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Nonlinear effects produced by continuous electromagnetic waves propagating in weakly dissipative plasmas

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A general perturbation method, founded on the multiple-space-scales technique, is applied to the kinetic analysis of the nonlinear effects generated by continuous transverse electromagnetic waves of finite amplitude propagating in warm and partially ionized plasmas. The collisional and weakly collisional cases are successively studied for a typical ordering of the significant parameters. In each case, it is first shown that the zero-order electron distribution function is determined by a set of two coupled equations and that it becomes inhomogeneous and generally non-Maxwellian under the effect of the collisional heating due to the wave absorption in the medium. The electron distribution function and the electric field inside the plasma are then completely determined up to second order in terms of the zero-order quantities by exact formulas which allow discussion of the influence of the microscopic interaction law on the nonlinear behavior of the medium. Some other applications of this method are outlined, particularly for the case of amplitude-modulated electric fields.

I. INTRODUCTION

This paper is devoted to the study of the nonlinear effects associated with the propagation of continuous electromagnetic waves in weakly dissipative plasmas. Although problems of this type have been already considered in many works,[1-8] there does not exist to my knowledge any complete calculation of these effects explicitly taking into account the nonlinear modifications of the kinetic state of the medium background. The difficulty of such a formulation lies in the fact that the zero-order distribution functions and the primary fields are determined, in the stationary case, by coupled nonlinear equations which can be solved only by a perturbative procedure. One is thus led to assume that the wave absorption in the plasma is weak, and to develop a general scheme of approximations to uncouple the set of matter and field equations.

As shown in a previous paper,[9] designated by I in the following, this can be done in the framework of the multiple-space-scales formalism, which yields a coherent determination both of the space scales and of the order of magnitude of the various nonlinear effects involved. Particularly, one can thus obtain equations to compute the zero-order non-Maxwellian distribution function in terms of the zero-order electric field, as well as exact kinetic formulas for the various harmonics of the electric field inside the plasma.

So, the purpose of this paper is to apply the general perturbation methods of I, set up for partially ionized, warm, and weakly dissipative plasmas, to the analysis of the nonlinear effects generated in such a medium by imposed transverse electromagnetic waves of finite amplitude. More specifically, let us consider a uniform, semi-infinite, weakly dissipative plasma, in the precise sense of I, in which is propagating a continuous transverse wave whose amplitude is fixed on the plane $x=0$. Moreover it is also assumed (i) that there is no external magnetic field; (ii) that the plasma state is well described by the electronic component only, with a uniform positive background of density N; and (iii) that the ionization degree is such that the collisional terms of the electron kinetic equation are those defined by the formulas (2.8) of I, with $NY/\bar{v}^3\bar{\nu}\simeq\delta$. Then, in order to describe the kinetic state of such a plasma, one is led to introduce as in I the dimensionless kinetic equation for the electrons, with the reduced variables $\tau,\vec{x},\vec{w},\vec{e}$, and to expand the electronic distribution function $F_e(\tau,\vec{x};\vec{w})$ in terms of the irreducible Cartesian tensors of the velocity space by putting

$$F_e=F^{(0)}(\tau,\vec{x};w)+\vec{w}\cdot\vec{F}^{(1)}(\tau,\vec{x};w)+\cdots+(\vec{w}\vec{w}\cdots\vec{w})^{(l)}:\vec{F}^{(l)}(\tau,\vec{x};w)+\cdots. \tag{1.1}$$

One thus obtains, to determine $F^{(0)}$ and the anisotropies $\vec{F}^{(l)}$, the fundamental set of kinetic equations

$$\frac{\partial F^{(0)}}{\partial\tau}+\eta'^{1/2}\frac{w^2}{3}\vec{\nabla}_x\cdot\vec{F}^{(1)}+\frac{\alpha'^{1/2}}{3w^2}\frac{\partial}{\partial w}(w^3\vec{e}\cdot\vec{F}^{(1)})=(\delta\bar{\nu}/\omega)[I(F^{(0)})+C_{ee}(F^{(0)})], \tag{1.2}$$

$$\frac{\partial\vec{F}^{(l)}}{\partial\tau}+\eta'^{1/2}[\vec{\nabla}_x\cdot\vec{F}^{(l-1)}]^0+\frac{\alpha'^{1/2}}{w}\left[\vec{e}\,\frac{\partial\vec{F}^{(l-1)}}{\partial w}\right]^0+\frac{l+1}{2l+3}\left[\eta'^{1/2}w^2\vec{\nabla}_x\cdot\vec{F}^{(l+1)}+\alpha'^{1/2}\frac{1}{w^{2l+2}}\frac{\partial}{\partial w}(w^{2l+3}\vec{e}\cdot\vec{F}^{(l+1)})\right]$$
$$-\alpha'^{1/2}\eta'^{1/2}l\left(\int^{\tau}(\vec{\nabla}_x\times\vec{e})d\tau'\right)\times\vec{F}^{(l)}=-\frac{\bar{\nu}}{\omega}\nu_l'\vec{F}^{(l)},\quad l\geqslant1 \tag{1.3}$$

in which ν_l' and the collisional operators $I(F^{(0)})$ and $C_{ee}(F^{(0)})$ have been defined in I, as well as the inhomogeneity and nonlinearity parameters $\eta'^{1/2}=\bar{v}/\lambda_0\omega$ and $\alpha'^{1/2}=\Gamma_0/\bar{v}\omega$. On the other hand, these equations are coupled with Maxwell's field equations which may be written

$$\frac{1}{\lambda_0^2}\vec{\nabla}_x\times(\vec{\nabla}_x\times\vec{e})+\frac{\omega^2}{c^2}\frac{\partial^2\vec{e}}{\partial\tau^2}=-\frac{4\pi e}{mc^2\bar{v}\alpha'^{1/2}}\frac{\partial\vec{j}}{\partial\tau}, \quad (1.4)$$

$$\alpha'^{1/2}\eta'^{1/2}\omega^2\vec{\nabla}_x\cdot\vec{e}=\frac{4\pi e}{m}\rho, \quad (1.5)$$

where the electric charge and current densities ρ and $\vec{j}$, defined by

$$\rho=4\pi e\int_0^\infty w^2F^{(0)}\,dw-Ne=e(n-N),$$
$$\vec{j}=\frac{4\pi e\bar{v}}{3}\int_0^\infty w^4\vec{F}^{(1)}\,dw \quad (1.6)$$

satisfy the continuity equation

$$\frac{\partial\rho}{\partial\tau}+\frac{\eta'^{1/2}}{\bar{v}}\vec{\nabla}_x\cdot\vec{j}=0. \quad (1.7)$$

This set of coupled equations has to be solved in the stationary case by applying a perturbation technique in which the parameters η' and α' are assumed small. As shown in I, one must at first define the relevant small parameter ϵ of the considered problem, by ordering between them the four physical parameters α',η',δ $(=2m/M)$, and $\bar{v}/\omega$. Then one is led to introduce the various time scales $\tau_0=\tau$, $\tau_1=\epsilon\tau_0$, $\tau_2=\epsilon^2\tau^0,\ldots$, and the corresponding space scales $\vec{x}_0=\vec{x}$, $\vec{x}_1=\epsilon\vec{x}_0$, $\vec{x}_2=\epsilon^2\vec{x}_0$, $\ldots$, and to assume that the physical quantities become functions of these multiple variables; however, as we are dealing with stationary solutions, the time dependence of the system involves only the fast time scale τ_0, so that the $\vec{F}^{(l)}$ $(l=0,1,2,\ldots)$ and the reduced electric field $\vec{e}$ depend on τ_0 and on the multiple space variables $\vec{x}_0,\vec{x}_1,\vec{x}_2,\ldots$. This being done, these quantities are expanded in powers of ϵ and the successive approximations of the solution have to be calculated by applying the multiple-space-scales formalism.

Let us now specify which ordering scheme is used in this paper. At first, it is assumed as in I that α', η', and δ satisfy the condition $\alpha'\simeq\eta'\simeq\delta$, so that the relevant small parameter is $\epsilon\simeq\alpha'^{1/2}\simeq\eta'^{1/2}\simeq\delta^{1/2}$; it must be observed that this condition has a simple physical interpretation, namely, that the electron–wave-energy exchanges due to the nonlinear effects are of the same order as the thermal effects as well as the electron–neutral-energy transfer due to collisions. Then, one has to look at the collisional parameter $\bar{v}/\omega$. We consider successively in this paper the so-called collisional case, with $\bar{v}/\omega\simeq 1$, and the weakly collisional case, with $\bar{v}/\omega\simeq\delta\simeq\epsilon^2$, which are studied, respectively, in Secs. II and III. Finally, in order to satisfy the condition of weak dissipativity, it remains to fix the order of magnitude of the absorption coefficient which is determined, according to I, by the ratio $K_{0I}/K_{0R}\propto(\omega_{pe}^2/\omega^2)(\bar{v}/\omega)$. In Secs. II and III, it is generally assumed that $K_{0I}/K_{0R}\simeq\epsilon^2$, a condition which is fulfilled in the weakly collisional case without any assumption on the ratio ω_{pe}^2/ω^2; for the collisional case, on the other hand, it is seen that the further condition $\omega_{pe}^2/\omega^2\simeq\epsilon^2$ is needed. In these two cases, it then follows that the electronic distribution function and the electric fields depend only on the even space variables $\vec{x}_0,\vec{x}_2,\ldots$, and that they can be developed according to the ϵ^2 expansions (2.1) given below. It can thus be shown that it is in fact $\vec{x}_2$ which is the characteristic length for the space variation of the medium properties; but, when the high-frequency condition $\omega_{pe}^2/\omega^2\simeq\epsilon^2$ holds for a weakly collisional plasma, the absorption coefficient is of order ϵ^4 and the characteristic length becomes $\vec{x}_4$.

