s_i^-\rangle_t = -\tfrac{1}{2}\langle a^+ s_i^-\rangle_t ,$$

which can be derived using the well-known properties of the atomic spin operators $\langle s_{3i}^2\rangle = \frac{1}{4}$, $\langle s_i^+ s_{3i}\rangle = -\frac{1}{2}\langle s_i^+\rangle$, etc. (ii) For $|t-t'| \gg \gamma^{-1}$, $s_{3i,t'}$, and $s_{i,t}^\pm$ are uncorrelated, i.e., they have their stationary $(t, t' \gg \gamma^{-1})$ values averaged over the atomic and reservoir states; in this limit $s_{3i,t}$ becomes $\frac{1}{2}\sigma_0$ and $s_{i,t}^\pm$ becomes $\mp i\lambda V^{-1/2}\sigma_0 a_t^\pm$, to lowest order. We can satisfy both conditions by assuming

$$\begin{aligned} \langle a_{t'}^+ s_{3i,t'} s_{i,t}^-\rangle = &-\tfrac{1}{2}\langle a_t^+ s_{i,t}^-\rangle e^{-\gamma|t-t'|} \\ &+ \tfrac{1}{2} i\lambda V^{-1/2}\sigma_0^2 \langle a_t^+ a_t\rangle (1-e^{-\gamma|t-t'|}) . \end{aligned} \qquad (2.19)$$

The substitution of (2.18) and (2.19) in (2.17) leads to

$$\langle s_i^+ s_i^-\rangle_t = \tfrac{1}{2}(1+\sigma_0) + N^{-1}\zeta\langle a^+a - \bar{n}\rangle_t \qquad (2.20)$$

and thus

$$\langle S^+S^-\rangle_t - \frac{N}{2}(1+\sigma_0) \equiv \langle N_2\rangle_t - N_{2,\mathrm{eq}} = \zeta\langle a^+a - \bar{n}\rangle_t , \qquad (2.21)$$

where N_2 denotes the excited-atom-number operator, $N_{2,\mathrm{eq}} = \frac{1}{2}N(1+\sigma_0) = (-\bar{n}\sigma_0)$ is the average number of excited atoms at thermal equilibrium, and

$$\zeta = \zeta_0(-\sigma_0) = \kappa\gamma^{-1} . \qquad (2.22)$$

In (2.20) memory effects are neglected in agreement with the condition $t \gg \gamma^{-1}$. The higher-order factorial moments $\langle N_2^{(\nu)}\rangle_t \equiv \langle S^{+\nu} S^{-\nu}\rangle_t$ are evaluated in the same way. In terms of the coherent state representation for the field variables, we finally obtain

$$\begin{aligned} \langle N_2^{(\nu)}\rangle_t = &\frac{N!\,\nu!}{(N-\nu)!\,N^\nu}(-\sigma_0\bar{n})^\nu (1-\zeta_0 e^{-2\kappa t}) \\ &\times \int d^2\alpha\, W_0(\alpha) L_\nu\left(-\frac{|\alpha|^2\zeta_0 e^{-2\kappa t}}{\bar{n}(1-\zeta_0 e^{-2\kappa t})}\right) \end{aligned} \qquad (2.23)$$

with $W_0(\alpha)$ denoting the initial field distribution as introduced in Eq. (2.14).

III. PHOTOCOUNTING PROBABILITY

The essential observable in a photocounting experiment is the number N_2 of excited atoms. We calculate the photocounting probability from the factorial moments of the number of photocounts in the time interval $t_0 < t < t_0 + \tau$, whose average value is defined as

$$N_A(t_0, t_0+\tau) = 2\gamma \int_{t_0}^{t_0+\tau} \langle N_2 \rangle_t \, dt . \quad (3.1)$$

The corresponding factorial moments, as shown in Paper I are given by

$$N_A^{(\nu)}(t_0, t_0+\tau) = (2\gamma)^\nu \int_{t_0}^{t_0+\tau} dt_1 \cdots \int_{t_0}^{t_0+\tau} dt_\nu \langle N_{2,t_1}(N_{2,t_2}-1)\cdots (N_{2,t_\nu}-\nu+1)\rangle . \quad (3.2)$$

The integral in Eq. (3.2) is evaluated by generalizing the relation (2.24) to different times. As already discussed in I, the Markovian property of the field and atomic process is essential for this generalization.

