Optical levitation and partial-wave resonances

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Sharp peaks observed recently in a levitation-power experiment are caused by partial-wave resonances. The theoretical results computed from the Mie-Debye theory are compared with experimental observations. The first resonances in each partial wave are extremely narrow and they were not detected in the experiment. We conjecture that the peaks in levitation power do not exist if levitated particles are irregularly shaped.

Optical levitation[1] (the suspension of small particles by the force of radiation pressure) has recently been used in the study of photoelectric effect[2] and resonance phenomena in radiation pressure.[3] The technique is useful whenever it is desirable to suspend a small particle in a given position without mechanical support. This proves particularly convenient in studying single-particle light scattering, evaporation of liquid droplets, evolution of water droplets in cloud physics, etc.

The observed variation of the radiation pressure force as a function of the wavelength and/or the size of a dielectric spherical particle shows a regular series of sharp resonancelike peaks.[3] In this study we show that the resonant structure of the light pressure force is caused by partial wave resonances in the partial-wave expansion of the scattering amplitude. Our analysis shows that the first resonance in each partial wave has a halfwidth of the order of $\Delta x/x \doteq 10^{-7}-10^{-8}$ (for the refractive index $n=1.47$ and the size parameter $x=2\pi r/\lambda \sim 40$, where r is a radius of an observed droplet and λ is the wavelength of the incident radiation). Since the reported experimental resolution was $\Delta\lambda/\lambda = \Delta x/x \simeq 10^{-5}$, such narrow resonances were not observed in the experimental work of Ashkin and Dziedzic.[3] The secondary resonances in each partial wave have a halfwidth of the order of $\Delta x/x \simeq 10^{-5}$ and all of them were observed (in the case of $n=1.47$ and $x\sim 40$).

The problem of scattering of a plane monochromatic electromagnetic wave on a small spherical particle has been solved by Mie[4] and Debye.[5] The two nonzero components of the scattering matrix can be written (using the notation standard in optics) in the form of partial-wave expansions

$$S_1 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n), \tag{1}$$

$$S_2 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n). \tag{2}$$

The explicit form of the complex partial-wave scattering amplitudes a_n and b_n and of the angular functions π_n and τ_n can be found in available monographs.

The dimensionless normalized extinction, scattering and absorption cross sections (efficiencies for extinction, scattering, and absorption) are related to the partial-wave amplitudes by

$$Q_{\text{ext}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)\,\mathrm{Re}(a_n + b_n), \tag{3}$$

$$Q_{\text{sct}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2), \tag{4}$$

$$Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sct}}. \tag{5}$$

Finally the normalized radiation pressure is

$$Q_{\text{pr}} = Q_{\text{ext}} - \frac{4}{x^2} \sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} \mathrm{Re}(a_n^* a_{n+1} + b_n^* b_{n+1}) + \frac{2n+1}{n(n+1)} \mathrm{Re}(a_n^* b_n), \tag{6}$$

where the asterisk indicates complex conjugate. The levitation power is inversely proportional to Q_{pr}.

It has been shown[6] that the ripple structure of the extinction curve is caused by resonances in the partial-wave amplitudes a_n and b_n occurring in the neighborhood of $x \sim n$ for a sufficiently large x. It is therefore natural to expect the resonant structure in Q_{sct}, Q_{abs}, and Q_{pr} to be caused by the same partial-wave resonances in a_n and b_n. Numerical calculation confirms this conjecture.

For a nonabsorbing particle (real index of refraction), at the center of a resonance peak, we have[6] $\mathrm{Re}A_n = 1$ and $\mathrm{Im}a_n = 0$ or $\mathrm{Re}b_n = 1$ and $\mathrm{Im}B_n = 0$. The distance in x between the two a_n or b_n resonances is

$$\Delta x = \frac{\arctan(n^2-1)^{1/2}}{(n^2-1)^{1/2}}, \tag{7}$$

where n is the index of refraction.