The main subject matter of this paper is thus divided into Secs. II and III, which are devoted to the collisional and weakly collisional cases, respectively. In each case, first an equation is obtained for determining the zero-order distribution function $F_{(0)}^{(0)}$, which is shown to be inhomogeneous and generally non-Maxwellian owing to the thermoeffect of the zero-order electric field $\vec{e}_{(0)}'$. Then, the electronic distribution function and the electric field inside the plasma are calculated up to ϵ^2 order in terms of these zero-order quantities; the various components of the field are thus derived from exact kinetic formulas involving the e-n interaction law through the collision frequencies $\nu_1(v)$ and $\nu_2(v)$. These expressions allow one to describe in detail the kinetic behavior of the plasma and the first harmonics generated at this approximation, and to analyze the role played by the collisions in the various nonlinear contributions. Finally, some possible extensions of these methods are briefly outlined in Sec. IV, particularly for physical systems with another ordering of the fundamental parameters and for the case of amplitude-modulated fields.

II. CONTINUOUS WAVES IN COLLISIONAL MEDIA

We consider in this section a semi-infinite collisional plasma in the sense defined in the Introduction $(\bar{v}/\omega\simeq 1)$, in which a continuous transverse electromagnetic wave of frequency ω_1 is propagating along the direction $x>0$. As one has to do with a system in a stationary state, the only time variable occurring is the short time scale τ_0; one has thus to solve a boundary-value problem in which the field amplitude is given on the plane $x=0$.

As previously shown in I, this system will be weakly dissipative only if $\omega_{pe}^2/\omega_1^2 \ll 1$; in the following, it is assumed that $\omega_{pe}^2/\omega_1^2 \simeq \epsilon^2 (\simeq \delta)$, so that the wave absorption takes place at the space scale $\vec{x}_2 = \epsilon^2 \vec{x}_0$. In this case, it can then be shown that the electronic distribution function F_e and the electric field $\vec{e}$ depend only (besides the time variable τ_0) on the even space scales $\vec{x}_0, \vec{x}_2, \ldots$, and that the anisotropies $\vec{F}^{(l)}(\tau_0 \vec{x}_0, \vec{x}_2, \ldots; w)$ and the field $\vec{e}(\tau_0 \vec{x}_0, \vec{x}_2, \ldots)$ can be expanded in ϵ^2 according to

$$\vec{F}^{(l)} = \epsilon^l (\vec{F}^{(l)}_{(l)} + \epsilon^2 \vec{F}^{(l)}_{(l+2)} + \cdots), \quad l = 0, 1, 2, \ldots \tag{2.1a}$$

$$\vec{e} = \vec{e}_{(0)} + \epsilon^2 \vec{e}_{(2)} + \cdots. \tag{2.1b}$$

By (2.1a), one has also for the electronic charge and current densities

$$\rho = \rho_{(0)} + \epsilon^2 \rho_{(2)} + \cdots = e(n_{(0)} - N) + \epsilon^2 e n_{(2)} + \cdots, \tag{2.2a}$$

$$\vec{j} = \epsilon \vec{j}_{(1)} + \epsilon^3 \vec{j}_{(3)} + \cdots, \tag{2.2b}$$

where N is the positive background density. As we know, these expansions allow one to solve the fundamental set (1.2)–(1.7) by a well-defined perturbation scheme for which we get the following equations.

A. Field equations

They are obtained by putting the expansions (2.1b) and (2.2) into the equations (1.4) and (1.5). In order to make the parameter $\omega_{pe}^2/\omega^2 = \epsilon^2$ appear in the source term of (1.4), it is convenient to introduce the current density $\vec{J}$ defined by $-(4\pi e/m\bar{v}\epsilon)\vec{j} = -\omega_{pe}^2 \vec{J}$. With this notation and according to the multiple-space-scales formalism, Eq. (1.4) splits into the following equations:

$$\mathcal{L}^{(0)}(\vec{e}_{(0)}) \equiv \frac{\partial^2 \vec{e}_{(0)}}{\partial \tau_0^2} - \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial^2 \vec{e}_{(0)\perp}}{\partial x_0^2} = 0, \tag{2.3a}$$

$$\begin{aligned}\mathcal{L}^{(2)}(\vec{e}_{(0)}, \vec{e}_{(2)}) &\equiv \left(\frac{\partial^2 \vec{e}_{(2)}}{\partial \tau_0^2} - \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial^2 \vec{e}_{(2)\perp}}{\partial x_0^2}\right) \\ &\quad - 2 \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial}{\partial x_2} \frac{\partial \vec{e}_{(0)\perp}}{\partial x_0} \\ &= -\frac{\partial \vec{J}_1}{\partial \tau_0},\end{aligned} \tag{2.3b}$$

$$\begin{aligned}\mathcal{L}^{(4)}(\vec{e}_{(0)}, \vec{e}_{(2)}, \vec{e}_{(4)}) &\equiv \left(\frac{\partial^2 \vec{e}_{(4)}}{\partial \tau_0^2} - \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial^2 \vec{e}_{(4)\perp}}{\partial x_0^2}\right) \\ &\quad - 2 \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial}{\partial x_2} \frac{\partial \vec{e}_{(2)\perp}}{\partial x_0} \\ &\quad - \frac{c^2}{\lambda_0^2 \omega^2} \left(\frac{\partial^2 \vec{e}_{(0)\perp}}{\partial x_2^2} + 2 \frac{\partial}{\partial x_4} \frac{\partial \vec{e}_{(0)\perp}}{\partial x_0}\right) \\ &= -\frac{\partial \vec{J}_{(3)}}{\partial \tau_0}, \ldots,\end{aligned} \tag{2.3c}$$

where it has been put $\vec{\nabla}_x = \hat{x}\, \partial/\partial x$ and $\vec{e} = \vec{e}_\parallel + \vec{e}_\perp$, with $\vec{\nabla}_x \cdot \vec{e}_\perp = 0$. In the same way, the Poisson equation (1.5) and the continuity equation (1.7) give, respectively,

$$\vec{\nabla}_{x0} \cdot \vec{e}_{(0)} = (n_{(0)} - N)/N, \tag{2.4a}$$

$$\vec{\nabla}_{x0} \cdot \vec{e}_{(2)} + \vec{\nabla}_{x_2} \cdot \vec{e}_{(0)} = n_{(2)}/N, \tag{2.4b}$$

$$\frac{\partial \rho_{(0)}}{\partial \tau_0} = 0, \tag{2.5a}$$

$$\frac{\partial \rho_{(2)}}{\partial \tau_0} = -\frac{1}{\bar{v}} \vec{\nabla}_{x0} \cdot \vec{j}_{(1)}, \ldots. \tag{2.5b}$$

B. Kinetic equations

By the definitions (1.6), the source terms of the field equations (2.3) and (2.4) are to be drawn from the successive approximations of $F^{(0)}$ and $\vec{F}^{(1)}$. These are deduced from the fundamental set (1.2)–(1.7) which splits, according to (2.1a), into the following sequence of kinetic equations:

$$\frac{\partial F^{(0)}_{(0)}}{\partial \tau_0} = 0, \tag{2.6a}$$

$$\begin{aligned}\frac{\partial F^{(0)}_{(2)}}{\partial \tau_0} &= \frac{\bar{\nu}}{\omega} [I(F^{(0)}_{(0)}) + C_{ee}(F^{(0)}_{(0)})] - \gamma' \frac{w^2}{3} \vec{\nabla}_{x_0} \cdot \vec{F}^{(1)}_{(1)} \\ &\quad - \frac{\gamma'}{3w^2} \frac{\partial}{\partial w} (w^3 \vec{e}_{(0)} \cdot \vec{F}^{(1)}_{(1)}),\end{aligned} \tag{2.6b}$$