By straightforward integration of (2.23) we obtain $N_A^{(\nu)}$ in terms of the initial field distribution $W_0(\alpha)$, viz.,

$$N_A^{(\nu)}(\tau) = \nu!\, u^\nu \int d^2\alpha\, W_0(\alpha) L_\nu\left(-\frac{\eta|\alpha|^2}{u}\right), \quad \eta = 1 - \exp(-2\kappa\tau) , \quad (3.3)$$

where we have chosen $t_0 = 0$, where κ is given by (2.9), and

$$u = 2\gamma\tau N_{2,\mathrm{eq}} - \eta\bar{n} \quad (3.4)$$

with $N_{2,\mathrm{eq}} \equiv -\bar{n}\sigma_0$ denoting the number of excited atoms in thermal equilibrium. We observe that u is always positive because of $\gamma \gg \kappa_0$. The average number of photocounts ($\nu = 1$) reads

$$N_A(\tau) = u + \eta\langle a^\dagger a\rangle_0 . \quad (3.5)$$

Thus u can be interpreted as the average number of photocounts due to the thermal noise. We note that u is made up of two terms: the first term is due to the stationary flow of excitations from the atomic system to the reservoir while the second represents the thermal emission of the atoms inside the cavity.

In order to connect the result (3.3) with the usual formula for the moments, we consider the integrated moments of

$$\hat{N}_2(\alpha) \equiv N_2(\alpha) - N_2(\alpha = 0) ,$$

where by $N_2(\alpha)$ we stress the dependence of the number operator N_2 on the field amplitude. $\hat{N}_2(\alpha)$ is the occupation number operator biased with respect to zero incoming field; it may be interpreted as the noise-free occupation number operator. Proceeding as before we obtain the well-known expression[18]

$$\hat{N}_A^{(\nu)}(\tau) = \int d^2\alpha\, W_0(\alpha)\eta^\nu |\alpha|^{2\nu} , \quad (3.6)$$

but with temperature-dependent attenuation factor η. We remark that $\hat{N}_A^{(\nu)}$ is the leading term of Eq. (3.3) for $(\eta|\alpha|^2/u) \gg 1$, i.e., in the limit of large signal-to-noise ratio. Thus from Eq. (3.3) we obtain the deviation from the noiseless detection procedure to all orders of noise-to-signal ratio, viz.,

$$N_A^{(\nu)} = \int d^2\alpha\, W_0(\alpha)\eta^\nu |\alpha|^{2\nu} \times \left[1 + \nu^2\left(\frac{u}{\eta|\alpha|^2}\right) + O\left(\left(\frac{u}{\eta|\alpha|^2}\right)^2\right)\right].$$

From (3.3), we now construct the generating function

$$Q(x,\tau) = \sum_\nu \frac{1}{\nu!}(-x)^\nu N_A^{(\nu)}(\tau) = (1+xu)^{-1}\int d^2\alpha\, W_0(\alpha)\exp\left(-\frac{x\eta|\alpha|^2}{1+xu}\right) \quad (3.7)$$

which finally leads to the photocounting probability

$$p(n,\tau) = \int d^2\alpha\, W_0(\alpha)\frac{u^n}{(1+u)^{n+1}}\exp\left(-\frac{\eta|\alpha|^2}{1+u}\right) \times L_n\left(-\frac{\eta|\alpha|^2}{u(1+u)}\right) . \quad (3.8)$$

We point out that this result has been derived without imposing any restriction on τ. The formula (3.8) can be directly applied to experimental counting rates provided that the time interval τ used in the experiment has been judiciously chosen small compared with the coherence time τ_c of the intensity fluctuation. In order to account for correlation effects in the case $\tau \gtrsim \tau_c$ the experimental data must previously be processed in the spirit of Refs. 18 and 19.