We have calculated the real and imaginary parts

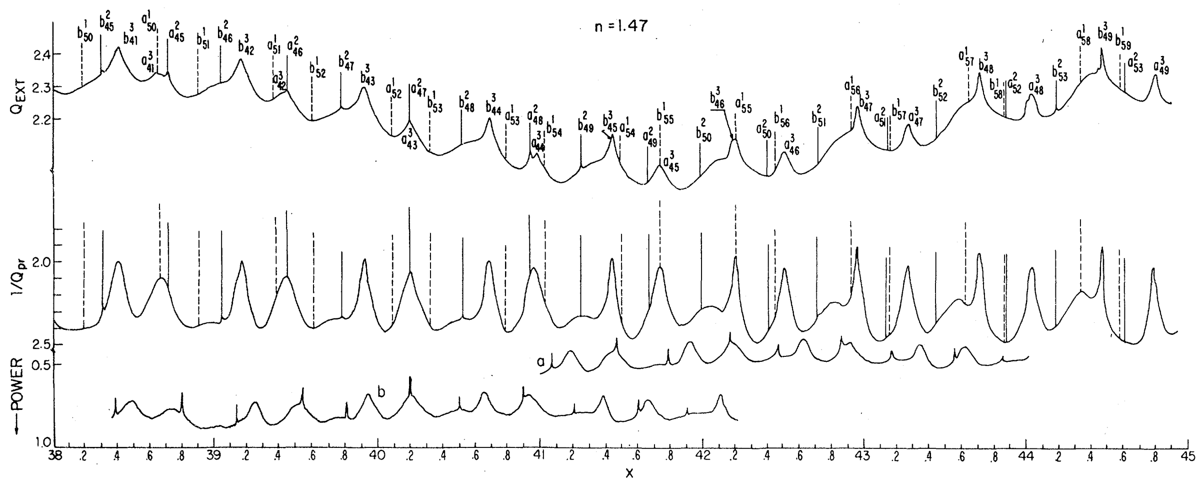

FIG. 1. Each sharp peak in the normalized extinction cross section Q_{ext} and in $1/Q_{pr}$ is caused by a resonance in one of the partial waves a_n or b_n. The first resonance in each partial wave (dashed line) is very narrow and the instrumental resolution $\Delta\lambda/\lambda = \Delta x/x$ at least of the order of 10^{-7} is required for their detection. The experimental measurement of the levitation power by Ashkin and Dziedzic[3] is shown in a curve a. From the peaks morphology it can be deduced that the curve a is misplaced toward the larger size parameter x by approximately two units in x. The correct position of the experimental curve is given by the curve b. We notice that the radius of a droplet is changing during the measurement. From the position of each measured peak we can calculate a radius of a droplet at the moment of measurement around considered peak.