$$\begin{aligned}\frac{\partial F^{(0)}_{(4)}}{\partial \tau_0} &= \frac{\bar{\nu}}{\omega} [I(F^{(0)}_{(2)}) + C_{ee}(F^{(0)}_{(2)})] \\ &\quad - \gamma' \frac{w^2}{3} (\vec{\nabla}_{x_0} \cdot \vec{F}^{(1)}_{(3)} + \vec{\nabla}_{x_2} \cdot \vec{F}^{(1)}_{(1)}) \\ &\quad - \frac{\gamma'}{3w^2} \frac{\partial}{\partial w} [w^3 (\vec{e}_{(0)} \cdot \vec{F}^{(1)}_{(3)} + \vec{e}_{(2)} \cdot \vec{F}^{(1)}_{(1)})], \ldots,\end{aligned} \tag{2.6c}$$

$$\frac{\partial \vec{F}^{(1)}_{(1)}}{\partial \tau_0} = -\frac{\nu_1}{\omega} \vec{F}^{(1)}_{(1)} - \vec{\nabla}_{x_0} F^{(0)}_{(0)} - \frac{\vec{e}_{(0)}}{w} \frac{\partial F^{(0)}_{(0)}}{\partial w}, \tag{2.7a}$$

$$\begin{aligned}\frac{\partial \vec{F}^{(1)}_{(3)}}{\partial \tau_0} &= -\frac{\nu_1}{\omega} \vec{F}^{(1)}_{(3)} - (\vec{\nabla}_{x_0} F^{(0)}_{(2)} + \vec{\nabla}_{x_2} F^{(0)}_{(0)}) \\ &\quad - \frac{1}{w} \left(\vec{e}_{(0)} \frac{\partial F^{(0)}_{(2)}}{\partial w} + \vec{e}_{(2)} \frac{\partial F^{(0)}_{(0)}}{\partial w}\right) \\ &\quad - \frac{2}{5} \left(w^2 \vec{\nabla}_{x_0} \cdot \vec{F}^{(2)}_{(2)} + \frac{1}{w^4} \frac{\partial}{\partial w} (w^5 \vec{e}_{(0)} \cdot \vec{F}^{(2)}_{(2)})\right) \\ &\quad + \left(\int^{\tau_0} (\vec{\nabla}_{x_0} \times \vec{e}_{(0)} d\tau_0')\right) \times \vec{F}^{(1)}_{(1)}, \ldots,\end{aligned} \tag{2.7b}$$

$$\frac{\partial \vec{F}^{(2)}_{(2)}}{\partial \tau_0} = -\frac{\nu_2}{\omega} \vec{F}^{(2)}_{(2)} - [\vec{\nabla}_{x_0} \vec{F}^{(1)}_{(1)}]^0 - \frac{1}{w} \left[\vec{e}_{(0)} \frac{\partial \vec{F}^{(1)}_{(1)}}{\partial w}\right]^0, \ldots, \tag{2.8a}$$

where $\gamma' = \alpha'/\delta \simeq O(1)$. The successive approximations $\vec{e}_{(0)}, \vec{e}_{(2)}, \ldots$ of the electric field can then be calculated by solving step by step these sequences of coupled equations according to the methods of the multiple-space-scales formalism.

C. Electric field at zero order

If we put

$$\vec{e}_{(0)}=\vec{e}'_{(0)}(\tau_0\vec{x}_0,\vec{x}_2,\ldots)+\vec{e}''_{(0)}(\vec{x}_0,\vec{x}_2,\ldots), \tag{2.9}$$

where $\vec{e}'_{(0)}\equiv\vec{e}_{(0)\perp}$ is the oscillating transverse component (at time scale τ_0) of $\vec{e}_{(0)}$ and $\vec{e}''_{(0)}$ is its stationary longitudinal component (depending only on the space coordinates), it is immediately seen by (2.3a) that $\vec{e}'_{(0)}$ satisfies the field equation in the vacuum [of course, this is due to the condition $\omega_{pe}^2/\omega^2=\epsilon^2$ from which it results that the electric field does not "see" the plasma at the scale $\vec{x}_0$]. Thus it can be written for the electric field $\vec{e}'_{(0)}$

$$\vec{e}'_{(0)}\equiv\vec{e}_{(0)\perp}=\vec{e}'^{\,0}_{(0)}(\vec{x}_2,\ldots)e^{i\varphi}+\text{c.c.}, \tag{2.10a}$$

with

$$\varphi=(\omega_1/\omega)\tau_0-\vec{K}_0\cdot\vec{x}_0+\varphi_0,$$
$$K_0=|\vec{K}_0|=\lambda_0\omega_1/c,$$
$$\vec{K}_0\cdot\vec{e}'^{\,0}_{(0)}=0, \tag{2.10b}$$

where φ_0 is an initial phase and $\vec{K}_0$ the real wave vector at zero order.

We then have to determine $\vec{e}'^{\,0}_{(0)}(\vec{x}_2,\ldots)$ by using the second-order field equation (2.3b) and the kinetic equations (2.6a), (2.6b), and (2.7a). As, by (2.6a), $F^{(0)}_{(0)}$ is independent of τ_0, we can use for $\vec{F}^{(1)}_{(1)}$ the same decomposition as for $\vec{e}_{(0)}$; we thus write

$$\vec{F}^{(1)}_{(1)}=\vec{F}^{(1)\prime}_{(1)}(\tau_0\vec{x}_0,\vec{x}_2,\ldots;w)+\vec{F}^{(1)\prime\prime}_{(1)}(\vec{x}_0,\vec{x}_2,\ldots;w) \tag{2.11}$$

from which it follows that (2.7a) splits into the two following equations:

$$\frac{\partial\vec{F}^{(1)\prime}_{(1)}}{\partial\tau_0}+\frac{\nu_1}{\omega}\vec{F}^{(1)\prime}_{(1)}=-\frac{\vec{e}'_{(0)}}{w}\frac{\partial F^{(0)}_{(0)}}{\partial w}, \tag{2.12a}$$

$$\frac{\nu_1}{\omega}\vec{F}^{(1)\prime\prime}_{(1)}=-\vec{\nabla}_{x_0}F^{(0)}_{(0)}-\frac{\vec{e}''_{(0)}}{w}\frac{\partial F^{(0)}_{(0)}}{\partial w}, \tag{2.12b}$$

obtained by separating in (2.7a) the terms dependent and independent of τ_0, respectively.

From (2.12b), it is seen that the stationary field $\vec{e}''_{(0)}$ is linked to the inhomogeneity of $F^{(0)}_{(0)}$ at the space scale $\vec{x}_0$; as the medium is assumed homogeneous in the absence of the wave, it will be shown below that $\vec{e}''_{(0)}=0$. On the other hand, by multiplying (2.12a) by $w^4\,dw$, integrating and neglecting the transient contributions (at the time scale τ_0), we get

$$\vec{J}_{(1)}=\omega[\sigma(\omega_1)\vec{e}'^{\,0}_{(0)}e^{i\varphi}+\sigma^*(\omega_1)\vec{e}'^{\,0*}_{(0)}e^{-i\varphi}], \tag{2.13}$$

where the conductivity $\sigma(\omega_1)$ is given by the usual expression

$$\sigma(\omega_1)\equiv(1/\omega_1^2)(\bar{\nu}\sigma''_R-i\omega_1\sigma''_I)$$
$$=-\frac{4\pi}{3N}\int_0^\infty\frac{w^3}{\nu_1+i\omega_1}\frac{\partial F^{(0)}_{(0)}}{\partial w}dw. \tag{2.14}$$

Putting then these results in (2.3b), we obtain

$$\frac{\partial^2\vec{e}_{(2)}}{\partial\tau_0^2}-\frac{c^2}{\lambda_0^2\omega^2}\frac{\partial^2\vec{e}_{(2)\perp}}{\partial x_0^2}=2\frac{c^2}{\lambda_0^2\omega^2}\frac{\partial}{\partial x_2}\frac{\partial\vec{e}_{(0)\perp}}{\partial x_0}-i[\omega_1\sigma(\omega_1)\vec{e}'^{\,0}_{(0)}e^{i\varphi}-\text{c.c.}] \tag{2.15}$$

which is to be solved by applying the multiple-space-scales techniques. By putting $\vec{e}_{(2)}=\vec{e}_{(2)\parallel}+\vec{e}_{(2)\perp}$, it is seen that (2.15) gives $\partial^2\vec{e}_{(2)\parallel}/\partial\tau_0^2=0$, since the field $\vec{e}'_{(0)}$ is transverse; therefore $\vec{e}_{(2)\parallel}\equiv\vec{e}''_{(2)}$ is a stationary field, viz., $\vec{e}_{(2)\parallel}=\vec{e}''_{(2)}(\vec{x}_2)$, which will be computed below. On the other hand, the transverse part of (2.15) is a wave equation for $\vec{e}_{(2)\perp}$ with a right-hand resonating member, since $e^{i\varphi}$ (or $e^{im\varphi}$, m being an integer) is a solution of the homogeneous equation. According to the multiple-space-scales technique, the secularities must be canceled at each stage of the calculation, so that one is led to annul the resonating term in (2.15). One thus obtains two equations: one for $\vec{e}_{(2)\perp}$,

$$\frac{\partial^2\vec{e}_{(2)\perp}}{\partial\tau_0^2}-\frac{c^2}{\lambda_0^2\omega^2}\frac{\partial^2\vec{e}_{(2)\perp}}{\partial x_0^2}=0, \tag{2.16}$$

whose solutions are of the form

$$\vec{e}_{(2)\perp}\equiv\vec{e}'_{(2)}={}^m\vec{e}'^{\,0}_{(0)}(\vec{x}_2,\ldots)e^{im\varphi}+\text{c.c.}; \tag{2.17}$$

the other for determining the variation of $\vec{e}'^{\,0}_{(0)}$ at the space scale $\vec{x}_2$,

$$-2iK_0\frac{c^2}{\lambda_0^2\omega^2}\left(\frac{\partial\vec{e}'^{\,0}_{(0)}}{\partial x_2}e^{i\varphi}-\frac{\partial\vec{e}'^{\,0*}_{(0)}}{\partial x_2}e^{-i\varphi}\right)=i[\omega_1\sigma(\omega_1)\vec{e}'^{\,0}_{(0)}e^{i\varphi}-\text{c.c.}]\,. \tag{2.18}$$