We note that Eq. (3.8) is analogous to Eq. (43) in I, but with the Poisson distribution substituted by the convolution of a Poisson with a Gaussian distribution of width u. Indeed, in the limit of zero temperature, $u \to 0$ and

$$p(n,\tau)\to\int d^2\alpha\, W_0(\alpha)\frac{\eta^n|\alpha|^{2n}}{n!}e^{-\eta|\alpha|^2}, \tag{3.9}$$

which is the result of I. For zero field intensity ($\alpha=0$), from (3.8) we obtain the photocounting probability of the dark current,

$$p(n,\tau)=u^n/(1+u)^{n+1}, \tag{3.10}$$

which corresponds to the statistics of a thermal field with mean occupation number

In order to get more insight into the physical meaning of (3.8), let us assume that the incoming field is a coherent field with an initial distribution $W_0(\alpha)=\delta^{(2)}(\alpha-\alpha_0)$. In this case (3.8) reduces to the well-known[18] photocounting distribution for the superposition of a coherent field and a Gaussian field. We thus arrive at the following interpretation of the detection process at finite temperature: the detector acts as a source of an intrinsic Gaussian field, which is coherently superimposed to the incoming field. The mean number of photons of the intrinsic field is given by Eq. (3.4) and depends on the number of thermal photons $\bar{n}$, on the corresponding unsaturated inversion σ_0 and on the decay constants of the atoms-reservoir and atoms-field interactions. Of course the model becomes meaningless in the limit $\sigma_0\to 0$. In this case the system is initially in a highly excited state and, as is known from laser theory,[15] a linear theory is invalid.

IV. APPLICATIONS

Let us now consider a number of specific statistical distributions for the incoming field. For formal simplicity we restrict our discussion to the factorial moments $N_A^{(\nu)}(\tau)$ as given by Eq. (3.3). We compare the cases of zero and nonzero detector temperature in terms of the ratio $R_\nu \equiv N_A^{(\nu)}(\tau;\bar{n})/N_A^{(\nu)}(\tau;\bar{n}=0)$. We consider also the ratio $Q_\nu=N_A^{(\nu)}(\tau)/\hat{N}_A^{(\nu)}(\tau)$ which is a measure of the deviation from a zero-noise detector. The ratios Q_ν and R_ν are related by $Q_\nu/R_\nu=(\eta_0/\eta)^\nu$, where η_0 is the attenuation constant at zero temperature. Particularly simple results are found in the case of low temperature, i.e., $\bar{n}\ll 1$, and small attenuation, i.e., $2\kappa_0\tau\ll 1$.

A. Coherent radiation

Let us begin with a coherent incident field of intensity $|\alpha_0|^2$ leading to

$$Q_\nu=\nu!\,(u/\eta|\alpha_0|^2)^\nu L_\nu(-\eta|\alpha_0|^2/u). \tag{4.1}$$

For large intensity $\eta|\alpha_0|^2\gg u$ we have

$$Q_\nu\cong 1+\nu^2(u/\eta|\alpha_0|^2). \tag{4.2}$$

For low temperature, $\bar{n}\ll 1$, and small attenuation, $2\kappa_0\tau\ll 1$, $R_\nu\equiv(\eta/\eta_0)^\nu Q_\nu$ reduces to

$$R_\nu=1-2\nu\bar{n}+\nu^2\gamma\kappa_0^{-1}(\bar{n}/|\alpha_0|^2). \tag{4.3}$$

The temperature effect manifests itself in R_ν through $\bar{n}$, the photon number at thermal equilibrium. For example, the measurement of the intensity ($\nu=1$) of a CO_2 laser beam (wavelength 10.6 μm) with the detector at *room temperature* is found to be affected by a relative error of about 2%.

B. Chaotic radiation

We continue with Gaussian distributed fields characterized by

$$W_0(\alpha)=(\pi\mathfrak{N})^{-1}\exp(-|\alpha|^2/\mathfrak{N}). \tag{4.4}$$

The corresponding factorial moments read

$$N_A^{(\nu)}=\nu!\,(u+\eta\mathfrak{N})^\nu \tag{4.5}$$

leading to

$$Q_\nu=(1+u/\eta\mathfrak{N})^\nu. \tag{4.6}$$

For $\eta\mathfrak{N}\gg u$, $\bar{n}\ll 1$, and $2\kappa_0\tau\ll 1$, we find

$$R_\nu=1-2\nu\bar{n}+\nu\gamma\kappa_0^{-1}(\bar{n}/\mathfrak{N}). \tag{4.7}$$

C. Superposition of coherent and chaotic radiation

Let us now finally consider the superposition of a coherent field of intensity $|\alpha_0|^2$ with Gaussian radiation of intensity $\mathfrak{N}$, as described by

$$W_0(\alpha)=(\pi\mathfrak{N})^{-1}\exp(-|\alpha-\alpha_0|^2/\mathfrak{N}). \tag{4.8}$$

