Generation of coherent radiation in a resonant medium by a relativistic charged particle

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The resonant radiation due to the coherent excitation of the electric dipole moments of the atoms which are embedded in a host medium with real dielectric constant ϵ has been studied. These dipole moments are induced by the electric field of a relativistic charge particle moving through the resonant medium. In this context, coherent excitation refers to the correlation of phases of the dipole moments, which are induced by a single relativistic charged particle. The phases of dipole moments which are induced by different charged particles are, of course, uncorrelated. The density $\mathfrak{N}$ of the embedded atoms in the host medium is assumed to be sufficiently small so that $\mathfrak{N}|\alpha(\omega_r)| \ll 1$, where $\alpha(\omega_r)$ is the polarizability of the atoms at the resonance frequency ω_r. Under this assumption, the electric field acting upon each atom is approximately equal to the electric field of the incident relativistic charged particle. This field is analyzed into its frequency spectrum and the effect of each frequency component on the embedded atoms is determined. Expressions for the fields, far away from the resonant medium, and the radiated energy are derived for transitions between bound states. Two cases are considered. In the first case, it is assumed that $\zeta = (1-\epsilon\beta^2)^{-1/2}$ is real, where $\beta = v/c$, v being the velocity of the incident charged particle and c the velocity of light. It is shown that if $\zeta \gg 1$, the radiation is emitted in the direction of motion of the incident charged particle. In the second case, it is assumed that ζ is imaginary. Since the Čerenkov condition is satisfied, i.e., $\beta\sqrt{\epsilon} > 1$, there is a primary Čerenkov radiation in the host medium which excites coherently the embedded atoms at their resonance frequency. In spite of the fact that both the primary and stimulated radiations are emitted in the direction of the Čerenkov cone, it is shown that it is possible to detect the presence of the latter because of the finite lifetime of the embedded atoms. Therefore, the possibility arises for the manifestation of their coherent excitation by relativistic charged particles.

I. INTRODUCTION

This paper is concerned with the radiation due to the coherent excitation of the electric dipole moments of the atoms which are embedded in a host medium with real dielectric constant ϵ. These dipole moments are induced by the electric field of a relativistic charged particle moving through the resonant medium. In this context, coherent excitation refers to the correlation of the phases of the dipole moments which are induced by a single relativistic charged particle. The phases of dipole moments which are induced by different charged particles are, of course, uncorrelated. In a previous paper,[1] the generation of coherent x rays by a relativistic charged particle moving through a crystal was studied and it was shown that the coherent radiation is emitted very close to the Bragg directions. The basic assumption in that paper was that the density $\mathfrak{N}$ of the atoms in the crystal is such that $\mathfrak{N}|\alpha(\omega)| \ll 1$, where $\alpha(\omega)$ is the polarizability of the atoms. In this case, the electric field acting upon each atom is approximately equal to the electric field of the incident relativistic charged particle. This assumption is valid for x rays, but it is not, in general, true for light frequencies if the density $\mathfrak{N}$ is that of the atoms in a crystal (i.e., $\mathfrak{N} \sim 10^{22}$ atoms/cm^3). On the other hand, in a resonant medium the density $\mathfrak{N}$ of the embedded atoms may be sufficiently small (e.g., $\mathfrak{N} \sim 10^{18}$ atoms/cm^3) so that the above assumption is valid for light frequencies. In such a case, the electric field acting upon each embedded atom can be taken approximately equal to the electric field of the incident relativistic charged particle. Based upon this assumption then, the characteristics of the emitted coherent radiation, namely, the radiation pattern, the behavior of the radiation fields in the time domain, and the radiated energy, for resonance frequencies in the optical region, will be presented in this paper. It should be pointed out that extensive work[2-15] has been done in the past, on the problem of an incident charged particle in a dielectric medium. This work refers either to the energy loss per unit length by the charged particle or to the density effect.[16] The theory which relates the work in this paper with the work that has been done in the past is presented in the Appendix. It is shown there that the coherently radiated energy and the absorbed energy by the embedded atoms in the resonant medium, under the influence of the external electric field of the incident charged particle, are distinct physical quantities. The total energy supplied by the incident particle is equal to the sum of the coherently radiated and absorbed energies. This paper is concerned with the coherently radiated energy while the work that has been done in the past is concerned with the absorbed energy. It is demonstrated in the Appendix that the absorbed energy is identical with the "energy loss" by the incident particle. The latter term is commonly used in the literature. Finally, in the Appendix, the conditions are stated under which the effective electric field acting upon each embedded atom can be replaced by the external field of the incident charged particle. This constitutes the first-order approximation. This approximation is valid under the condition $\mathfrak{N}|\alpha(\omega)| \ll 1$ stated above, and also under the condition that the dimensions of the resonant medium in the directions of the coherently emitted radiation is small in comparison with the inverse of the absorption coefficient.

Since this paper is based on the first-order approximation, it will be assumed in the following that the dimensions of the resonant medium are sufficiently small so that negligible absorption takes place. In addition, it will be assumed that the resonant medium is in the form of a thin slab, so that the amount of energy lost by the incident relativistic charged particle as it moves through the slab is negligible. The radiation properties of the embedded atoms will be studied close to a single resonance frequency. In the past, multifrequency absorption in a dielectric medium has been considered.[6,10] In frequency regions of a resonant medium where there is no overlap of the resonant lines, each one of these lines can be treated independently of the others. This paper is confined to such frequency regions, so that the single-frequency model studied here is justified.

The following approach to the problem will be used. The electromagnetic field of the incident relativistic charged particle will be analyzed into its frequency spectrum and the effect of each Fourier component on the embedded atoms will be determined.

Two cases will be examined. In the first case, it will be assumed that $\beta^2\epsilon<1$, where $\beta=v/c$, v is the velocity of the incident charged particle and c is the velocity of light. The electric field of the charged particle decreases exponentially for large distances from its trajectory. Thus the effective distance from the trajectory of the charged particle over which its electric field is sufficiently large to induce radiation by the embedded atoms is approximately equal to $l_{\text{eff}}=\zeta\beta\lambda\!\!\!^{-}$, where $\zeta=(1-\beta^2\epsilon)^{-1/2}$ and $\lambda\!\!\!^{-}$ is the wavelength of the radiation in vacuum, i.e., $\lambda\!\!\!^{-}=c/\omega$, where ω is the frequency. For example, if $\zeta=100$ and $\lambda\!\!\!^{-}=10^{-5}$ cm, then $l_{\text{eff}}=10^{-3}$ cm, i.e., atoms as far as 10^3 atomic distances away from the trajectory of the incident charged particle will be coherently excited by its electric field (it was assumed that the average distance between the embedded atoms is of the order of 10^{-6} cm which corresponds to a density of 10^{18} atoms per cm^3). It will be shown that if $\zeta \gg 1$, the induced radiation is emitted in the direction of motion of the incident charged particle.

In the second case, it will be assumed that $\beta^2\epsilon>1$. Since the Čerenkov condition is satisfied, i.e., $\beta\sqrt{\epsilon}>1$, there will be a primary Čerenkov radiation in the host medium which will excite coherently the embedded atoms at their resonance frequency. As a consequence of this coherent excitation, there will be a secondary stimulated resonant radiation which is also emitted in the direction of the Čerenkov cone. Although both the primary and secondary radiations are emitted in the same direction, it is possible to detect the presence of the secondary radiation because of the finite lifetime of the embedded atoms. Therefore, it may be possible to observe the coherent excitation of the embedded atoms in a resonant medium by relativistic charged particles moving through it.

II. THE MODEL

For simplicity, it will be assumed that the whole space is filled with a medium with real dielectric constant ϵ and the region in which the embedded atoms are located is a slab of width L. The z axis is chosen normal to the two surfaces of the slab and the origin of the coordinate system is chosen so that these plane surfaces lie at $z=\pm\frac{1}{2}L$. A charged particle moves with constant velocity v along the z axis, in the positive direction. The origin of time is chosen so that at time $t=0$ the particle is at $z=0$. Its electric field, in the frequency domain, is equal to[17]

$$\vec{E}_0(\vec{R},\omega)=-(2Q/\epsilon\beta c)[\vec{\nabla}_\rho+i(\lambda_0/\zeta)\hat{z}] \times K_0(\lambda_0\rho)e^{i(\omega/\beta c)z}, \tag{2.1}$$

where

$$\vec{\nabla}_\rho=\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}. \tag{2.2}$$

The symbols β, ζ, c, and ϵ have already been defined, while Q is the charge of the particle. $K_0(x)$ is the modified Bessel function of order zero, $\lambda_0=\omega/\zeta\beta c$, and $\hat{x},\hat{y},\hat{z}$ are unit vectors along the x, y, z axis, respectively. The position vector $\vec{R}$ in Eq. (2.1) has cylindrical coordinates (ρ,ϕ,z).

It will be assumed that the amount of energy the charged particle loses is negligible compared to its total energy. In a real situation, however, the particle does lose energy as it travels through the slab, mostly due to ionization and excitation of the atoms in the crystal and also due to bremsstrahlung. If coherent emission of radiation is to take place,[18] the difference in the times for the incident charged particle to traverse successive distances $2\pi\lambdabar/\sqrt{\epsilon}$ should be small compared with the period $2\pi/\omega$ of the emitted wave.