From (2.10) and (2.14), thus it is obtained for $\vec{e}'^{\,0}_{(0)}$:

$$\vec{e}'^{\,0}_{(0)}(\vec{x}_2)=\vec{u}_{(0)}e^{-\beta(\vec{x}_2)+i\varphi_1(\vec{x}_2)},\ \ (\vec{K}_0\cdot\vec{u}_{(0)}=0)\,, \tag{2.19}$$

where $\vec{u}_{(0)}(\vec{x}_4,\ldots)$ is to be determined by the wave amplitude on the plane $x=0$ and where the absorption coefficient $\beta(\vec{x}_2)$ and the phase shift $\varphi_1(\vec{x}_2)$ are given by

$$\beta(\vec{x}_2)=\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\frac{\bar{\nu}}{\omega_1}\int^{x_2}\sigma''_R\,dx'_2, \tag{2.20a}$$

$$\varphi_1(\vec{x}_2)=\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\int^{x_2}\sigma''_I\,dx'_2, \tag{2.20b}$$

respectively. Let us note that we have not mentioned in (2.20) the dependence on the higher-order variables $\vec{x}_4, \ldots,$ because it will be shown with the next-order equations that $\vec{u}_{(0)}$ is a constant vector (which can be taken real) and that $F_{(0)}^{(0)}$ and $\vec{e}'_{(0)}$ are in fact independent of these space scales.

From (2.19) and (2.20), the zero-order field $\vec{e}'_{(0)}$ is thus well-defined in terms of the conductivities σ_R'' and σ_I'' which depend on $\vec{x}_2$ through $F_{(0)}^{(0)}$, as it is shown below. The phase shift φ_1 expresses the variation of the wave-number vector at the same scale and allows one to obtain the local dispersion equation at order ϵ^2 for transverse waves under the form

$$K_0' = K_0\left[1 - \frac{\omega_{pe}^2}{2\omega_1^2}\left(\sigma_I'' + i\,\frac{\overline{\nu}}{\omega_1}\,\sigma_R''\right)\right] \;. \tag{2.21}$$

In order to determine completely $\vec{e}'_{(0)}$, it remains to compute $F_{(0)}^{(0)}$ which occurs in the expressions for σ_R'' and σ_I''. For this purpose, we have to consider Eq. (2.6b) which, from (2.6a), (2.9), and (2.11), includes terms both dependent and independent of τ_0. In order to eliminate the secularities, these two kinds of terms separately have to be written equal to zero; we thus obtain the two equations

$$\frac{\partial F_{(2)}^{(0)}}{\partial \tau_0} = \frac{\omega u_{(0)}^2 e^{-2\beta}}{3w^2}\,\frac{\partial}{\partial w} \times\left[w^2\,\frac{\partial F_{(0)}^{(0)}}{\partial w}\left(\frac{e^{2i(\varphi+\varphi_1)}}{\nu_1 + i\,\omega_1} + \text{c.c.}\right)\right] \tag{2.22}$$

which allows to compute $F_{(2)}^{(0)}$, and

$$I(F_{(0)}^{(0)}) + C_{ee}(F_{(0)}^{(0)}) = -\,\frac{2\gamma'\omega^2 u_{(0)}^2 e^{-2\beta}}{3w^2}\,\frac{\partial}{\partial w}\left(\frac{w^2\nu_1'}{\nu_1^2+\omega_1^2}\,\frac{\partial F_{(0)}^{(0)}}{\partial w}\right) - \gamma'\,\frac{w^2}{3}\,\vec{\nabla}_{x_0}\cdot\vec{F}_{(1)}^{(1)\prime\prime} - \frac{\gamma'}{3w^2}\,\frac{\partial}{\partial w}\left(w^3\,\vec{e}''_{(0)}\cdot\vec{F}_{(1)}^{(1)\prime\prime}\right) , \tag{2.23}$$

because the products $\vec{e}'_{(0)}\cdot\vec{F}_{(1)}^{(1)\prime\prime}$ and $\vec{e}''_{(0)}\cdot\vec{F}_{(1)}^{(1)\prime}$ are zero (transversality of $\vec{e}'_{(0)}$). It is seen that the space variable occurs in (2.23) only at the scale $\vec{x}_2$ through the factor $e^{-2\beta}$ in the term corresponding to the energy borrowed from the field by the electrons through collisions. As the plasma is assumed homogeneous in the absence of the wave, we retain for $F_{(0)}^{(0)}$ only solutions independent of $\vec{x}_0$. In this case, $\vec{F}_{(1)}^{(1)\prime\prime}$ and $\vec{e}''_{(0)}$ depend only on $\vec{x}_2, \ldots,$ and are linked between them by (2.12b) in which $\vec{\nabla}_{x_0}F_{(0)}^{(0)} = 0$; as, on the other hand, $\vec{j}''_{(1)} = 0$ by virtue of continuity equation, it results that $\vec{e}''_{(0)} = \vec{F}_{(1)}^{(1)\prime\prime} = 0$. Moreover, as $\vec{e}'_{(0)}$ is transverse, the Poisson equation (2.4a) gives

$$n_{(0)} = N , \tag{2.24a}$$

so that $F_{(0)}^{(0)}$ is determined by

$$I(F_{(0)}^{(0)}) + C_{ee}(F_{(0)}^{(0)}) = -\,\frac{2\gamma'\omega^2 u_{(0)}^2\,e^{-2\beta}}{3w^2} \times\frac{\partial}{\partial w}\left(\frac{w^2\nu_1'}{\nu_1^2+\omega_1^2}\,\frac{\partial F_{(0)}^{(0)}}{\partial w}\right) , \tag{2.25}$$

with the normalization condition

$$4\pi\int_0^\infty w^2 F_{(0)}^{(0)}\,dw = N ; \tag{2.24b}$$

this is the relevant equation of our problem.

Thus it is seen that $F_{(0)}^{(0)}$ depends on the space variable $\vec{x}_2$ through the absorption $\beta(\vec{x}_2)$ which itself is given by (2.20a). So, the plasma state is determined at zero order by the two equations (2.20a) and (2.25) which allow one to compute the non-Maxwellian stationary distribution $F_{(0)}^{(0)}(\vec{x}_2; w)$. For the special case of an imperfectly Lorentzian plasma, in which the Coulomb collision term $C_{ee}(F_{(0)}^{(0)})$ can be neglected before $I(F_{(0)}^{(0)})$, (2.25) is easily integrable and gives

$$F_{(0)}^{(0)}(\vec{x}_2; w) = C(\vec{x}_2)\exp\left\{-\int^w 3w\,dw\Big/\left[1 + 4\gamma'\omega^2 u_{(0)}^2 \exp\left(-K_0\,\frac{\overline{\nu}}{\omega_1}\,\frac{\omega^2}{\omega_1^2}\int^{x_2}\sigma_R''\,dx_2'\right)\Big/(\nu_1^2+\omega_1^2)\right]\right\} \tag{2.26}$$

in which $C(\vec{x}_2)$ is to be determined by the condition (2.24b).

In conclusion, thus it is seen that the plasma, homogeneous in the absence of the wave, becomes inhomogeneous at the scale $\vec{x}_2$ under the effect of the wave absorption defined by (2.20a). As a consequence, it follows that the "first-order" conductivity depends in fact nonlinearly on the field amplitude at $\vec{x}_2$. As, on the other hand, the electronic density remains nevertheless constant at this order of approximation, it results that the effect of the field $\vec{e}'_{(0)}$ on the plasma appears as a local temperature rise due to the factor $\gamma'\omega^2 u_{(0)}^2 e^{-2\beta}/(\nu_1^2+\omega_1^2)$ in (2.26): This thermoeffect is thus a collisional heating linked with the wave absorption in the plasma. Moreover, it will be seen below that this inhomogeneity induces also at the next approximation a longitudinal stationary field $\vec{e}''_{(2)}(\vec{x}_2)$ at order ϵ^2.