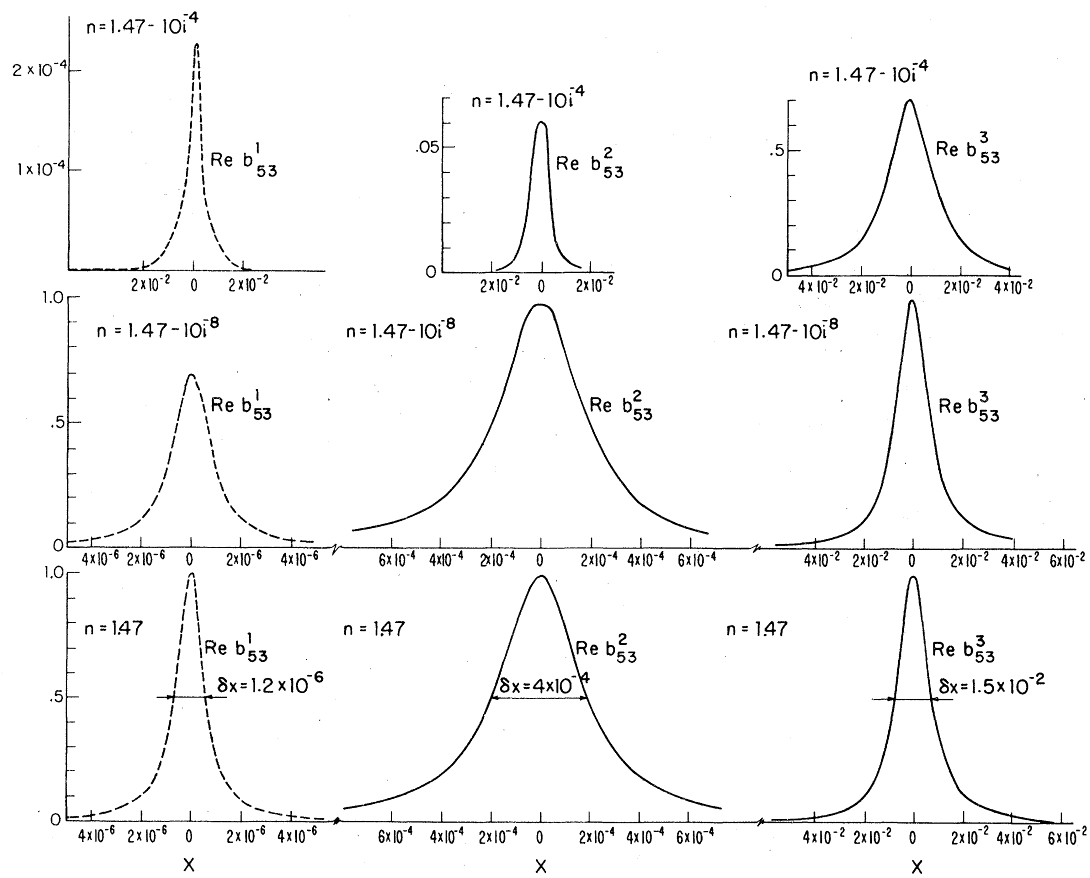

FIG. 2. To demonstrate the shape and the width of partial-wave resonances, the real part of the first three resonances of b_{53} is shown for three different imaginary parts of refractive index. The scale in x is measured from the position of a resonance peak.

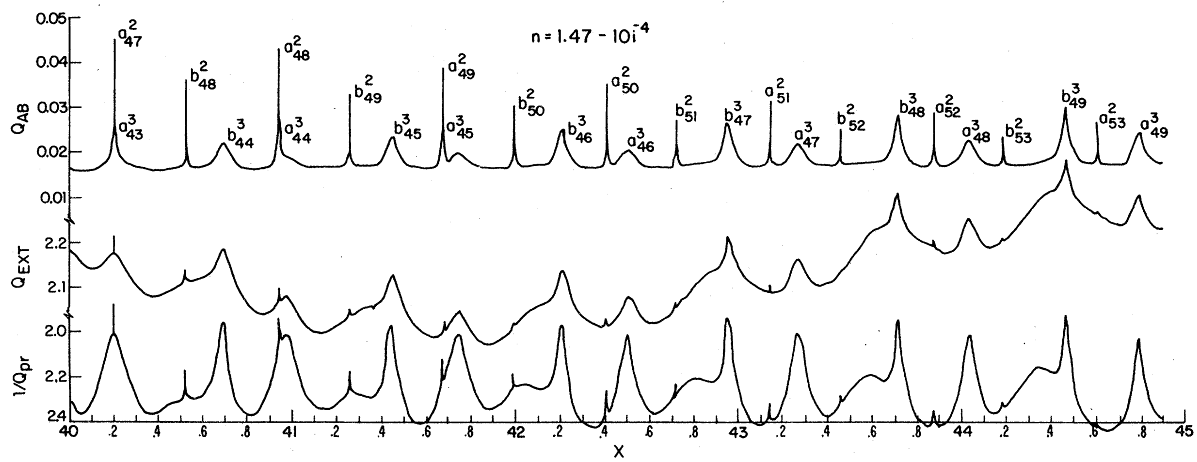

FIG. 3. Imaginary part of refractive index of the order of 10^{-4} reduces the strength of the first-order resonances by a factor of 10^{-4} so that they are beyond detection. The second- and the third-order resonances are now responsible for a resonant character of extinction, absorption, and radiation power.

of a_n and b_n, Q_{ext} and $1/Q_{pr}$ for refractive index $n = 1.47$ and the size parameter $38 \leq x \leq 45$. To find all the resonances in the considered region, we have used, in selected regions of x, an increment $\Delta x = 10^{-7}$ which gives a resolution of $\Delta x/x = \Delta\lambda/\lambda \simeq 2.5 \times 10^{-9}$. The fact that Im$a_n$ (or Imb_n) changes sign at the center of a resonance peak provides a convenient check that we have indeed found all resonances. The results of our calculations are shown on Fig. 1. Each resonant peak (the ripple structure) in the Q_{ext} curve is denoted by the symbol a_n or b_n corresponding to the resonating partial wave. In the investigated range of the size parameter x there are several resonances in each partial wave which can be associated with peaks in the extinction curve. The first, second, and the third