Each atom in the slab has an induced dipole moment equal to

$$\vec{P}_i(\omega)=\alpha_d(\omega)\vec{E}(\vec{R}_i,\omega), \tag{2.3}$$

where $\vec{R}_i$ is the position of the ith atom, $\vec{E}(\vec{R}_i,\omega)$ is the electric field acting upon it, and $\alpha_d(\omega)$ is the atomic polarizability (for simplicity, the embedded atoms are assumed to be identical). The expression for $\alpha_d(\omega)$ depends on the relation between the frequency under consideration and the ionization potentials, divided by $\hbar$, of the electrons bound to each atom. Thus if the frequency component associated with the electric field of the incident charged particle is less than the ionization potentials, divided by $\hbar$, of the electrons in each atom, then only resonant transitions between bound states are possible and the atomic polarizability is equal to[19]

$$\alpha_d(\omega)=\frac{1}{2\pi}\sigma_d(\omega_r)\sqrt{\epsilon}\,c\frac{\Delta\omega_r}{\omega_r^2-\omega^2-i2\omega\Delta\omega_r}, \tag{2.4}$$

where

$$\sigma_d(\omega_r)=(\pi/\epsilon)\lambdabar_r^2A_{21}/\Delta\omega_r\,. \tag{2.5}$$

Here, $\sigma_d(\omega_r)$ is the atomic absorption cross section at resonance, ω_r is the resonance frequency of the transition, $\lambdabar_r=c/\omega_r$, $\Delta\omega_r$ is the broadening of the resonant line, and A_{21} is the spontaneous rate of radiation between the bound states. In this paper, only such transitions will be considered.

III. RADIATION FIELDS

It will be assumed that the inequality $\mathfrak{N}|\alpha_d(\omega_r)|\ll 1$ is satisfied, where $\mathfrak{N}$ is the density of the embedded atoms in the slab. In this case, the effective field $\vec{E}(\vec{R}_i,\omega)$ in Eq. (2.3) may be replaced by the external field $\vec{E}_0(\vec{R}_i,\omega)$, since the electric field at the ith atom which is due to its neighbors is negligible in comparison to the external electric field.

Under this assumption, the electromagnetic field of the induced radiation at the point of observation $\vec{R}$ outside the slab is equal to

$$\vec{E}(\vec{R},\omega)=\frac{1}{\epsilon}\sum_{i=1}^{M}\alpha_d(\omega)\vec{\nabla}_R\times\vec{\nabla}_R\times[G(\vec{R}-\vec{R}_i;\sqrt{\epsilon}\,\omega/c)\vec{E}_0(\vec{R}_i,\omega)], \tag{3.1a}$$

$$\vec{B}(\vec{R},\omega)=\sum_{i=1}^{M}-\frac{i\omega}{c}\alpha_d(\omega)\vec{\nabla}_R\times[G(\vec{R}-\vec{R}_i;\sqrt{\epsilon}\,\omega/c)\vec{E}_0(\vec{R}_i,\omega)], \tag{3.1b}$$

where

$$G(\vec{R},k)=e^{ikR}/R, \tag{3.2}$$

is the retarded Green's function and $R=|\vec{R}|$. Also, M is the total number of embedded atoms in the slab. Now, for a very large number of atoms M, the summation in Eqs. (3.1a) and (3.1b) can be replaced by an integral. Moreover, since the radiation field far away from the slab is of interest, use will be made of the far-field approximation of the retarded Green's function, which is given by the expression

$$G(\vec{R}-\vec{R}';\sqrt{\epsilon}\,\omega/c)\sim(e^{i\sqrt{\epsilon}(\omega/c)R}/R)\,e^{-i\sqrt{\epsilon}(\omega/c)\hat{R}\cdot\vec{R}'}. \tag{3.3}$$

The unit vector $\hat{R}$ is in the direction of the point of observation $\vec{R}$. With the help of Eqs. (3.1) and (3.3) the following expressions are obtained for the radiation fields:

$$\vec{E}_{\rm rad}(\vec{R},\omega)\sim\chi(\omega)(\omega/c)^2(e^{i\sqrt{\epsilon}(\omega/c)R}/R)\times\hat{R}\times[\hat{R}\times\vec{\pi}(\sqrt{\epsilon}(\omega/c)\hat{R},\omega)], \tag{3.4a}$$

$$\vec{B}_{\rm rad}(\vec{R},\omega)\sim\sqrt{\epsilon}\,\chi(\omega)(\omega/c)^2(e^{i\sqrt{\epsilon}(\omega/c)R}/R)\times\hat{R}\times\vec{\pi}(\sqrt{\epsilon}(\omega/c)\hat{R},\omega), \tag{3.4b}$$

where

$$\vec{\pi}(\vec{k},\omega)=\int_V\vec{E}_0(\vec{R}',\omega)e^{-i\vec{k}\cdot\vec{R}'}d^3R', \tag{3.5}$$

and

$$\chi(\omega)=\mathfrak{N}\alpha_d(\omega). \tag{3.6}$$

Here V is the volume occupied by the embedded atoms, i.e., by the slab. It may be noted that although the field $\vec{E}_0(\vec{R},\omega)$ diverges at $\rho=0$, the integral in Eq. (3.5) is finite. It will be assumed in the following that $\beta^2\epsilon<1$, so that the parameter ζ in Eq. (2.1) is real and, therefore, the electric field of the incident charged particle has an exponentially decreasing asymptotic behavior[20] for large distances from the trajectory of the particle. Thus, if the slab extends a few effective lengths $l_{\rm eff}=\zeta\beta\lambdabar$ from the trajectory of the charged particle, its volume may be considered to be of infinite extent on the x-y plane without introducing a significant error in the evaluation of the integral in Eq. (3.5). When Eq. (2.1) is substituted into Eq. (3.5), the integration can be performed analytically and it leads to the relation[21]

$$\vec{\pi}(\vec{k},\omega)=-i\,\frac{4\pi Q}{\epsilon\beta^2c^2}\,\omega\,\frac{(\beta c/\omega)\vec{k}_\rho+(1/\xi^2)\hat{z}}{k_\rho\cdot k_\rho+(\omega/\xi\beta c)^2}\times\frac{\sin(k_z-\omega/\beta c)\,L/2}{\frac{1}{2}(k_z-\omega/\beta c)}\,, \tag{3.7}$$

where $\vec{k}_\rho=k_x\hat{x}+k_y\hat{y}$. It is appropriate to express the vector $\vec{\pi}(\vec{k},\omega)$ in spherical coordinates with components π_r, π_θ, and π_ϕ. It follows from Eqs. (3.4a), (3.4b), and the cylindrical symmetry of the problem that only the π_θ component contributes to the radiation fields and is given by the relation

$$\pi_\theta(\sqrt{\epsilon}\,(\omega/c)\hat{R},\omega)=i\,\frac{4\pi Q}{\epsilon\omega}\sin\theta\,\frac{1-\beta^2\epsilon-\beta\sqrt{\epsilon}\,\cos\theta}{1-\beta^2\epsilon\cos^2\theta}\times\frac{\sin(1-\beta\sqrt{\epsilon}\,\cos\theta)(\omega/\beta c)L/2}{\frac{1}{2}(1-\beta\sqrt{\epsilon}\,\cos\theta)\omega/\beta c}\,, \tag{3.8}$$

where (θ,ϕ) are the spherical coordinates of the unit vector $\hat{R}$ in the direction of observation. The π_ϕ component vanishes. When Eq. (3.8) is substituted into Eqs. (3.4a) and (3.4b), their Fourier transforms with respect to the frequency ω provide the radiation fields in the time domain. In spherical coordinates, these fields are equal to

$$\hat{E}_{\mathrm{rad}\,\theta}(\vec{R},t)\sim-\frac{1}{R}\frac{2Q}{\sqrt{\epsilon}}\tau_d(\omega_r)\frac{\Delta\omega_r}{c}\times\sin\theta\,\frac{1-\beta^2\epsilon-\beta\sqrt{\epsilon}\,\cos\theta}{1-\beta^2\epsilon\cos^2\theta}\,F(\tau), \tag{3.9a}$$

$$\hat{B}_{\mathrm{rad}\,\phi}(\vec{R},t)=\sqrt{\epsilon}\,\hat{E}_{\mathrm{rad}\,\theta}(\vec{R},t), \tag{3.9b}$$

where

$$F(\tau)=\int_{-L/2}^{L/2}e^{-\Delta\omega_r x}\cos\omega_r x\,\Theta(x)\,dz'\,, \tag{3.10}$$

and

$$x=\tau-(1-\beta\sqrt{\epsilon}\,\cos\theta)z'/\beta c. \tag{3.11}$$

Also $\tau=t-\sqrt{\epsilon}\,(\omega/c)R$ and $\tau_d(\omega_r)=\mathfrak{N}\sigma_d(\omega_r)$ is the absorption coefficient of the slab at the resonance frequency ω_r. Finally, $\Theta(x)=1$ if $x\geq0$ and $\Theta(x)=0$ if $x<0$. All other components of the radiation fields vanish. Since $x\geq0$ in Eq. (3.10), it follows that, at all times t, the inequality

$$\tau\geq(z'/L)\Delta t, \tag{3.12}$$

must hold, where

$$\Delta t=(1-\beta\sqrt{\epsilon}\,\cos\theta)L/\beta c. \tag{3.13}$$

Also, z' must lie in the interval $(-\frac{1}{2}L,\frac{1}{2}L)$ [cf., the limits of integration in Eq. (3.10)]. Thus, there are three cases to distinguish in the evaluation of the function $F(\tau)$:

(i) If $\tau\leq-\frac{1}{2}\Delta t$, then it follows from Eq. (3.12) that $z'\leq\frac{1}{2}L$, i.e., z' lies outside the interval $(-\frac{1}{2}L,\frac{1}{2}L)$ and, therefore, $F(\tau)\equiv0$. In this case, the radiation fields at the point of observation are zero. The equality $\tau=-\frac{1}{2}\Delta t$ determines the time of arrival of the electromagnetic wave at the point of observation if the origin of time is taken as the instant the charged particle enters the slab. Hence, if $\tau<-\frac{1}{2}\Delta t$, the electromagnetic wave has not arrived at the point of observation and, of course, the radiation fields are zero there.