D. Electric field at order ϵ^2

As its oscillating part $\vec{e}'_{(2)}$ is of the form (2.17), one has to compute the amplitudes corresponding to the different possible values of m by using the ϵ^4 order equation (2.3c). In the latter, the nonlinear source term $\partial\vec{J}_{(3)}/\partial\tau_0$ is deduced from (2.7b), (2.8), and (2.22) in which $\vec{e}'_{(0)}$ and $F^{(0)}_{(0)}$ are expressed as before.

1. Determination of $\vec{J}_{(3)}$

As the right-hand member of (2.7b) contains terms dependent on $\tau_0\vec{x}_0$ and terms such as $\vec{\nabla}_{x_2}F^{(0)}_{(0)}$, which depend only on $\vec{x}_2$, we are led to use again decompositions similar to (2.11) by writing

$$\vec{F}^{(1)}_{(3)} = \vec{F}^{(1)\prime}_{(3)}(\tau_0\vec{x}_0;\vec{x}_2;w) + \vec{F}^{(1)\prime\prime}_{(3)}(\vec{x}_2;w)\,, \tag{2.27}$$

$$\vec{e}_{(2)} = \vec{e}'_{(2)}(\tau_0\vec{x}_0;\vec{x}_2) + \vec{e}''_{(2)}(\vec{x}_2) = \vec{e}_{(2)\perp} + \vec{e}_{(2)\parallel}\,, \tag{2.28}$$

with $\vec{e}'_{(2)} \equiv \vec{e}_{(2)\perp}$ and $\vec{e}''_{(2)}(\vec{x}_2) = \vec{e}_{(2)\parallel}$ by (2.15). Moreover, by integrating (2.22), one gets also

$$F^{(0)}_{(2)} = F^{(0)\prime}_{(2)}(\tau_0\vec{x}_0,\vec{x}_2;w) + F^{(0)\prime\prime}_{(2)}(\vec{x}_2;w)\,, \tag{2.29}$$

in which $F^{(0)\prime\prime}_{(2)}$ is to be determined by the ϵ^4 order equation (2.6c) (cf. the Appendix), and where $F^{(0)\prime}_{(2)}$ is given by

$$F^{(0)\prime}_{(2)} = -\frac{i\omega^2 u^2_{(0)}e^{-2\beta}}{6\omega_1}\left[O_1(F^{(0)}_{(0)})e^{2i(\varphi+\varphi_1)} - \text{c.c.}\right], \tag{2.30}$$

the operator $O_1(F^{(0)}_{(0)})$ being defined in the Appendix by (A7a). Finally, we need also the expression of $\vec{F}^{(2)}_{(2)}$ which is obtained by integrating (2.8a). By using (A1) and the definitions (A7) of the Appendix, one thus gets

$$\begin{aligned}\vec{\vec{F}}^{(2)}_{(2)} = {}&-i\omega^2 e^{-\beta(\vec{x}_2)}\left(\frac{\vec{K}_0\vec{u}_{(0)}+\vec{u}_{(0)}\vec{K}_0}{2}\right)\left(\frac{e^{i(\varphi+\varphi_1)}}{(\nu_1+i\omega_1)(\nu_2+i\omega_1)} - \text{c.c.}\right)\frac{1}{w}\,\frac{\partial F^{(0)}_{(0)}}{\partial w} + \omega^2 e^{-2\beta(\vec{x}_2)}[\vec{u}_{(0)}\vec{u}_{(0)}]^0\\ &\times\frac{1}{w}\left(\frac{O'_1(F^{(0)}_{(0)})e^{2i(\varphi+\varphi_1)}}{\nu_2+2i\omega_1} + \text{c.c.}\right) + \omega^2 e^{-2\beta(\vec{x}_2)}[\vec{u}_{(0)}\vec{u}_{(0)}]^0\,\frac{1}{w\nu_2}\,O'(F^{(0)}_{(0)})\,,\end{aligned} \tag{2.31}$$

in which the third term is a stationary contribution independent of $\tau_0,\vec{x}_0$. We are now able to calculate with these expressions the various contributions to $\vec{F}^{(1)\prime}_{(3)}$ and $\vec{F}^{(1)\prime\prime}_{(3)}$. By looking at the integrated version (A2) of (2.7b), it is easily seen that $\vec{F}^{(1)\prime\prime}_{(3)}$ is given by

$$\begin{aligned}\vec{F}^{(1)\prime\prime}_{(3)} = {}&-\frac{\omega}{\nu_1}\vec{\nabla}_{x_2}F^{(0)}_{(0)} - \frac{\omega}{\nu_1 w}\vec{e}''_{(2)}\frac{\partial F^{(0)}_{(0)}}{\partial w} + 2\vec{K}_0\omega^3 u^2_{(0)}e^{-2\beta}\left\{-\frac{1}{\omega_1(\nu_1^2+\omega_1^2)w}\,\frac{\partial F^{(0)}_{(0)}}{\partial w}\right.\\ &\left.+\frac{i}{10\nu_1 w^4}\,\frac{\partial}{\partial w}\left[w^4\,\frac{\partial F^{(0)}_{(0)}}{\partial w}\left(\frac{1}{(\nu_1+i\omega_1)(\nu_2+i\omega_1)} - \text{c.c.}\right)\right]\right\}\end{aligned} \tag{2.32}$$

and that $\vec{F}^{(1)\prime}_{(3)}$ is the sum of three terms in $e^{i\varphi}$, $e^{2i\varphi}$, and $e^{3i\varphi}$, respectively. Because of (2.17), one is thus led to split $\vec{e}'_{(2)}$ into three terms, by writing

$$\vec{e}'_{(2)} = {}^1\vec{e}'_{(2)} + {}^2\vec{e}'_{(2)} + {}^3\vec{e}'_{(2)} = ({}^1\vec{e}'^0_{(2)}e^{i\varphi} + \text{c.c.}) + ({}^2\vec{e}'^0_{(2)}e^{2i\varphi} + \text{c.c.}) + ({}^3\vec{e}'^0_{(2)}e^{3i\varphi} + \text{c.c.})\,, \tag{2.33}$$

so that one has also for $\vec{F}^{(1)\prime}_{(3)}$

$$\vec{F}^{(1)\prime}_{(3)} = {}^1\vec{F}^{(1)\prime}_{(3)} + {}^2\vec{F}^{(1)\prime}_{(3)} + {}^3\vec{F}^{(1)\prime}_{(3)} \tag{2.34}$$

with, according to (A2), (2.31), and (2.33),

$$\begin{aligned}{}^1\vec{F}^{(1)\prime}_{(3)} = {}&-\frac{\omega}{w}\,\frac{\partial F^{(0)}_{(0)}}{\partial w}\left(\frac{{}^1\vec{e}'^0_{(2)}e^{i\varphi}}{\nu_1+i\omega_1} + \text{c.c.}\right) - \frac{\omega\vec{u}_{(0)}e^{-\beta}}{w}\,\frac{\partial F^{(0)\prime\prime}_{(2)}}{\partial w}\left(\frac{e^{i(\varphi+\varphi_1)}}{\nu_1+i\omega_1} + \text{c.c.}\right)\\ &+\omega^3\vec{u}_{(0)}e^{-\beta}\left(\frac{e^{i(\varphi+\varphi_1)}}{\nu_1+i\omega_1}\left[u^2_{(0)}e^{-2\beta}A_1(F^{(0)}_{(0)}) + \tfrac{1}{5}K_0^2A'_1(F^{(0)}_{(0)})\right] + \text{c.c.}\right),\end{aligned} \tag{2.35a}$$

$${}^2\vec{F}^{(1)\prime}_{(3)} = -\frac{\omega}{w}\,\frac{\partial F^{(0)}_{(0)}}{\partial w}\left(\frac{{}^2\vec{e}'^0_{(2)}e^{2i\varphi}}{\nu_1+2i\omega_1} + \text{c.c.}\right) + \omega^3u^2_{(0)}e^{-2\beta}\vec{K}_0\left(\frac{e^{2i(\varphi+\varphi_1)}}{\nu_1+2i\omega_1}A_2(F^{(0)}_{(0)}) + \text{c.c.}\right), \tag{2.35b}$$

$${}^3\vec{F}^{(1)\prime}_{(3)} = -\frac{\omega}{w}\,\frac{\partial F^{(0)}_{(0)}}{\partial w}\left(\frac{{}^3\vec{e}'^0_{(2)}e^{3i\varphi}}{\nu_1+3i\omega_1} + \text{c.c.}\right) + \omega^3\vec{u}_{(0)}u^2_{(0)}e^{-3\beta}\left(\frac{e^{3i(\varphi+\varphi_1)}}{\nu_1+3i\omega_1}A_3(F^{(0)}_{(0)}) + \text{c.c.}\right), \tag{2.35c}$$

in which the various operators A are defined by (A3)–(A6) in the Appendix.