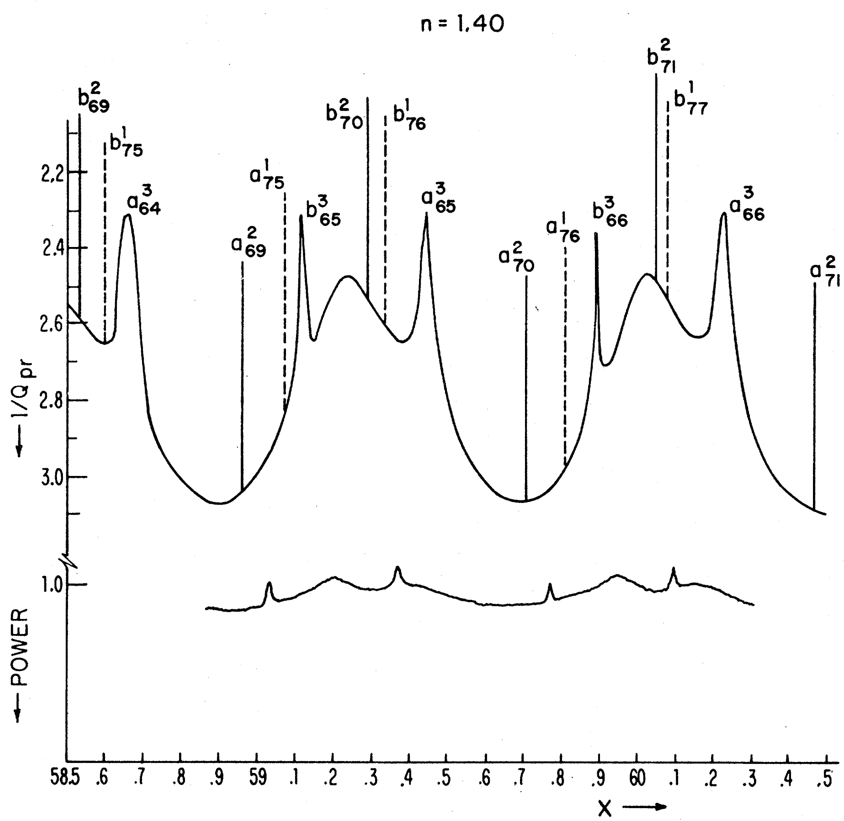

FIG. 4. In the size parameter region $x \sim 60$ the detection of the first- and second-order resonances requires the instrumental resolution of the order of $\Delta\lambda/\lambda = 10^{-9}$ and 10^{-7}, respectively. Consequently, with resolution of the order of 10^{-5} only the third- and higher-order resonances were detected[3] (the lower curve).

TABLE I. Position of resonance peaks for refractive index $n = 1.47$.

First-order resonances			
b_{50}^1	38.194 757	a_{50}^1	38.663 506
b_{51}	38.905 705	a_{51}	39.375 079
b_{52}	39.616 241	a_{52}	40.086 213
b_{53}	40.326 381	a_{53}	40.796 930
b_{54}	41.036 136	a_{54}	41.507 225
b_{55}	41.745 521	a_{55}	42.217 133
b_{56}	42.454 545 3	a_{56}	42.926 659 8
b_{57}	43.163 221 4	a_{57}	43.635 817 9
b_{58}	43.871 559 8	a_{58}	44.344 619 3
b_{59}	44.579 570 5		
Second-order resonances			
b_{45}^2	38.314	a_{45}^2	38.719
b_{46}	39.052	a_{46}	39.460
b_{47}	39.788	a_{47}	40.199
b_{48}	40.524	a_{48}	40.938
b_{49}	41.259	a_{49}	41.675
b_{50}	41.993	a_{50}	42.412
b_{51}	42.726 2	a_{51}	43.147 4
b_{52}	43.458 6	a_{52}	43.882 0
b_{53}	44.190 3	a_{53}	44.615 7
Third-order resonances			
b_{41}^3	38.41	a_{41}^3	38.67
b_{42}	39.17	a_{42}	39.44
b_{43}	39.93	a_{43}	40.18
b_{44}	40.67	a_{44}	40.98
b_{45}	41.45	a_{45}	41.75
b_{46}	42.21	a_{46}	42.50
b_{47}	42.96	a_{47}	43.27
b_{48}	43.71	a_{48}	44.04
b_{49}	44.47	a_{49}	44.79

resonances are denoted by the corresponding superscript. Thus, for example, b_{46}^2 denotes the second resonance in the b_{46} partial wave. It turns out that the halfwidth of the first resonances is of the order of $\delta x = 10^{-6}$. The second resonances are broader with $\delta x \simeq 10^{-4}$. The resolution of Fig. 1 is not sufficient to show the structure of these resonances (the structure is shown in Fig. 2) and consequently the first-order resonances are then represented by a dashed line and the second-order resonances by a solid line. The same resonances appear in the $1/Q_{pr}$ curve.