(ii) If $-\frac{1}{2}\Delta t\leq\tau\leq\frac{1}{2}\Delta t$, then if follows from Eq. (3.12) that $-\frac{1}{2}L\leq z'\leq(\tau/\Delta t)L$ and, therefore,

$$F(\tau)=\frac{\omega_r}{\omega_r^2+\Delta\omega_r^2}\,\frac{\beta c}{1-\beta\sqrt{\epsilon}\,\cos\theta}\times[e^{-\Delta\omega_r\tau_+}\sin\omega_r\tau_++(\Delta\omega_r/\omega_r)(1-e^{-\Delta\omega_r\tau_+}\cos\omega_r\tau_+)]\,, \tag{3.14}$$

where

$$\tau_\pm=\tau\pm\tfrac{1}{2}\Delta t. \tag{3.15}$$

The equality $\tau=\frac{1}{2}\Delta t$ determines the time of arrival of the electromagnetic wave at the point of observation if the origin of time is taken as the instant the charged particle leaves the slab. Hence, if $-\frac{1}{2}\Delta t\leq\tau\leq\frac{1}{2}\Delta t$, Eqs. (3.9a), (3.9b), and (3.14) provide the electromagnetic field, as it reaches the point of observation at a retarded time, from the instant the incident charged particle enters the slab to the instant it exits.

(iii) If $\tau\geq\frac{1}{2}\Delta t$, then it follows from Eq. (3.12) that $-\frac{1}{2}L\leq z'\leq\frac{1}{2}L$ and, therefore,

$$F(\tau)=\frac{\omega_r}{\omega_r^2+\Delta\omega_r^2}\,\frac{\beta c}{1-\beta\sqrt{\epsilon}\,\cos\theta}e^{-\Delta\omega_r\tau_-}\times[e^{-\Delta\omega_r\Delta t}\sin\omega_r\tau_+-\sin\omega_r\tau_-+(\Delta\omega_r/\omega_r)(\cos\omega_r\tau_--e^{-\Delta\omega_r\Delta t}\cos\omega_r\tau_+)], \tag{3.16}$$

where τ_+, τ_-, and Δt have already been defined by Eqs. (3.15) and (3.13), respectively. In this case, Eqs. (3.9a), (3.9b), and (3.16) provide the electromagnetic field, as it reaches the point of observation at a retarded time, after the incident charged particle leaves the slab.

IV. RADIATION PATTERN AND RADIATED ENERGY

The average amount of energy radiated per unit time and per unit solid angle is equal to

$$\frac{d^2W_{\mathrm{rad}}}{dt\,d\Omega_{\hat{R}}}=\frac{c}{4\pi}\langle\hat{E}_{\mathrm{rad}\,\theta}(\vec{R},t)\hat{B}_{\mathrm{rad}\phi}(\vec{R},t)\rangle R^2, \tag{4.1}$$

where the brackets denote a time averaging over the fast oscillating terms with a period $2\pi/\omega_r$. In order to obtain simple expressions for the radiated energy, it will be assumed that $\Delta\omega_r/\omega_r\ll1$, which is valid for resonant transitions, and also that $\Delta\omega_r\Delta t\ll1$. Physically, the second assumption means that the incident charged particle traverses the slab before the embedded atoms have sufficient time to decay to their ground state. Then the main contribution of the radiation comes for times such that $\tau \geq \frac{1}{2}\Delta t$ [case (iii) in previous Sec. III]. Substitution of Eqs. (3.9a), (3.9b), and (3.16) into Eq. (4.1) and the time averaging lead to expression

$$\frac{d^2 W_{\rm rad}}{dt\, d\Omega_{\hat{R}}} = \hbar\omega_r \frac{\Delta\omega_r}{\pi} N_0 e^{-2\Delta\omega_r \tau_-}(1-\cos\omega_r \Delta t)\Phi(\theta), \qquad (4.2)$$

where

$$N_0 = \frac{1}{\sqrt{\epsilon}} \frac{Q^2}{\hbar c} \beta^2 \tau_d^2(\omega_r) \lambda\!\!\!^-_r{}^2 \frac{\Delta\omega_r}{\omega_r}, \qquad (4.3)$$

and

$$\Phi(\theta) = \left(\frac{\sin\theta}{1-\beta\sqrt{\epsilon}\cos\theta} \frac{1-\beta^2\epsilon - \beta\sqrt{\epsilon}\cos\theta}{1-\beta^2\epsilon\cos^2\theta}\right)^2. \qquad (4.4)$$

Also τ_- is greater or equal to zero. It is seen from Eq. (4.2) that the average radiated power per unit solid angle decreases exponentially with time and its decay constant is equal to the lifetime of the embedded atoms (for homogeneous broadening).

Integration over time of Eq. (4.2) yields the following expression for the number of radiated photons per unit solid angle:

$$\frac{dN_{\rm rad}}{d\Omega_{\hat{R}}} = \frac{1}{2\pi} N_0 \{1-\cos[\delta(1-\beta\sqrt{\epsilon}\cos\theta)]\}\Phi(\theta), \qquad (4.5)$$

where

$$\delta = L/\beta\lambda\!\!\!^-_r\,. \qquad (4.6)$$

The gross characteristics of the radiation pattern are determined by the function $\Phi(\theta)$, while the expression within square brackets in Eq. (4.5) provides a fringe structure as the polar angle θ varies. It follows from Eqs. (4.4) and (4.5) that significant radiation occurs only when $\beta\sqrt{\epsilon}$ is very close to unity. In this case, the function $\Phi(\theta)$ becomes maximum when $\theta = \theta_{\rm max}$, where either

$$\theta_{\rm max} = [\tfrac{2}{3}(1-\beta\sqrt{\epsilon})]^{1/2}, \qquad (4.7a)$$

or

$$\theta_{\rm max} = \pi - [2(1-\beta\sqrt{\epsilon})]^{1/2}. \qquad (4.7b)$$

Therefore, maximum radiation occurs in the direction of motion of the incident charged particle (forward radiation), but there is also weak radiation in the opposite direction (backward radiation).

Integration over the solid angle of Eq. (4.5) leads to the following expression for total number of radiated photons:

$$N_{\rm rad} = N_0 Q(\delta), \qquad (4.8)$$

where

$$Q(\delta) = \int_{-1}^{1} \{1-\cos[\delta(1-\beta\sqrt{\epsilon}\,x)]\} f(x)\,dx, \qquad (4.9)$$

and

$$f(x) = \frac{1-x^2}{(1-\beta\sqrt{\epsilon}\,x)^4} \frac{1-\beta^2\epsilon - \beta\sqrt{\epsilon}\,x^2}{1+\beta\sqrt{\epsilon}\,x}. \qquad (4.10)$$

The function $f(x)$, being the ratio of two polynomials, can be expressed in terms of partial fractions, i.e., in the form

$$f(x) = \sum_{n=1}^{4} \frac{A_n}{(1-\beta\sqrt{\epsilon}\,x)^n} + \sum_{n=1}^{2} \frac{B_n}{(1+\beta\sqrt{\epsilon}\,x)^n}, \qquad (4.11)$$

where the coefficients A_n, B_n depend on $\beta\sqrt{\epsilon}$. In particular, when $\beta\sqrt{\epsilon}$ is very close to unity, these coefficients are equal to $A_1 = \frac{1}{8}$, $A_2 = -\frac{3}{4}$, $A_3 = \frac{1}{2}$, $A_4 = -\frac{1}{2}(1-\beta\sqrt{\epsilon})$ and $B_1 = \frac{1}{8}$, $B_2 = -\frac{1}{8}(1-\beta\sqrt{\epsilon})$. When Eq. (4.11) is substituted into Eq. (4.9), it is rather easy to show, through partial integrations, that the function $Q(\delta)$ can be expressed in terms of the sine and cosine integrals[22] Si(z) and Ci(z), respectively, as well as other elementary functions. The general expression for $Q(\delta)$ is too lengthy and will not be given here. On the other hand, various cases will be considered when $\beta\sqrt{\epsilon}$ is very close to unity. The terms proportional to A_n in Eq. (4.11) are associated with the forward radiation and the terms proportional to B_n with the backward radiation. Hence, the function $Q(\delta)$ can be written as the sum of two functions $Q_f(\delta)$ and $Q_b(\delta)$ associated with the forward and backward radiations, respectively. These functions are given as follows:

(i) If $\delta(1-\beta\sqrt{\epsilon}) \ll 1$, then

$$Q_f(\delta) = \frac{\delta^2}{4}\left(\frac{1}{2} - \ln\frac{1-\beta\sqrt{\epsilon}}{2}\right) + \tfrac{1}{8}(1-2\delta^2)[\gamma + \ln 2\delta - \mathrm{Ci}(2\delta)] - \tfrac{3}{4}\delta\,\mathrm{Si}(2\delta) - \tfrac{1}{8}\delta\sin 2\delta + \tfrac{5}{16}(1-\cos 2\delta), \qquad (4.12a)$$

$$Q_b(\delta) = -\frac{1}{8}\left(1 + \ln\frac{1-\beta\sqrt{\epsilon}}{2}\right) + \tfrac{1}{64}[\gamma + 1 + \ln\delta(1-\beta\sqrt{\epsilon}) - \mathrm{Ci}(2\delta)]\cos 2\delta - \tfrac{1}{64}\mathrm{Si}(2\delta)\sin 2\delta, \qquad (4.12b)$$

where γ is Euler's constant ($\gamma = 0.5772\cdots$).