It then results from (2.27) and (2.34)–(2.35c) that the current density can be written in the form

$$\vec{J}_{(3)}=\vec{J}'_{(3)}(\tau_0\vec{x}_0,\vec{x}_2)+\vec{J}''_{(3)}(\vec{x}_2), \tag{2.36}$$

in which the stationary component $\vec{J}''_{(3)}$ depends only on $\vec{x}_2$ and where $\vec{J}'_{(3)}$ is split up into three terms in $e^{i\varphi}$, $e^{2i\varphi}$, and $e^{3i\varphi}$, respectively. One thus gets

$$\vec{J}'_{(3)}={}^1\vec{J}'_{(3)}+{}^2\vec{J}'_{(3)}+{}^3\vec{J}'_{(3)}, \tag{2.37}$$

with

$${}^1\vec{J}'_{(3)}=\omega[\sigma(\omega_1)\,{}^1\vec{e}'^{0}_{(2)}\,e^{i\varphi}+\sigma_{(2)}(\omega_1)\vec{u}_{(0)}e^{-\beta}e^{i(\varphi+\varphi_1)}+\omega^2\vec{u}_{(0)}e^{-\beta}(u^2_{(0)}e^{-2\beta}\sigma_1+K_0^2\sigma'_1)e^{i(\varphi+\varphi_1)}+\text{c.c.}], \tag{2.38a}$$

$${}^2\vec{J}'_{(3)}=\omega[\sigma(2\omega_1)\,{}^2\vec{e}'^{0}_{(2)}e^{2i\varphi}+\omega^2u^2_{(0)}e^{-2\beta}\vec{K}_0\sigma_2e^{2i(\varphi+\varphi_1)}+\text{c.c.}], \tag{2.38b}$$

$${}^3\vec{J}'_{(3)}=\omega[\sigma(3\omega_1)\,{}^3\vec{e}'^{0}_{(2)}e^{3i\varphi}+\omega^2\vec{u}_{(0)}u^2_{(0)}e^{-3\beta}\sigma_3e^{3i(\varphi+\varphi_1)}+\text{c.c.}], \tag{2.38c}$$

in which $\sigma(\omega_1)$, $\sigma(2\omega_1)$, and $\sigma(3\omega_1)$ are the usual conductivities defined by (2.14), and where the following higher-order conductivities have been introduced:

$$\sigma_1(\omega_1)=\frac{4\pi}{3N}\int_0^\infty\frac{w^4}{\nu_1+i\omega_1}A_1(F^{(0)}_{(0)})dw, \tag{2.39a}$$

$$\sigma'_1(\omega_1)=\frac{4\pi}{3N}\int_0^\infty\frac{w^4}{5(\nu_1+i\omega_1)}A'_1(F^{(0)}_{(0)})dw, \tag{2.39b}$$

$$\sigma_{(2)}(\omega_1)=-\frac{4\pi}{3N}\int_0^\infty\frac{w^3}{\nu_1+i\omega_1}\frac{\partial F^{(0)''}_{(2)}}{\partial w}dw, \tag{2.40}$$

$$\sigma_2(\omega_1)=\frac{4\pi}{3N}\int_0^\infty\frac{w^4}{\nu_1+2i\omega_1}A_2(F^{(0)}_{(0)})dw$$
$$=\sigma'_2+\sigma''_2+\sigma^L_2, \tag{2.41a}$$

$$\sigma_3(\omega_1)=\frac{4\pi}{3N}\int_0^\infty\frac{w^4}{\nu_1+3i\omega_1}A_3(F^{(0)}_{(0)})dw. \tag{2.41b}$$

It must be noted about the nonlinear conductivities $\sigma_2(\omega_1)$ and $\sigma_3(\omega_1)$ that, by virtue of (A6), σ_3 includes only collisional contributions which cancel out when $\nu_{1,2}(v)=cte$ (or $\nu_{1,2}=0$), while from (A5), σ_2 is composed of three terms of various nature which are defined in the Appendix; it is thus seen that only σ''_2, given by (A8c), is a purely collisional term which cancels out when $\nu_{1,2}=cte$ (or $\nu_{1,2}=0$), while σ'_2 and σ^L_2, defined by (A8a) and (A8b), are linked, respectively, to the gradient of the pressure tensor induced by the zero-order field $\vec{e}'_{(0)}$ and to the Lorentz force due to the magnetic field of the wave. In the same way, it can be checked from (A3) and (A4) that $\sigma_1(\omega_1)$ is of collisional nature and that $\sigma'_1(\omega_1)$ is also linked to the gradient of the pressure tensor. As for $\sigma_{(2)}(\omega_1)$, its expression results from the calculation of the stationary component $F^{(0)''}_{(2)}$ which must satisfy Eq. (A9), with the condition (A11). In the special case of an imperfectly Lorentzian plasma, $F^{(0)''}_{(2)}$ is explicitly given by (A12) so that $\sigma_{(2)}(\omega_1)$ takes then the particular form

$$\sigma_{(2)}(\omega_1)=\omega^2\sigma^1_{(2)}({}^1\vec{e}'^{0}_{(2)}\cdot\vec{e}'^{0*}_{(0)}+\text{c.c.})$$
$$+\omega^4u^2_{(0)}e^{-2\beta}(\sigma^{2\prime}_{(2)}u^2_{(0)}e^{-2\beta}+\sigma^{2\prime\prime}_{(2)}K_0^2) \tag{2.40′}$$

where the conductivities $\sigma^1_{(2)}$, $\sigma^{2\prime}_{(2)}$, and $\sigma^{2\prime\prime}_{(2)}$ are given by (A18). With these definitions and with the condition (A11), it is easily checked that these three coefficients are of collisional type and vanish when $\nu_1=cte$.

2. *Field equations*

The field equations are deduced from (2.3c), (2.4b), and (2.5b) in which the current density $\vec{J}_{(3)}$ is given by the previous expressions. First of all, let us remark that $\vec{e}''_{(2)}$ can be drawn from (2.32) by observing that one must put $\vec{J}''_{(3)}=0$, since one has $\vec{\nabla}_{x_0}\cdot\vec{J}''_{(3)}=0$ and because the current has to be zero on the frontier $x=0$. Besides, it follows from (2.38) that the odd current components ${}^1\vec{J}'_{(3)}$ and ${}^3\vec{J}'_{(3)}$ give transverse source terms in the field equations, whereas the even component ${}^2\vec{J}'_{(3)}$ gives a longitudinal one; therefore, the components ${}^1\vec{e}'^{0}_{(2)}$ and ${}^3\vec{e}'^{0}_{(2)}$ will be transverse, while ${}^2\vec{e}'^{0}_{(2)}$ will be longitudinal. But, as we know by (2.15) that $\vec{e}_{(2)\parallel}$ has no oscillating contribution, one must have

$${}^2\vec{e}'_{(2)}=0, \tag{2.42}$$

so that it can be written from (2.33)

$$\vec{e}'_{(2)}=\vec{e}_{(2)\perp}={}^1\vec{e}'_{(2)}+{}^3\vec{e}'_{(2)}. \tag{2.43}$$

This result can yet be confirmed by considering the Poisson and continuity equations (2.4b) and (2.5b). Indeed, by (2.5b), one has $\partial\rho_{(2)}/\partial\tau_0=0$ by virtue of transversality of $\vec{J}_{(1)}$, so that $n_{(2)}=n''_{(2)}$ which is independent of τ_0. As one has also $\vec{\nabla}_{x_2}\cdot\vec{e}'_{(0)}=0$ (transverse wave) and $\vec{\nabla}_{x_0}\cdot\vec{e}''_{(2)}=0$($\vec{e}''_{(2)}$ function of $\vec{x}_2$ alone), Eq. (2.4b) reduces to $\vec{\nabla}_{x_0}\cdot{}^2\vec{e}'_{(2)}=n''_{(2)}/N$, which can be satisfied only if $n''_{(2)}=0$ and ${}^2\vec{e}'_{(2)}=0$, because the lefthand side is oscillating in τ_0.