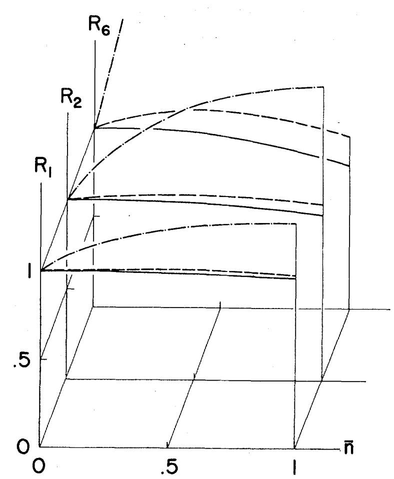

FIG. 1. Relative factorial moments R_1, R_2, R_6 plotted against the average thermal photon number $\bar{n}$ with $2\gamma\tau=10$ $(2\kappa_0\tau)=10^2$ for Gaussian field with $\mathfrak{N}=10^3$ (dashed) or $\mathfrak{N}=10^2$ (dot-dashed), coherent field with $|\alpha_0|^2=10^5$ (full), and their superpositions (same full curves).

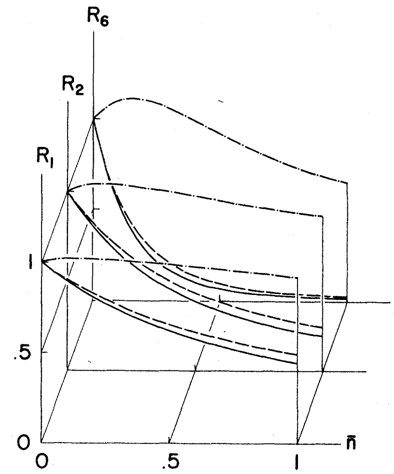

FIG. 2. Same as Fig. 1, but $2\gamma\tau = 10(2\kappa_0\tau) = 10$ for Gaussian fields with $\mathfrak{N} = 100$ (dashed) or $\mathfrak{N} = 10$ (dot-dashed) and coherent field with $|\alpha_0|^2 = 10^5$ or superposition field (full).

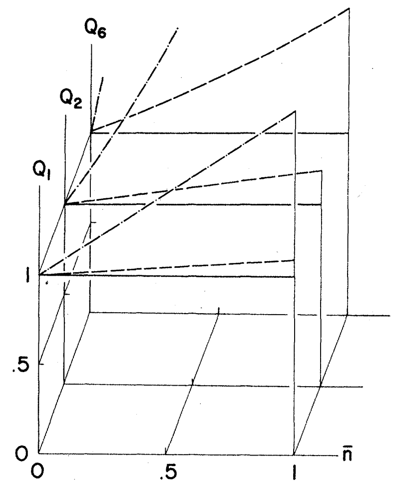

FIG. 4. Relative factorial moments Q_1, Q_2, Q_6, for $2\gamma\tau = 10(2\kappa_0\tau) = 1$: Gaussian fields with $\mathfrak{N} = 100$ (dashed) or $\mathfrak{N} = 10$ (dot-dashed) and coherent field with $|\alpha_0|^2 = 10^5$ or superpositions (full).

For this distribution we obtain

$$Q_\nu = \left(1 + \frac{u}{\eta\mathfrak{N}}\right)^\nu L_\nu\left(-\frac{\eta|\alpha_0|^2}{u+\eta\mathfrak{N}}\right) \Big/ L_\nu\left(-\frac{|\alpha_0|^2}{\mathfrak{N}}\right). \quad (4.9)$$

In the limit $\eta|\alpha_0|^2 \gg \eta\mathfrak{N} \gg u$ we obtain

$$Q_\nu = 1 + \nu^2(\mathfrak{N}/|\alpha_0|^2) + \nu^2(u/\eta|\alpha_0|^2). \quad (4.10)$$

Thus for $\bar{n} \ll 1$ and $2\kappa_0\tau \ll 1, R_\nu$ reduces to

$$R_\nu = 1 + \nu^2(\mathfrak{N}/|\alpha_0|^2) - 2\nu\bar{n} + \nu^2\gamma\kappa_0^{-1}(\bar{n}/|\alpha_0|^2). \quad (4.11)$$

D. Numerical examples and discussion

In Figs. 1–4 we compare the temperature effects on the factorial moments for Gaussian, coherent,