(ii) If $\delta(1-\beta\sqrt{\epsilon}) \ll 1$ and $2\delta \gg 1$, then

$$Q_f(\delta) = \tfrac{1}{4}\delta^2[-\ln\delta(1-\beta\sqrt{\epsilon}) + \tfrac{1}{2} - \gamma] - \tfrac{3}{8}\pi\delta + \tfrac{1}{8}(\gamma + \ln 2\delta) + \tfrac{5}{16}, \qquad (4.13a)$$

$$Q_b(\delta) = -\tfrac{1}{8}(1-\cos 2\delta)[1 + \ln\delta(1-\beta\sqrt{\epsilon})] + \tfrac{1}{8}\ln 2\delta + \tfrac{1}{8}\gamma\cos 2\delta - \tfrac{1}{16}\pi\sin 2\delta. \qquad (4.13b)$$

(iii) If $\delta(1-\beta\sqrt{\epsilon})\gg 1$, then

$$Q_f(\delta)=\frac{1}{12}\,\frac{1}{(1-\beta\sqrt{\epsilon})^2}-\frac{3}{4}\,\frac{1}{1-\beta\sqrt{\epsilon}}+\frac{5}{16}-\frac{1}{8}\ln\frac{1-\beta\sqrt{\epsilon}}{2}, \tag{4.14a}$$

$$Q_b(\delta)=-\frac{1}{8}\left(1+\ln\frac{1-\beta\sqrt{\epsilon}}{2}\right). \tag{4.14b}$$

As a numerical example, the following numbers are chosen: $\tau_d(\omega_r)\sim 1\ \mathrm{cm}^{-1}$, $\lambda\!\!\!^{-}_r\sim 10^{-5}$ cm, $\Delta\omega_r/\omega_r\sim 10^{-6}$, $1-\beta\sqrt{\epsilon}\sim 10^{-4}$. For a slab of width $L\sim 10^{-2}$ cm and for a 1-mA incident beam of electrons, which corresponds to 10^{16} incident electrons per second, there will be radiated approximately 10^4 photons per second in the forward direction. On the other hand, for a slab of width $L\sim 1$ cm and for the same incident beam of electrons there will be radiated 10^5 photons per second in the forward direction. The backward radiation is insignificant in either case.

The other case to be considered is when $\beta^2\epsilon>1$. Since the parameter ζ in Eq. (2.1) is imaginary, there will be a primary Čerenkov radiation in the host medium which will excite coherently the embedded atoms in the slab. Therefore, there will be a secondary stimulated resonant radiation emitted by the atoms. The same procedure is followed and the same assumptions made as before to calculate this radiation, but it is more convenient to work in the frequency domain. Since the primary Čerenkov radiation decreases very slowly with the distance from the trajectory of the incident charged particle, the slab must have finite dimensions in order to obtain a finite secondary radiation. It will be assumed that the slab has the shape of a cylinder with radius ρ_0 from the z axis and width L, as before, along this axis. After a rather lengthy calculation, which will not be given, the following expression is obtained for the energy radiated by the embedded atoms per unit frequency interval and per unit solid angle:

$$\frac{d^2W_{\mathrm{rad}}}{d\omega\, d\Omega_{\hat{R}}}=\hbar\omega_r\,\frac{\sqrt{\epsilon}}{4\pi^3}N_c\,\frac{\lambda\!\!\!^{-}_r L}{\rho_0^2}\,\frac{\Delta\omega_r}{(\omega-\omega_r)^2+\Delta\omega_r^2}\times\left(\frac{\sin XL}{X}\right)^2\left(\frac{\sin|\zeta|X\rho_0}{|\zeta|X}\right)^2, \tag{4.15}$$

where

$$N_c=\frac{\pi}{4}\,\frac{Q^2}{\hbar c}\tau_d^2(\omega_r)\rho_0^2\,\frac{L}{\lambda\!\!\!^{-}_r}\,\frac{\Delta\omega_r}{\omega_r}, \tag{4.16}$$

$$X=(1-\beta\sqrt{\epsilon}\cos\theta)\omega_r/2\beta c, \tag{4.17}$$

and $|\zeta|=(\beta^2\epsilon-1)^{-1/2}$. It is seen that the secondary radiation is also emitted in the Čerenkov cone and it has a very small angular spread.

Integration of Eq. (4.15) over the solid angle[23] leads to the following expression for the number of resonant photons radiated by the embedded atoms per unit frequency interval:

$$\frac{dN_{\mathrm{rad}}}{d\omega}=N_c\,\frac{L}{|\zeta|\rho_0}\left(1-\frac{L}{3|\zeta|\rho_0}\right)\frac{1}{\pi}\,\frac{\Delta\omega_r}{(\omega-\omega_r)^2+\Delta\omega_r^2}, \tag{4.18a}$$

if $L\leq|\zeta|\rho_0$, and

$$\frac{dN_{\mathrm{rad}}}{d\omega}=N_c\left(1-\frac{|\zeta|\rho_0}{3L}\right)\frac{1}{\pi}\,\frac{\Delta\omega_r}{(\omega-\omega_r)^2+\Delta\omega_r^2}, \tag{4.18b}$$

if $L\geq|\zeta|\rho_0$. Finally, one obtains the following expressions for the number of radiated photons by integrating Eqs. (4.18a) and (4.18b) over the frequency domain:

$$N_{\mathrm{rad}}=N_c(L/|\zeta|\rho_0)(1-L/3|\zeta|\rho_0), \tag{4.19a}$$

if $L\leq|\zeta|\rho_0$, and

$$N_{\mathrm{rad}}=N_c(1-|\zeta|\rho_0/3L), \tag{4.19b}$$

if $L\geq|\zeta|\rho_0$.

As a numerical example, the following numbers are chosen: $\tau_d(\omega_r)\rho_0\sim 0.1$, $L/\lambda\!\!\!^{-}_r\sim 10^5$, $\Delta\omega_r/\omega_r\sim 10^{-6}$, $|\zeta|\sim 1$. For a sample with $L>|\zeta|\rho_0$ and for a 1-mA incident beam of electrons, which corresponds to 10^{16} incident electrons per second, there will be radiated approximately 10^{11} resonant photons per second in the direction of the Čerenkov cone. This is a much larger radiated energy as compared to that obtained when $\beta^2\epsilon<1$.

V. DISCUSSION

The treatment given above represents a rather idealized situation. For example, the absorption that takes place in the slab, when resonant radiation is emitted, was neglected. This is a good approximation, if the dimensions of the slab in the direction of the emitted radiation are small in comparison with the inverse of the absorption coefficient at resonance. For example, if the length of the slab in the direction of the emitted radiation is equal to $\frac{1}{10}$ of the inverse of the absorption coefficient, approximately 10% of the emitted radiation will be absorbed. This follows from the fact that, inside the resonant medium, the emitted radiation decays exponentially with the length of the slab. Also the slab should not be very large in the direction that the incident charged particle moves, if the loss of energy by the particle is to be negligible. Finally, the scattering that the incident particle undergoes, due to its interaction with the nuclei, was not taken into consideration. As a result of it, there should be an additional broadening of the angular spread of the emitted radiation.

As it may be seen from Eqs. (4.8) and (4.19), the total radiated energy is proportional to the density of the embedded atoms squared which indicates that the radiation is coherently emitted.

In Secs. III and IV, the case for which $\beta^2\epsilon<1$ was considered for resonant frequencies in the visible region. But the same formalism can be applied for the generation of x rays by a relativistic charged particle moving through an amorphous solid (as was mentioned in the Introduction, the case of a crystal has been treated elsewhere[1]). If in the relations obtained in Secs. III and IV, under the condition that $\beta^2\epsilon<1$, the host medium is vacuum, the atoms in the slab form an amorphous solid, and the resonant transition is in the x-ray region, then these relations are valid. Thus, if $\zeta=(1-\beta^2)^{-1/2}$ is much greater than unity, the induced x-ray radiation is emitted in the direction of motion of the incident charged particle. As a numerical example, the following numbers are chosen: $\tau(\omega_r)\sim10^3\ \text{cm}^{-1}$, $\lambdabar_r\sim10^{-8}$ cm, $\Delta\omega_r/\omega_r\sim10^{-6}$. For a slab of width $L\sim10^{-3}$ cm and for a 1-mA incident beam of 50-MeV electrons, which corresponds to $\zeta\simeq10^2$ and 10^{16} incident electrons per second, there will be radiated approximately 10^6 x-ray resonant photons per second in the forward direction.

In Sec. IV, it was shown that when $\beta^2\epsilon>1$, the primary Čerenkov radiation as well as the secondary stimulated radiation are emitted in the same direction of the Čerenkov cone. The problem arises then how to detect the presence of the secondary radiation. This should be possible, for example, if the time duration of the stimulated radiation was much longer than that of the primary Čerenkov radiation. The time duration of the former radiation is determined by the lifetime of the embedded atoms (it is assumed that the time it takes the charged particle to traverse the slab is much shorter than the lifetime). The time duration of the latter radiation is determined by the time it takes the incident charged particle to traverse the slab and by the inverse of the frequency width within which the dielectric constant ϵ of the host medium is a real constant and such that $\beta^2\epsilon>1$ (in the previous sections it was implied that this width is infinite but in a real situation it is finite). For all practical purposes, this frequency width is much larger than the inverse of the lifetime of the embedded atoms. Therefore, the stimulated radiation will last for a much longer time than the primary Čerenkov radiation and, since it is emitted in the Čerenkov cone, it can be referred as "delayed Čerenkov radiation," the delay determined by the lifetime of the embedded atoms. The possibility arises then for the manifestation of their coherent excitation by relativistic charged particles. The numerical examples given above indicate that the delayed Čerenkov radiation is the most intense of all cases considered.