Let us now consider the field equation (2.3c); by writing again $\vec{e}_{(4)} = \vec{e}_{(4)\parallel} + \vec{e}_{(4)\perp}$, it splits up into two equations: one for the longitudinal component,

$$\frac{\partial^2 \vec{e}_{(4)\parallel}}{\partial \tau_0^2} = -\frac{\partial \vec{J}_{(3)\parallel}}{\partial \tau_0} = -\frac{\partial^2 \vec{J}'_{(3)}}{\partial \tau_0}$$

$$= -2i\omega_1 \omega^2 u^2_{(0)} e^{-2\beta} \vec{K}_0 (\sigma_2 e^{2i(\varphi+\varphi_1)} - \text{c.c.}) , \tag{2.44}$$

which shows that the oscillating part of $\vec{e}_{(4)\parallel}$ is the first even harmonic of the field inside the plasma; the other for the transverse component,

$$\frac{\partial^2 \vec{e}_{(4)\perp}}{\partial \tau_0^2} - \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial^2 \vec{e}_{(4)\perp}}{\partial x_0^2}$$

$$= \frac{c^2}{\lambda_0^2 \omega^2} \left(2 \frac{\partial}{\partial x_2} \frac{\partial \vec{e}_{(2)}}{\partial x_0} + \frac{\partial^2 \vec{e}'_{(0)}}{\partial x_2^2} + 2 \frac{\partial}{\partial x_4} \frac{\partial \vec{e}'_{(0)}}{\partial x_0} \right)$$

$$- \frac{\partial\, {}^1\vec{J}'_{(3)}}{\partial \tau_0} - \frac{\partial\, {}^3\vec{J}'_{(3)}}{\partial \tau_0} , \tag{2.45}$$

which is again a wave equation with a resonating right-hand member. By canceling the secularities, one obtains as previously two equations: on the one hand,

$$\frac{\partial^2 \vec{e}_{(4)\perp}}{\partial \tau_0^2} - \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial^2 \vec{e}_{(4)\perp}}{\partial x_0^2} = 0 , \tag{2.46}$$

whose solution is of the form (2.17), and on the other hand,

$$\frac{c^2}{\lambda_0^2 \omega^2} \left(2 \frac{\partial}{\partial x_2} \frac{\partial ({}^1\vec{e}'_{(2)} + {}^3\vec{e}'_{(2)})}{\partial x_0} + \frac{\partial^2 \vec{e}'_{(0)}}{\partial x_2^2} + 2 \frac{\partial}{\partial x_4} \frac{\partial \vec{e}'_{(0)}}{\partial x_0} \right)$$

$$= \frac{\partial\, {}^1\vec{J}'_{(3)}}{\partial \tau_0} + \frac{\partial\, {}^3\vec{J}'_{(3)}}{\partial \tau_0} , \tag{2.47}$$

which allows one to compute ${}^1\vec{e}'_{(2)}$ and ${}^3\vec{e}'_{(2)}$. According to (2.43) and from (2.19) and (2.38), we finally derive from (2.47) the following two equations for ${}^1\vec{e}'_{(2)}$ and ${}^3\vec{e}'_{(2)}$, respectively,

$$\frac{c^2}{\lambda_0^2 \omega^2} \left(2 \frac{\partial}{\partial x_2} \frac{\partial\, {}^1\vec{e}'_{(2)}}{\partial x_0} + \frac{\partial^2 \vec{e}'_{(0)}}{\partial x_2^2} + 2 \frac{\partial}{\partial x_4} \frac{\partial \vec{e}'_{(0)}}{\partial x_0} \right)$$

$$= i\omega_1 [\sigma(\omega_1)\, {}^1\vec{e}'^0_{(2)} e^{i\varphi} + \sigma_{(2)}(\omega_1) \vec{u}_{(0)} e^{-\beta} e^{i(\varphi+\varphi_1)}$$

$$+ \omega^2 \vec{u}_{(0)} e^{-\beta} (u^2_{(0)} e^{-2\beta} \sigma_1 + K_0^2 \sigma'_1)$$

$$\times e^{i(\varphi+\varphi_1)} - \text{c.c.}] , \tag{2.48}$$

$$2 \frac{c^2}{\lambda_0^2 \omega^2} \frac{\partial}{\partial x_2} \frac{\partial\, {}^3\vec{e}'_{(2)}}{\partial x_0}$$

$$= 3i\omega_1 [\sigma(3\omega_1)\, {}^3\vec{e}'^0_{(2)} e^{3i\varphi} + \omega^2 \vec{u}_{(0)} u^2_{(0)} e^{-3\beta} \sigma_3$$

$$\times e^{3i(\varphi+\varphi_1)} - \text{c.c.}] \tag{2.49}$$

in which ${}^1\vec{e}'_{(2)}$ is the order-ϵ^2 perturbation in $e^{i\varphi}$ whereas ${}^3\vec{e}'_{(2)}$ is the first odd harmonic.

3. Complete expression of the electric field at order ϵ^2

As the first even harmonic is of order ϵ^4 from (2.42) and (2.44), it follows from the previous results that the electric field inside the plasma can be written at order ϵ^2, in the form

$$\vec{e} = \vec{e}_{(0)} + \epsilon^2 \vec{e}_{(2)} = \vec{e}'_{(0)} + \epsilon^2 ({}^1\vec{e}'_{(2)} + {}^3\vec{e}'_{(2)} + \vec{e}''_{(2)})$$

$$= [(\vec{u}_{(0)} e^{-\beta(\vec{x}_2) + i\varphi_1(\vec{x}_2)} + \epsilon^2\, {}^1\vec{e}'^0_{(2)}) e^{i\varphi}$$

$$+ \epsilon^2\, {}^3\vec{e}'^0_{(2)} e^{3i\varphi} + \text{c.c.}] + \epsilon^2 \vec{e}''_{(2)}(\vec{x}_2) , \tag{2.50}$$

from which it is seen that the field is composed, at this approximation, of three components, one being stationary and the two others being oscillating in $e^{i\varphi}$ and $e^{3i\varphi}$, respectively.

The order-ϵ^2 stationary field, $\epsilon^2 \vec{e}''_{(2)}$, is of longitudinal type and well defined by (2.32) with the condition $\vec{j}''_{(3)} = 0$, as previously seen; it is due to the medium inhomogeneity at scale $\vec{x}_2$ induced by the field $\vec{e}'_{(0)}$ and, as it has been shown in connection with the discussion of the $F^{(0)}_{(0)}$ equation, it is in fact linked to the collisional heating of the plasma by the primary wave.

The oscillating component in $e^{i\varphi}$ is composed of the zero-order field $\vec{e}'_{(0)}$, given by (2.19) and (2.20), and of the ϵ^2-order contribution ${}^1\vec{e}'_{(2)}$ which is determined by (2.48). This latter equation first allows one to show that $\partial \vec{u}_{(0)}/\partial x_4 = 0$, so that the plasma and field variables are independent of $\vec{x}_4$ and of higher-order scales. Indeed, as we have

$$\frac{\partial \vec{e}'^0_{(0)}}{\partial x_4} = \left(\frac{1}{u_{(0)}} \frac{\partial u_{(0)}}{\partial x_4} + \frac{\partial}{\partial x_4} (-\beta + i\varphi_1) \right) \vec{u}_{(0)} e^{-\beta + i\varphi_1} ,$$

it is possible, by dividing (2.48) by $e^{-\beta+i\varphi_1}$, to isolate the term in $\partial \vec{u}_{(0)}/\partial x_4$ which depends only on $\vec{x}_4$ (and on higher scales eventually); now, all the other terms in (2.48) depend both on $\vec{x}_2$ and on $\vec{x}_4$, etc. We are thus led to set $\partial \vec{u}_{(0)}/\partial x_4 = 0$, so that $\vec{u}_{(0)}$ is a constant vector determined by the wave amplitude on the frontier $x = 0$. As previously stated, it also follows that $F^{(0)}_{(0)}$ and the field $\vec{e}'_{(0)}$ are independent of $\vec{x}_4$, $\vec{e}'^0_{(0)}$ being given by (2.19) with $\vec{u}_{(0)}$ constant.

This being the case, the equation for ${}^1\vec{e}'^0_{(2)}$ is drawn from (2.48) by expressing the term in $\partial^2 \vec{e}'^0_{(0)}/\partial x_2^2$ and by computing the conductivity $\sigma_{(2)}$ from (2.40) and (A9). For an imperfectly Lorentzian plasma, one thus finds from (2.40′) and (A18)

$$\frac{\partial\, {}^1\vec{e}\,'^{0}_{(2)}}{\partial x_2}+\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\sigma(\omega_1)\,{}^1\vec{e}\,'^{0}_{(2)}+\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\omega^2\vec{u}_{(0)}e^{-\beta+i\varphi_1}\sigma^1_{(2)}({}^1\vec{e}\,'^{0}_{(2)}\cdot\vec{e}\,'^{0*}_{(0)}+\text{c.c.})$$
$$=-\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\vec{u}_{(0)}e^{-\beta+i\varphi_1}\left[\omega^4\sigma^{2\prime}_{(2)}u^4_{(0)}e^{-4\beta}+\omega^2u^2_{(0)}e^{-2\beta}(\omega^2K_0^2\sigma^{2\prime\prime}_{(2)}+\sigma_1)+\omega^2K_0^2\sigma_1'-i\left(\frac{1}{2K_0}\frac{\partial\sigma}{\partial x_2}-\frac{\omega_1}{4}\frac{\omega^2}{\omega_1^2}\sigma^2\right)\right], \tag{2.51}$$

so that the electric field perturbation of order ϵ^2, ${}^1\vec{e}\,'_{(2)}$, is determined in this case by the two coupled linear equations (2.51) and its complex conjugate, whose coefficients depend on $\vec{x}_2$ through $F^{(0)}_{(0)}$.