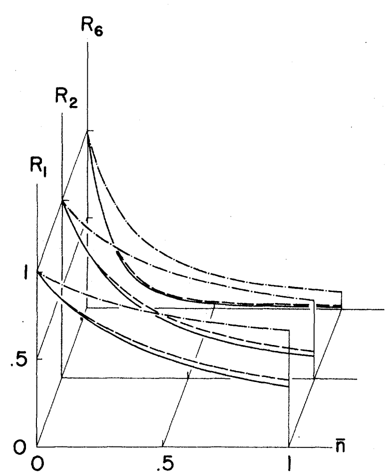

FIG. 3. Same as Fig. 2, but $2\gamma\tau = 10\ (2\kappa_0\tau) = 1$.

and mixed incident radiation. To this end we plot the moment ratios R_ν (Fig. 1–3) and Q_ν (Fig. 4) given by Eqs. (4.1)–(4.11) as functions of $\bar{n}$. We recall that R_ν is defined with respect to the limit of zero temperature, whereas Q_ν is defined with respect to noiseless detection, but accounting for the temperature dependence of the attenuation factor $\eta = 1 - \exp(-2\kappa\tau)$.

The figures are organized as follows. A fixed value of the damping parameter $2\kappa_0\tau$ is chosen for each figure, viz. $2\kappa_0\tau = 10$ for Fig. 1, $2\kappa_0\tau = 1$ for Fig. 2, and $2\kappa_0\tau = 0.1$ for Figs. 3 and 4. In agreement with the model condition $\gamma \gg \kappa_0$, the atomic relaxation parameter $2\gamma\tau$ is always chosen ten times larger than $2\kappa_0\tau$. Each figure shows three plots of the moment ratios corresponding to the order $\nu = 1$, $\nu = 2$, and $\nu = 6$, respectively.

Each plot displays curves corresponding to different incident fields, namely the Gaussian fields with $\mathfrak{N} = 10$, 100, and 1000, the coherent field with $|\alpha_0|^2 = 10^5$, and the superposition of the coherent field with $|\alpha_0|^2 = 10^5$ and the Gaussian fields with $\mathfrak{N} = 10$, 100, and 1000. The coherent field and the superposition fields lead to closely similar results which cannot be distinguished in the plots. The physical realization of the above fields is discussed in, e.g., Ref. 18.

Figures 1–3 illustrate how the photoelectron moments are increasingly affected with increasing temperature (measured by $\bar{n}$) and order ν.

From Fig. 4 we learn that the deviations from ideal (noiseless) detection are negligible in the case of coherent or superposition sources with $|\alpha_0|^2 \gtrsim 10^5$. This result is due to the very large signal-to-noise ratio occuring for reasonable temperatures. In this case we are entitled to use the classical formula for the photocounting probability, provided that we insert the appropriate (temperature dependent) attenuation factor η. Appreciable deviations from the ideal moments, however, occur in the case of Gaussian fields with $\mathfrak{N} \lesssim 100$. In the low-temperature limit $\bar{n}/\mathfrak{N} \ll 1$, the corresponding curves show the linear behavior of the relative deviation $Q_\nu - 1 \propto \nu\bar{n}$.

To conclude, we recall that three interrelated parameters are essential in our analysis of temperature effects in photodetection. These are the temperature dependent attenuation factor η, the average noise intensity u, and the average thermal photon number $\bar{n}$ measuring the effective temperature. The general result (3.3) presents the deviations from noiseless detection in terms of polynomials of u.

In the case of very large signal-to-noise ratio, the effects of the intrinsic thermal noise are essentially described by the temperature dependence of the attenuation constant η. Allowing for the temperature correction of η, we can interpret counting experiments in terms of the usual formula. The resulting counting statistics is, however, modified in the case of smaller signal-to-noise ratio, where the dependence of both η and u is essential. The corresponding experiments have to be interpreted in terms of the complete expression (3.3)

We finally remark that the thermal effects in photodetection are likewise important in the case of low temperatures ($\bar{n} \ll 1$) in the case of extremely small signal-to-noise ratio as can be inferred from the third term on right-hand side of Eqs. (4.3), (4.7), and (4.11), i.e., for $|\alpha_0|^2 \ll \bar{n}$ or $\mathfrak{N} \ll \bar{n}$.

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