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APPENDIX

The purpose of the Appendix is to provide the theory and the approximations involved in this paper and to relate the present work with the work that has been done in the past. As it has already been stated, this paper is concerned with the radiation due to the coherent excitation of the electric dipole moments of the atoms which are embedded in a host medium with real dielectric constant ϵ. These dipole moments are induced by the external electric field of an incident relativistic charged particle moving through the resonant medium. On the other hand, most of the work in the past refers to the energy loss by an incident charged particle in a dielectric medium. It will be shown here that this energy loss is identical to the energy absorbed by the embedded atoms in the presence of the external electric field of the incident charged particle. In addition, it will be shown that the total energy supplied by the external electric field of the incident charged particle is equal to the energy absorbed by the embedded atoms *and* to the energy radiated coherently by them. It should be pointed out that the energy absorbed by the atoms close to a particular resonance frequency may be due to nonradiative processes or to inhomogeneous broadening and, therefore, it should be considered as a distinct physical quantity from the energy which is coherently radiated close to that resonance frequency. To summarize then, the incident charged particle supplies energy for both absorption and coherent radiation by the embedded atoms. In the past, a detailed study[6,10] has been made of the absorption of energy by a dense resonant medium of infinite extent, under the influence of an incident charged particle. In this paper, however, a study is made of the coherent radiation by a dilute resonant medium of finite extent, under the influence of an incident charged particle.

For the sake of generality, it will be assumed that the external field, in the absence of any atoms embedded in the host medium, is determined by current sources with current density $\vec{J}(\vec{R},\omega)$, and is given by the relations

$$\vec{E}_0(\vec{R},\omega)=-\frac{c}{i\omega\epsilon}\left(\vec{\nabla}_R\times\vec{\nabla}_R\times\vec{A}_0(\vec{R},\omega)-\frac{4\pi}{c}\vec{J}(\vec{R},\omega)\right), \tag{A1}$$

$$\vec{B}_0(\vec{R},\omega)=\vec{\nabla}_R\times\vec{A}_0(\vec{R},\omega), \quad (A2)$$

where

$$\vec{A}_0(\vec{R},\omega)=\frac{1}{c}\int_{V_s}G(\vec{R}-\vec{R}';\sqrt{\epsilon}\,\omega/c)\vec{J}(\vec{R}',\omega)d^3R'. \quad (A3)$$

Here, $G(\vec{R},k)$ is the retarded Green's function given by Eq. (3.2) and $\vec{A}_0(\vec{R},\omega)$ is the vector potential which is due to the current density $\vec{J}(\vec{R},\omega)$. The host medium is of infinite extent and the embedded atoms occupy a volume V_0 with dielectric constant

$$\epsilon_1=\epsilon+4\pi\chi_d(\omega). \quad (A4)$$

The susceptibility $\chi_d(\omega)$ is due to the embedded atoms in the host medium, and is given by the Lorentz-Lorentz formula[24]

$$\chi_d(\omega)=\mathfrak{N}\alpha_d(\omega)/[1-(4\pi/3\epsilon)\mathfrak{N}\alpha_d(\omega)]. \quad (A5)$$

Here, $\mathfrak{N}$ is the density of the embedded atoms in the volume V_0 and $\alpha_d(\omega)$ their polarizability, which does not have to be limited to one resonance frequency.

The effective electric field at any point $\vec{R}$ in space is equal to the external electric field $\vec{E}_0(\vec{R},\omega)$ and the electric field of the embedded atoms, i.e.,

$$\vec{E}(\vec{R},\omega)=\vec{E}_0(\vec{R},\omega)+\vec{E}_d(\vec{R},\omega). \quad (A6)$$

The electric field of the embedded atoms is determined by the polarization $\vec{\pi}(\vec{R},\omega)$ of the volume V_0 occupied by them, as follows:

$$\vec{E}_d(\vec{R},\omega)=(1/\epsilon)\vec{\nabla}_R\times\vec{\nabla}_R \times\int_{V_0}G(\vec{R}-\vec{R}';\sqrt{\epsilon}\,\omega/c)\vec{\pi}(\vec{R}',\omega)d^3R' -(4\pi/\epsilon)\vec{\pi}(\vec{R},\omega)U(\vec{R}\epsilon V_0). \quad (A7)$$

The function $U(\vec{R}\epsilon V_0)$ is defined to be equal to 1 if $\vec{R}$ lies inside the volume V_0 and 0 if it lies outside V_0. In a linear theory, the polarization $\vec{\pi}(\vec{R},\omega)$ is assumed to be proportional to the effective field i.e.,

$$\vec{\pi}(\vec{R},\omega)=\chi_d(\omega)\vec{E}(\vec{R},\omega), \quad (A8)$$

for any point $\vec{R}$ inside the volume V_0. Substitution of $\vec{E}(\vec{R},\omega)$, given by Eqs. (A6) and (A7) into the above equation leads to the following integral equation for any point $\vec{R}$ inside the volume V_0:

$$\vec{\pi}(\vec{R},\omega)=\chi_d(\omega)\bigg(\vec{E}_0(\vec{R},\omega)+\frac{1}{\epsilon}\vec{\nabla}_R\times\vec{\nabla}_R \times\int_{V_0}G(\vec{R}-\vec{R}';\sqrt{\epsilon}\,\omega/c)\vec{\pi}(\vec{R}',\omega)d^3R' -\frac{4\pi}{\epsilon}\vec{\pi}(\vec{R},\omega)\bigg). \quad (A9)$$

When the polarization $\vec{\pi}(\vec{R},\omega)$ is obtained as a solution of the above integral equation, the electric field which is due only to $\vec{\pi}(\vec{R},\omega)$ (i.e., only to the embedded atoms) can be derived from Eq. (A7) at any point $\vec{R}$ in space. The effective electric field is obtained then from Eq. (A6) and the magnetic field is obtained from the relation

$$B(\vec{R},\omega)=(c/i\omega)\vec{\nabla}_R\times\vec{E}(\vec{R},\omega). \quad (A10)$$

In another paper,[25] it has been shown that the electromagnetic field, namely, the fields $\vec{E}(\vec{R},\omega)$ and $\vec{B}(\vec{R},\omega)$, obtained from the solution of the integral equation given above is identical to that obtained from the solution of Maxwell's equations and boundary conditions for a sample with the same volume V_0 and dielectric constant ϵ_1, given by Eq. (A4). The volume outside V_0 has a real dielectric constant ϵ, i.e., that of the host medium.

Now consider a volume V which lies inside the volume V_0 or may be as large as V_0. The energy absorbed by the embedded atoms which lie inside the volume V is equal to

$$\Delta E_{\text{abs}}=2\int_0^\infty\frac{d\omega}{2\pi}\int_V \text{Re}\{\vec{E}^*(\vec{R},\omega)\cdot[-i\omega\vec{\pi}(\vec{R},\omega)]\}\,d^3R. \quad (A11)$$

If use is made of Eq. (A8), it follows from Eq. (A11) that

$$\Delta E_{\text{abs}}=2\int_0^\infty\frac{d\omega}{2\pi}\text{Re}(-i\omega\chi_d(\omega))\int_V|\vec{E}(\vec{R},\omega)|^2\,d^3R. \quad (A12)$$

One may see from the above equation that in frequency regions where the susceptibility $\chi_d(\omega)$ of the embedded atoms is real, there is no energy absorbed by them. From the examples that will follow, it will become evident[26] that ΔE_{abs}, given by Eq. (A12), is identical to the energy loss by the incident charged particle. This is the physical quantity that has been studied extensively in the past.[6,10]

In addition to the energy absorbed by the embedded atoms, there is energy radiated coherently by them. This energy is determined by the Poynting vector of the electromagnetic field of the embedded atoms, at the surface S of the volume V and is equal to

$$\Delta E_{\text{rad}}=\frac{c}{2\pi}\int_0^\infty\frac{d\omega}{2\pi} \times\oint_S \text{Re}\{[\vec{E}_d^*(\vec{R},\omega)\times\vec{B}_d(\vec{R},\omega)]\cdot\vec{n}\}\,dS. \quad (A13)$$

The unit vector $\vec{n}$ is normal to the surface S and is directed from the inside towards the outside of the volume V. The magnetic field $\vec{B}_d(\vec{R},\omega)$ of the embedded atoms is equal to

$$\vec{B}_d(\vec{R}, \omega) = \vec{B}(\vec{R}, \omega) - \vec{B}_0(\vec{R}, \omega), \tag{A14}$$

where $\vec{B}(\vec{R}, \omega)$ and $\vec{B}_0(\vec{R}, \omega)$ are given by Eqs. (A10) and (A2), respectively. In this paper, a study is made of this coherently radiated energy by the embedded atoms in the first-order approximation that will be defined shortly. As it may be seen from Eqs. (A12) and (A13) the absorbed and coherently radiated energy are distinct physical quantities.