Curve a in Fig. 1 shows the measured power for levitation as reported by Ashkin and Dziedzic.[3] Note that the first-order resonances were not observed. Based on the results of our calculation, it seems probable that the scale of the size parameter x as given in Ref. 3 is not correct. However, the x scale in Ref. 3 was derived from the λ scale using an approximate value of the droplet radius determined by a microscope with the accuracy $\pm 5\%$. Thus the position of curve a in Fig. 1 with respect to the x coordinate may be shifted (due to the $\pm 5\%$ accuracy in the measurement of the droplet radius) by two units of x. From the morphology of the peaks it follows that a qualitative agreement (agreement between the peaks structure) between the $1/Q_{pr}$ calculation and the measurement of the levitation power can be achieved by shifting curve a by about two units of x toward smaller x values (curve b). Observing how the peak a_{47}^2 is superimposed on a_{43}^3; a_{46}^2 on a_{42}^3 and a_{48}^2 on a_{44}^3, we conclude that this identification (of the proper position of the measured curve with respect to the x scale) is unambiguous. We also note that the x scale on the measured curve is compressed compared to the theoretical curve (observed peaks at low x are shifted to the right from their correct position and observed peaks at high x are shifted to the left). This can happen only if the radius of an observed droplet is changing during the measurement. A simple calculation shows that the droplet's radius was not a constant but changed from $r_1 = 3.830$ μm when the b_{50}^2 resonance was observed to $r_2 = 3.793$ μm when the b_{45}^2 resonance was measured. This is consistent with the evaporation of the droplet during the experiment.

The effect of a small absorption of the droplet's material is shown in Fig. 2. Our calculations show that with an increasing imaginary part of the refractive index, the height of a resonant peak is reduced and the width of a resonance increases. The first-order resonances are affected the most by the change in the imaginary part of refractive index.

When the refractive index is changed to $n = 1.47 - 10^{-4}\,i$ the peak of the first-order resonances is reduced from 1 to 2×10^{-4} (see Fig. 2) and the first-order resonances practically disappear. The normalized extinction and absorption cross sections and $1/Q_{pr}$ for this case are shown in Fig. 3. Thus there is a chance that the first-order resonances will not be observed even with improved instrumental resolution if the droplet material is slightly absorbing.

Figure 4 shows another levitation-power measurement[3] and corresponding theoretical curve. In this case none of the first- and the second-order resonances were detected. The required instrumental resolution is $\Delta\lambda/\lambda \sim 10^{-9}$ for the first- and $\Delta\lambda/\lambda \sim 10^{-7}$ for the second-order resonances.

To summarize, we have shown that there are many sharp peaks in the Q_{ext}, Q_{abs}, Q_{sct}, and $1/Q_{pr}$ which are a manifestation of the resonances in the partial waves a_n and b_n. All resonances in the considered range of x with a halfwidth of the order of 10^{-4} and larger have been experimentally detected in the levitation experiment by Ashkin and Dziedzic.[3] However, there are many resonances with a smaller halfwidth which have not been observed with their present instrumental resolution. In Table I we have listed the approximate position of the resonance peaks in the investigated range of the size parameter x. The few first sharp resonances in each partial wave are connected with a surface wave[6] on a spherical droplet. It has been conjectured that these resonances do not exist if a particle is of an irregular shape.[7] Consequently, we predict that the peaks in the radiation pressure force or in the levitation pressure do not exist if a suspended particle is irregularly shaped. It would be of interest to confirm or to disprove this conjecture experimentally. By an irregular particle we mean a particle without axial symmetry whose surface curvature changes significantly over distances comparable to the wave height λ of incident radiation; and whose deviation from a sphere is not limited to a thin surface layer of a thickness $t<\lambda$. Thus, for example, a slightly deformed sphere or a spheroid does not represent an irregular particle. On the other hand typical dust or soil particles or laboratory prepared nonspherical aerosols[6] may represent irregular particles.

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