The total energy supplied by the external electric field $\vec{E}_0(\vec{R}, \omega)$ to the embedded atoms inside the volume V is equal to

$$\Delta E_{\text{ext}} = 2\int_0^\infty \frac{d\omega}{2\pi}\int_V \text{Re}\{\vec{E}_0^*(\vec{R}, \omega)\cdot[-i\omega\vec{\pi}(\vec{R}, \omega)]\}\, d^3R. \tag{A15}$$

Now, it will be shown that energy is supplied by the external field for both the absorbed and the coherently radiated energy by the embedded atoms inside the volume V, i.e.,

$$\Delta E_{\text{ext}} = \Delta E_{\text{abs}} + \Delta E_{\text{rad}}. \tag{A16}$$

In order to prove the above relation, use should be made of the identity

$$\Delta E_{\text{rad}} = 2\int_0^\infty \frac{d\omega}{2\pi}\int_V \text{Re}\{\vec{E}_d^*(\vec{R}, \omega)\cdot[i\omega\vec{\pi}(\vec{R}, \omega)]\}\, d^3R. \tag{A17}$$

This identity is derived from Eq. (A13) by a direct and lengthy computation which will not be given here. Then Eq. (A16) follows immediately from Eqs. (A11), (A15), and (A17), and use of Eq. (A6). Equation (A16) expresses conservation of energy. It may be noticed that even in frequency regions where energy is not absorbed, the external field does supply energy to the embedded atoms for coherent emission of radiation. For example, at the surface of a transparent medium the transmitted and reflected radiations are emitted in different directions than that of the external field $\vec{E}_0(\vec{R}, \omega)$. This is due to the fact that energy is supplied by the external field for the excitation of the embedded atoms in the transparent medium, and this energy is radiated coherently by the embedded atoms in directions prescribed by Maxwell's equations and boundary conditions. This explains the laws of reflection and refraction that occur at the boundary of a transparent medium.

The theory given above applies for a free external field as well as an external field which is due to current sources. Actually, Eq. (A16) has already been shown[27] to be valid in the case of an incident plane monochromatic wave. Also, in another paper,[28] the integral equation (A9) was solved for a spherical sample. The free external field was assumed to be a plane wave. In this special case too, the equivalence mentioned above was shown explicitly.

It is seen from Eqs. (A13) and (A17) that ΔE_{rad} can be expressed either as a surface integral or a volume integral. The same is true for ΔE_{abs} and ΔE_{ext}. The surface integrals for these physical quantities are equal to

$$\Delta E_{\text{abs}} = -\frac{c}{2\pi}\int_0^\infty \frac{d\omega}{2\pi}\oint_S \text{Re}\{[\vec{E}^*(\vec{R}, \omega)\times\vec{B}(\vec{R}, \omega)]\cdot\vec{n}\}\, dS - 2\int_0^\infty \frac{d\omega}{2\pi}\int_V \text{Re}[\vec{E}^*(\vec{R}, \omega)\cdot\vec{J}(\vec{R}, \omega)]\, d^3R, \tag{A18}$$

and

$$\Delta E_{\text{ext}} = -\frac{c}{2\pi}\int_0^\infty \frac{d\omega}{2\pi}\oint_S \text{Re}\{[\vec{E}_0^*(\vec{R}, \omega)\times\vec{B}_d(\vec{R}, \omega)]\cdot\vec{n}\}\, dS - \frac{c}{2\pi}\int_0^\infty \frac{d\omega}{2\pi}\oint_S \text{Re}\{[\vec{E}_d^*(\vec{R}, \omega)\times\vec{B}_0(\vec{R}, \omega)]\cdot\vec{n}\}\, dS - 2\int_0^\infty \frac{d\omega}{2\pi}\int_V \text{Re}[\vec{E}_d^*(\vec{R}, \omega)\cdot\vec{J}(\vec{R}, \omega)]\, d^3R. \tag{A19}$$

If the volume V does not contain any current sources, the volume integrals in Eqs. (A18) and (A19) should be omitted. In many cases, it is more convenient to evaluate the surface rather than the volume integrals of the energies given above. This will be the case with the two examples that will be presented.

The integral equation (A9) may be solved by an iterative method. In the first-order approximation, the polarization $\vec{\pi}(\vec{R}, \omega)$ is equal to

$$\vec{\pi}(\vec{R}, \omega) \simeq \chi_d(\omega)\vec{E}_0(\vec{R}, \omega). \tag{A20}$$

If this expression for $\vec{\pi}(\vec{R}, \omega)$ is substituted into the right-hand side of Eq. (A9), $\vec{\pi}(\vec{R}, \omega)$ is obtained in the second-order approximation. This procedure can be repeated to obtain the higher-order approximations. The iterative method provides the solution of Eq. (A9) as a Taylor series of the susceptibility $\chi_d(\omega)$. If $|\chi_d(\omega)| \ll 1$, the first-order approximation is sufficient, and $\vec{\pi}(\vec{R}, \omega)$ is given by Eq. (A20). This expression of $\vec{\pi}(\vec{R}, \omega)$ is used in Eq. (A7) in order to obtain the electric field due to the embedded atoms. The condition $|\chi_d(\omega)| \ll 1$ is satisfied if $\mathfrak{N}|\alpha_d(\omega)| \ll 1$ [cf., Eq. (A5)]. For a resonant medium, the latter condition is valid. It is this condition then, which justifies the first-order approximation used in this paper. This approximation is most appropriate when the embedded atoms occupy a volume V_0 whose space configuration is such that Maxwell's equations and boundary conditions cannot be solved exactly, so that one must resort to an approximate solution.

Two examples will be given to demonstrate the difference between the absorbed and coherently radiated energy and to illustrate the approximations made in this paper. In both examples, the external field is that of a charged particle which moves along the z axis with constant velocity v through a host medium of infinite extent and real dielectric constant ϵ. In the frequency domain, the external electromagnetic field in cylindrical coordinates is equal to[17]

$$E_{0_\rho}(\vec{R},\omega)=(2Q/\epsilon\beta c)\lambda_0 K_1(\lambda_0\rho)e^{i(\omega/\beta c)z}, \qquad (A21)$$

$$E_{0_z}(\vec{R},\omega)=-i(2Q/\epsilon\omega)\lambda_0^2 K_0(\lambda_0\rho)e^{i(\omega/\beta c)z}, \qquad (A22)$$

$$B_{0_\phi}(\vec{R},\omega)=(2Q/c)\lambda_0 K_1(\lambda_0\rho)e^{i(\omega/\beta c)z}, \qquad (A23)$$

where Q is the charge of the incident particle, c is the velocity of light, $\beta=v/c$, $K_0(z)$ and $K_1(z)$ are the modified Bessel functions of order zero and one, respectively, and (ρ,ϕ,z) are the cylindrical coordinates of the position vector $\vec{R}$. The parameter λ_0 is defined as the square root of

$$\lambda_0^2=(\omega/\beta c)^2(1-\beta^2\epsilon), \qquad (A24)$$

which lies either on the positive real axis when $\beta^2\epsilon<1$, or on the negative imaginary axis when $\beta^2\epsilon>1$. From the asymptotic behavior[20] of the modified Bessel function $K_n(z)$, it is seen from Eqs. (A21)–(A24) that for large distances from the trajectory of the incident charged particle the external field either decreases exponentially with the distance ρ when $\beta^2\epsilon<1$, or it oscillates and decreases slowly as ρ increases when $\beta^2\epsilon>1$.

In the first example, the volume V_0 occupied by the embedded atoms is taken to be the whole space, i.e., the resonant medium is of infinite extent. Maxwell's equations can be solved easily in this case and the electromagnetic field, in cylindrical coordinates, at any point $\vec{R}$ in space is equal to

$$E_\rho(\vec{R},\omega)=(2Q/\epsilon_1\beta c)\lambda_1 K_1(\lambda_1\rho)e^{i(\omega/\beta c)z}, \qquad (A25)$$

$$E_z(\vec{R},\omega)=-i(2Q/\epsilon_1\omega)\lambda_1^2 K_0(\lambda_1\rho)e^{i(\omega/\beta c)z}, \qquad (A26)$$

$$B_\phi(\vec{R},\omega)=(2Q/c)\lambda_1 K_1(\lambda_1\rho)e^{i(\omega/\beta c)z}, \qquad (A27)$$

where ϵ_1, the dielectric constant of the resonant medium, is given by Eq. (A4), and λ_1 is the square root of

$$\lambda_1^2=(\omega/\beta c)^2(1-\epsilon_1\beta^2), \qquad (A28)$$

chosen in the fourth quadrant of the complex plane. It can be shown directly that the polarization $\vec{\pi}(\vec{R},\omega)$, obtained from Eqs. (A8), (A25), and (A26), does satisfy the integral equation (A9).

Now the volume V in Eqs. (A12), (A15), and (A17) is chosen between the surfaces S_0 and S, of cylindrical shape and at distances ρ_0 and ρ, respectively, from the trajectory of the incident particle ($\rho_0<\rho$). Equation (A16) can be proven easily if use is made of Eqs. (A13), (A18), (A19), and the fact that the trajectory of the charged particle, where the current density is nonzero, lies outside the volume V. In particular, if the radius ρ_0 is so close to the trajectory of the charged particle that the condition $|\lambda_1|\rho_0\ll 1$ is satisfied, it follows from Eq. (A18) and from the values[29] of $K_0(z)$ and $K_1(z)$ for a small argument z, that

$$\frac{d}{dz}\Delta E_{\rm abs}=\frac{2}{\pi}\frac{Q^2}{\beta^2c^2}\int_0^\infty\frac{\omega}{|\epsilon_1|^2}\left(\mathrm{Im}\epsilon_1\ln\frac{1.123}{|\lambda_1|\rho_0}+\mathrm{Re}\epsilon_1-\beta^2|\epsilon_1|^2)\mathrm{Arg}\lambda_1\right)d\omega$$
$$+\frac{2}{\pi}\frac{Q^2}{\beta^2c^2}\int_0^\infty\omega\,\mathrm{Im}[(1/\epsilon_1-\beta^2)\lambda_1^*\rho K_0(\lambda_1\rho)K_1(\lambda_1^*\rho)]d\omega. \qquad (A29)$$

Here, $|\lambda_1|$ and $\mathrm{Arg}\lambda_1$ are the magnitude and phase of the complex parameter λ_1. It is easy to show from the above equation that in frequency regions where $\mathrm{Im}\epsilon_1=0$ [i.e., $\mathrm{Im}\chi_d(\omega)=0$], the energy absorbed by the embedded atoms inside the volume V is equal to zero either when $\beta^2\epsilon_1<1$ (i.e., $\mathrm{Arg}\lambda_1=0$) or when $\beta^2\epsilon_1>0$ (i.e., $\mathrm{Arg}\lambda_1=-\frac{1}{2}\pi$). On the other hand, consider frequency regions where $\mathrm{Im}\epsilon_1$ is greater than zero and the conditions $\beta^2\mathrm{Re}\epsilon_1>1$ and $\mathrm{Im}\epsilon_1\ll\mathrm{Re}\epsilon_1$ are satisfied. Then $\mathrm{Arg}\lambda_1$ in Eq. (A29) can be approximated by the expression

$$\mathrm{Arg}\lambda_1\simeq-\frac{\pi}{2}+\frac{1}{2}\frac{\beta^2\mathrm{Im}\epsilon_1}{\beta^2\mathrm{Re}\epsilon_1-1}. \qquad (A30)$$

Moreover, if the radius ρ of the outer surface S of the volume V is chosen so large that $\mathrm{Re}\lambda_1\rho\gg 1$, the term proportional to the modified Bessel functions in Eq. (A29) vanishes and this equation becomes equal to

$$\frac{d}{dz}\Delta E_{\rm abs}\simeq\frac{Q^2}{\pi\beta^2c^2}\int_0^\infty\frac{\omega\,\mathrm{Im}\epsilon_1}{|\epsilon_1|^2}\times\left(2\ln\frac{1.123}{|\lambda_1|\rho_0}-\beta^2\mathrm{Re}\epsilon_1\right)d\omega+\frac{Q^2}{\beta^2c^2}\int_0^\infty\left(\beta^2-\frac{\mathrm{Re}\epsilon_1}{|\epsilon_1|^2}\right)\omega\,d\omega. \qquad (A31)$$

In frequency regions where $\beta^2\mathrm{Re}\epsilon_1<1$, the second term in the above equation should be omitted. Equation (A31) gives the energy absorbed by the embedded atoms which occupy the entire space. It is identical to Eq. (20) in Budini's paper,[10] and thus, it can be identified as "the energy loss" of the incident charged particle. It should be noticed that the first term in Eq. (A31) is proportional to the density of the embedded atoms. Also, both Eqs. (A29) and (A31) are divergent as ρ_0 tends to zero.

Next, the coherently radiated energy by the atoms which are embedded in the volume V will be considered. If the conditions $|\lambda_0|\rho_0 \ll 1$ and $|\lambda_1|\rho_0 \ll 1$ are satisfied, it follows from Eq. (A13) that

$$\frac{d}{dz}\Delta E_{\text{rad}} = -\frac{2}{\pi}Q^2\int_0^\infty \text{Im}\left[\left(\frac{\lambda_1^2}{\epsilon_1}K_0(\lambda_1\rho) - \frac{\lambda_0^2}{\epsilon}K_0(\lambda_0\rho)\right) \times [\lambda_1^*\rho K_1(\lambda_1^*\rho) - \lambda_0^*\rho K_1(\lambda_0^*\rho)]\right]\frac{d\omega}{\omega}, \quad \text{(A32)}$$

where use was made of the values[29] of $K_0(z)$ and $K_1(z)$ for a small argument z. In particular, if $\beta^2\epsilon > 1$, $4\pi|\chi_d(\omega)| \ll \epsilon$, and if the outer surface S of the volume V lies at a distance ρ which is much larger than the wavelength[30] of the emitted radiation but much smaller than the inverse of the absorption coefficient, i.e., if also $|\lambda_0|\rho \gg 1$, $|\lambda_1|\rho \gg 1$, and $\text{Re}\lambda_1\rho \ll 1$, Eq. (A32) reduces to the following approximate expression

$$\frac{d}{dz}\Delta E_{\text{rad}} \simeq \frac{Q^2}{4\epsilon c^4}\rho^2\int_0^\infty |4\pi\chi_d(\omega)|^2\omega^3\,d\omega\,. \quad \text{(A33)}$$

The above equation could have been obtained directly from Eqs. (A20), (A7), and (A13), i.e., from the first-order approximation of the polarization. This indicates that this approximation is valid at distances ρ much larger than the wavelength of radiation and much smaller than the inverse of the absorption coefficient. It is seen from Eqs. (A32) and (A33) that in the expressions for the coherently radiated energy there is no divergence involved with the lower limit ρ_0 of the volume V as is the case with the absorbed energy. In addition, at distances ρ much larger than the wavelength and much smaller than the inverse of the absorption coefficient, the coherently radiated energy is proportional to the density of the embedded atoms squared and the distance ρ squared. In this example, then, it has been demonstrated that the coherently radiated energy and the absorbed energy by the embedded atoms are distinct physical quantities. The absorbed energy has been identified with the "energy loss" by the incident charged particle, where the latter term is commonly used in the literature. Finally, it was indicated under which conditions the first-order approximation, used in this paper, is valid.

The second example is more realistic. The external field is given by Eqs. (A21)–(A24) and the host medium is of infinite extent, but the embedded atoms occupy the volume V_0 of a cylinder with radius a and of infinite extent along its axis, which coincides with the z axis of the coordinate system. The dielectric constant of the resonant medium inside the cylinder is ϵ_1, given by Eq. (A4), and outside the cylinder it is equal to ϵ. The solution of Maxwell's equation and boundary conditions leads to the following expressions for the electromagnetic field in cylindrical coordinates:

$$E_{\rho_{\text{in}}}(\vec{R},\omega) = (2Q/\epsilon_1\beta c)\lambda_1[K_1(\lambda_1\rho) - A(\omega)I_1(\lambda_1\rho)]e^{i(\omega/\beta c)z}, \quad \text{(A34)}$$

$$E_{z_{\text{in}}}(\vec{R},\omega) = -i(2Q/\epsilon_1\omega)\lambda_1^2[K_0(\lambda_1\rho) + A(\omega)I_0(\lambda_1\rho)]e^{i(\omega/\beta c)z}, \quad \text{(A35)}$$

$$B_{\Phi_{\text{in}}}(\vec{R},\omega) = (2Q/c)\lambda_1[K_1(\lambda_1\rho) - A(\omega)I_1(\lambda_1\rho)]e^{i(\omega/\beta c)z}, \quad \text{(A36)}$$

inside the cylinder, and

$$E_{\rho_{\text{out}}}(\vec{R},\omega) = (2Q/\epsilon\beta c)B(\omega)\lambda_0K_1(\lambda_0\rho)e^{i(\omega/\beta c)z}, \quad \text{(A37)}$$

$$E_{z_{\text{out}}}(\vec{R},\omega) = -i(2Q/\epsilon\omega)B(\omega)\lambda_0^2K_0(\lambda_0\rho)e^{i(\omega/\beta c)z}, \quad \text{(A38)}$$

$$B_{\Phi_{\text{out}}}(\vec{R},\omega) = (2Q/c)B(\omega)\lambda_0K_1(\lambda_0\rho)e^{i(\omega/\beta c)z}, \quad \text{(A39)}$$

outside the cylinder, where

$$A(\omega) = -\frac{K_1(\lambda_0a)K_0(\lambda_1a) - (\epsilon_1\lambda_0/\epsilon\lambda_1)K_0(\lambda_0a)K_1(\lambda_1a)}{K_1(\lambda_0a)I_0(\lambda_1a) + (\epsilon_1\lambda_0/\epsilon\lambda_1)K_0(\lambda_0a)I_1(\lambda_1a)}, \quad \text{(A40)}$$

$$B(\omega) = \frac{1}{\lambda_0a}\,\frac{1}{K_1(\lambda_0a)I_0(\lambda_1a) + (\epsilon_1\lambda_0/\epsilon\lambda_1)K_0(\lambda_0a)I_1(\lambda_1a)}\,. \quad \text{(A41)}$$

The parameters λ_0, λ_1 have already been defined and $I_0(z)$, $I_1(z)$, $K_0(z)$, and $K_1(z)$ are modified Bessel functions. It can be verified directly that the polarization $\vec{\pi}(\vec{R},\omega)$, obtained from Eqs. (A8), (A34), and (A35), satisfies the integral equation (A9). Moreover, the field outside the cylinder, namely Eqs. (A37)–(A39) can be obtained from Eqs. (A6) and (A10). The field $\vec{E}_d(\vec{R},\omega)$ in Eq. (A6) is evaluated from Eq. (A7) for points $\vec{R}$ outside the cylinder and for a polarization $\vec{\pi}(\vec{R},\omega)$ which is the solution of the integral equation (A9).

The volume V in Eqs. (A12), (A15), and (A17) is chosen between the surfaces S_0 and S, of cylindrical shape and at distances ρ_0 and a, respectively, from the trajectory of the incident particle ($\rho_0 < a$). Equation (A16) can be proven easily if use is made of Eqs. (A13), (A18), (A19), and the fact that the trajectory of the charged particle lies outside the volume V. In particular, if ρ_0 is so small that the condition $|\lambda_1|\rho_0 \ll 1$ is satisfied, it follows from Eq. (A18) that

$$\frac{d}{dz}\Delta E_{\text{abs}} = \frac{2}{\pi}\frac{Q^2}{\beta^2 c^2}\int_0^\infty \frac{\omega}{|\epsilon_1|^2}\left(\text{Im}\epsilon_1 \ln\frac{1.123}{|\lambda_1|\rho_0} + (\text{Re}\epsilon_1 - \beta^2|\epsilon_1|^2)\text{Arg}\lambda_1\right)d\omega$$
$$+\frac{Q^2}{\beta^2 c^2}\int_0^\infty \omega\left[(1/\epsilon - \beta^2)|B(\omega)|^2\Theta(\beta^2\epsilon - 1) - \frac{2}{\pi}\text{Im}[(1/\epsilon_1 - \beta^2)A(\omega)]\right]d\omega\,. \quad \text{(A42)}$$

The function $\Theta(x)$ is defined to be $\Theta(x) = 1$ if $x > 0$ and $\Theta(x) = 0$ if $x < 0$. The above equation provides the energy absorbed by the embedded atoms in the cylinder. In frequency regions where $\text{Im}\epsilon_1 = 0$ [i.e., $\text{Im}\chi_d(\omega) = 0$] it can be shown that the integrand in the above equation vanishes for all four possible cases where λ_0 and λ_1 are real positive or purely imaginary negative. Thus in these frequency regions there is no energy absorbed by the embedded atoms. On the other hand, for frequency regions where the conditions $\beta^2\,\text{Re}\epsilon_1 > 1$, $\text{Im}\epsilon_1 \ll \text{Re}\epsilon_1$, $|\lambda_0|a \gg 1$, $|\lambda_1|a \gg 1$, and $\text{Re}\lambda_1 a \gg 1$ are satisfied, Budini's[10] Eq. (20) follows from Eq. (A42). Physically, the conditions just stated mean that the radius a of the cylinder occupied by the embedded atoms must be much larger than the wavelength of the radiation and also much larger than the inverse of the absorption coefficient. Only then the last term in Eq. (A42), proportional to $|B(\omega)|^2$ and $A(\omega)$, vanishes. One concludes then that Budini's Eq. (20) is valid only for resonant media with dimensions much larger than the inverse of the absorption coefficient. This was the case in the first example, where the resonant medium was of infinite extent, and Eq. (A31) was derived.

The coherently radiated energy follows from Eq. (A13) under the conditions $\beta^2\epsilon > 1$, $|\lambda_0|\rho_0 \ll 1$, $|\lambda_1|\rho_0 \ll 1$. It is equal to

$$\frac{d}{dz}\Delta E_{\text{rad}} = -\frac{Q^2}{\beta^2 c^2}\int_0^\infty \omega(1/\epsilon - \beta^2)|B(\omega) - 1|^2 d\omega\,. \quad \text{(A43)}$$

If $\beta^2\epsilon < 1$, the radiated energy vanishes, even when $\beta^2\,\text{Re}\epsilon_1 > 1$. This is due to the fact that total reflection occurs at the surface of the cylinder. In the special case, when $\beta^2\epsilon > 1$, $4\pi|\chi_d(\omega)| \ll \epsilon$, and also when the radius a of the cylinder is much larger than the wavelength of the radiation and much smaller than the inverse of the absorption coefficient, i.e., when $|\lambda_0|a \gg 1$, $|\lambda_1|a \gg 1$, and $\text{Re}\lambda_1 a \ll 1$, Eq. (A43) simplifies to the approximate expression

$$\frac{d}{dz}\Delta E_{\text{rad}} \simeq \frac{Q^2}{4\epsilon c^4}a^2\int_0^\infty |4\pi\chi_d(\omega)|^2\omega^3 d\omega\,. \quad \text{(A44)}$$

This equation could have been obtained from Eqs. (A20), (A7), and (A13), i.e., in the first-order approximation. This indicates that the first-order approximation is valid for resonant media with dimensions much smaller than the inverse of the absorption coefficient. This condition is assumed in this paper in addition to the condition $|\chi_d(\omega)| \ll 1$, which is true for a resonant medium. It is seen from Eqs. (A43) and (A44) that in the expressions for the radiated energy there is no logarithmic divergence involved with the lower limit ρ_0 of the volume V as is the case with the absorbed energy [cf., Eq. (A42)]. This proves to be the case also in the examples treated in this paper [cf., Eqs. (4.5) and (4.15)]. One concludes from Eq. (A44) that, when the first-order approximation is valid, the coherently radiated energy is proportional to the density of the embedded atoms squared and to the radius a of the resonant medium squared.

In the theory and examples presented above the only assumption made about the susceptibility of the embedded atoms was that it should be in magnitude much less than unity. In this case, the susceptibility is proportional to the atomic polarizability $\alpha_d(\omega)$, as may be seen from Eq. (A5) [cf., also Eq. (3.6)]. In this paper, the atomic polarizability and, hence, also the susceptibility, is due to the resonant transitions at a single resonance frequency ω_r[cf., Eq. (2.4)], while in the past[6,10] multifrequency absorption possibilities have been considered. But as it follows from Eq. (A31) for absorption and Eqs. (A33) and (A44) for coherent radiation, in frequency regions of a resonant medium where there is no overlap of the resonant lines, each one of these lines can be treated independently of the others. This paper is confined to such frequency regions, so that the single-frequency model studied here is justified.

[1]D. Dialetis, Phys. Rev. A 17, 1113 (1978).
[2]W. F. G. Swann, Franklin Inst. 226, 598 (1938).
[3]E. Fermi, Phys. Rev. 57, 485 (1940).
[4]O. Halpern and H. Hall, Phys. Rev. 57, 459 (1940).
[5]O. Halpern and H. Hall, Phys. Rev. 73, 477 (1948).
[6]R. M. Sternheimer, Phys. Rev. 88, 851 (1952).
[7]R. M. Sternheimer, Phys. Rev. 89, 1148 (1953).
[8]R. M. Sternheimer, Phys. Rev. 91, 256 (1953).
[9]P. Budini, Phys. Rev. 89, 1147 (1953).
[10]P. Budini, Nuovo Cimento 10, 236 (1953).
[11]P. Budini, Z. Naturforsch. A 7, 722 (1952).
[12]M. Shonberg, Nuovo Cimento 8, 159 (1951).
[13]M. Shonberg, Nuovo Cimento 9, 37 (1952).
[14]M. Huybrechts and M. Schonberg, Nuovo Cimento 9, 764 (1952).
[15]U. Fano, Phys. Rev. 103, 1202 (1956).
[16]Compare, for example, pp. 443–449 in J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).
[17]See, for example, Eqs. (13.64), (13.65), p. 445 in J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962). Equation (2.1) differs by a factor $(2\pi)^{1/2}$ from the above equations. The absence of the exponential term in Jackson is due to the fact that the point $\vec{R}$ is taken on the x-y plane, where $z=0$ (cf., Fig. 11.13, p. 381). Also, here use has been made of the identity $K_0'(z)=-K_1(z)$ [cf. Eq. (9.6.27), p. 376 in *Handbook of Mathematical Functions*, National Bureau of Standards (U. S. GPO, Washington, D.C., 1964)].
[18]For the case of Čerenkov radiation, cf., for example, p. 87 in J. V. Jelley, *Progress in Nuclear Physics* (Pergamon Press, London, 1953), Vol. 3, Chap. 4.
[19]Compare, Eq. (14), p. 108 in E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge U. P., London, 1964). It has been assumed that ω is so closely located to a particular resonant frequency that only one term of the sum in Eq. (14) contributes. Moreover, the broadening of the line was added as a phenomenological parameter and use was made of the relation $A_{21}=2\sqrt{\epsilon}\,(e^2/\mu c^3)\omega_r^{\,2}f_{21}$, where e, μ are the charge and mass of the electron and f_{21} is the oscillator strength of the transition.
[20]Compare, Eq. (9.7.2), p. 378 in the latter reference of footnote 17.
[21]For this purpose, use was made of Eq. (9.1.18), p. 360 in the latter reference of footnote 17, and also of Eq. (6.521.2), p. 672 in I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).
[22]For the definition of the sine and cosine integrals, see for example, Sec. 5.2 [in particular, Eqs. (5.2.1) and (5.2.27)], p. 231 in the latter reference of footnote 17.
[23]For that purpose, use is made of Eq. (3.828.10), p. 451 in I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).
[24]Compare, Eq. (15), p. 87 in M. Born and E. Wolf, *Principles of Optics*, 5th ed. (Pergamon, New York, 1975).
[25]D. Dialetis, J. Opt. Soc. Am. 68, 602 (1978).
[26]Compare, also, Eq. (13.68), p. 446 in reference of footnote 17, and Eqs. (13.69) and (13.70) for an incident charged particle moving through an infinite dielectric medium.
[27]Compare, Eq. (102), p. 657 in reference of footnote 24.
[28]D. Dialetis (unpublished).
[29]Compare, Eqs. (9.6.11) and (9.6.13), p. 375 in the latter reference of footnote 17.
[30]The wavelength c/ω should not be confused with the parameters λ_0 and λ_1 which are defined by Eqs. (A24) and (A28), respectively.