The other oscillating component is the first odd harmonic in $e^{3i\varphi}$; it is determined by (2.49) which can be written

$$\frac{\partial\, {}^3\vec{e}\,'^{0}_{(2)}}{\partial x_2}+\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\sigma(3\omega_1)\,{}^3\vec{e}\,'^{0}_{(2)}=-\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\omega^2\vec{u}_{(0)}u^2_{(0)}e^{-3\beta}\sigma_3e^{3i\varphi_1}, \tag{2.52}$$

where the nonlinear conductivity σ_3 is given by (2.41b). This is a linear wave equation for ${}^3\vec{e}\,'^{0}_{(2)}$ with a driving source term in $e^{3i\varphi_1}$ which, by virtue of the collisional nature of σ_3, cancels out when ν_1 and ν_2 are null or constant. It follows that this first odd harmonic is transverse and cancels out when $\nu_{1,2}=cte$ (or zero); by taking (2.20) into account and integrating (2.52) with the boundary condition ${}^3\vec{e}\,'^{0}_{(2)}=0$ for $x=0$, we get

$$ {}^3\vec{e}\,'^{0}_{(2)}(\vec{x}_2)=-\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\omega^2\vec{u}_{(0)}u^2_{(0)}\exp\left(-\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\int_0^{x_2}\sigma(3\omega_1)dx_2'\right)$$
$$\times\left[\int_0^{x_2}\sigma_3\exp\left(\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\,\omega_1\int_0^{x_2'}[\sigma(3\omega_1)-3\sigma(\omega_1)]dx_2''\right)dx_2'\right]. \tag{2.53}$$

This equation gives the variation at the space scale $\vec{x}_2$ of the amplitude of the first odd harmonic at order ϵ^2; let us remark that this variation is well determined by the expression of $F^{(0)}_{(0)}$ occurring in the conductivities $\sigma(\omega_1)$, $\sigma(3\omega_1)$, and $\sigma_3(\omega_1)$.

With regard to the first even harmonic, it is seen from (2.42) and (2.44) that it is of order ϵ^4 and of longitudinal type; it is a plasma oscillation driven by the $\vec{e}\,'_{(0)}$ wave and one has, by (2.44),

$$ {}^2\vec{e}\,'_{(4)\parallel}=i\,\frac{\omega_1}{2}\frac{\omega^2}{\omega_1^2}\,\omega^2u^2_{(0)}e^{-2\beta}\vec{K}_0(\sigma_2e^{2i(\varphi+\varphi_1)}-\text{c.c.}), \tag{2.54}$$

where the integration constant is taken equal to zero in order to have a zero field at the infinity $(x_2\to+\infty)$. The amplitude of this harmonic is seen to depend on x_2 through $\sigma_2(\omega_1)$ and the absorption factor $e^{-2\beta}$. From (2.41a) and (A8), it follows also that this harmonic is made up of three terms: one, proportional to σ_2'', has an essentially collisional character and disappears in the absence of collisions (or when $\nu_{1,2}=cte$); the two others, respectively, proportional to σ_2^L and σ_2', have a noncollisional origin, as previously discussed, but are of course altered by the presence of collisions (however, it must be pointed out that the contribution of σ_2' vanishes in the absence of collisions, with $\nu_{1,2}=0$, but remains different from zero when $\nu_{1,2}=cte$).

4. Conclusions

The previous formulas determine completely, at order ϵ^2, both the kinetic state of the electronic component and the electric field inside the plasma. They are given, respectively, by the electronic distribution function, which can be written as this approximation

$$F_e=F^{(0)}_{(0)}+\epsilon\vec{w}\cdot[\vec{F}^{(1)\prime}_{(1)}+\epsilon^2(\vec{F}^{(1)\prime}_{(3)}+\vec{F}^{(1)\prime\prime}_{(3)})]$$
$$+\epsilon^2[F^{(0)\prime}_{(2)}+F^{(0)\prime\prime}_{(2)}+(\vec{w}\vec{w})^0:\overleftrightarrow{F}^{(2)}_{(2)}], \tag{2.55}$$

and by the formula (2.50), in which all the kinetic quantities $\overleftrightarrow{F}^{(l)}_{(n)}$ and electric components $\vec{e}_{(n)}$ have been defined by general expressions in terms of the zero- and second-order stationary components of the isotropic distribution function, viz., $F^{(0)}_{(0)}(\vec{x}_2;w)$ and $F^{(0)\prime\prime}_{(2)}(\vec{x}_2;w)$. As $F^{(0)\prime\prime}_{(2)}$ expresses itself in terms of $F^{(0)}_{(0)}$ [cf. Eqs. (A9)–(A12)], it ensues that all the relevant physical quantities of such a system are well determined by the knowledge of $F^{(0)}_{(0)}$, for which one has to solve the two coupled equations (2.20a) and (2.25); for all these equations, explicit solutions have been given in the imperfectly Lorentzian case. Let us note that, in theory, this formalism could be applied at any order of approximation and would thus give higher-order contributions involving higher harmonics (the second odd harmonic appearing at order ϵ^4, the third one at order ϵ^6, etc).

It also must be emphasized that all the expressions thus obtained are closely dependent on the e-n interaction law which occurs, at order ϵ^2, through the two collision frequencies $\nu_1(v)$ and $\nu_2(v)$. It follows that the nonlinear effects involved are, in the collisional case, very sensitive to the nature of this interaction law.

As an example, let us consider the particular case of a Maxwellian interaction law (in $1/r^5$) for which the collision frequencies ν_1 and ν_2 are constant. Firstly, it is easily checked from (2.53) that the first odd harmonic cancels out for this law by virtue of the collisional nature of the nonlinear conductivity $\sigma_3(\omega_1)$ defined by (2.41b). Moreover, it can be seen by (2.14), (2.24a), and (2.24b) that one has $\sigma(\omega_1)=1/(\nu_1+i\omega_1)$, so that the "first-order" conductivity is now independent of $\vec{x}_2$; in the same way, one finds also, by (2.39), (A3), and (A4), that $\sigma_1=0$ and

$$\sigma_1' = -\frac{1}{(\nu_1+i\omega_1)^2(\nu_2+i\omega_1)}\frac{T_e'}{3T},$$

where T_e', defined by

$$\frac{T_e'}{3T}=\frac{4\pi}{3N}\int_0^\infty w^4 F_{(0)}^{(0)}\,dw, \qquad (2.56)$$

is the electronic temperature associated with the non-Maxwellian distribution $F_{(0)}^{(0)}$ and depends on the squared field amplitude $u_{(0)}^2$. Furthermore, for an imperfectly Lorentzian plasma, it can also be shown by (2.40′) and (A18) that $\sigma_{(2)}^1=\sigma_{(2)}^{2\prime}=\sigma_{(2)}^{2\prime\prime}=0$ and, by (2.14) and (2.26), that the distribution function $F_{(0)}^{(0)}$ becomes Maxwellian, with an electronic temperature $T_e'(\vec{x}_2)$ given by

$$T_e'(\vec{x}_2)=T\left[1+4\gamma'\omega^2u_{(0)}^2\exp\left(-K_0\frac{\omega^2}{\omega_1^2}\frac{\nu_1\omega_1}{\nu_1^2+\omega_1^2}x_2\right)\Big/(\nu_1^2+\omega_1^2)\right]. \qquad (2.57)$$

It thus follows that Eq. (2.51) for ${}^1\vec{e}_{(2)}'$ is considerably simplified in this case and that the only nonlinear term (in $u_{(0)}^2$) included in it proceeds from the electronic temperature through the conductivity $\sigma_1'(\omega_1)$.

It appears from these results that, as usual, the Maxwellian-interaction law gives particularly simple results: at order ϵ^2, all the nonlinear effects vanish except the thermal effect involving a term in $u_{(0)}^2\,e^{-2\beta(\vec{x}_2)}$; as regards the harmonics, they arise only at higher approximations (order ϵ^4, etc.). On the other hand, for the other types of e-n interaction, all the nonlinear effects, such as the first odd harmonic, arise from the order ϵ^2; this example allows one to show the important role played by the e-n interaction law in the nonlinear effects arising in collisional plasmas.

To conclude this section, let us remark that these results could be also obtained by using Fourier time expansions of the electronic distribution function and of the electric field. Indeed, as one has to do in the present case with a stationary system, it can be sought for the fundamental system (1.2)–(1.7) periodic solutions of the form

$$\vec{F}^{(l)}(\tau,\vec{x};w)=\sum_{-\infty}^{+\infty}\vec{F}_p^{(l)}(\vec{x};w)e^{ip(\omega_1/\omega)\tau}, \qquad (2.58)$$

$$\vec{e}(\tau,\vec{x})=\sum_{-\infty}^{+\infty}\vec{e}_p(\vec{x})e^{ip(\omega_1/\omega)\tau}, \qquad (2.59)$$

where the Fourier components $\vec{F}_p^{(l)}$ and $\vec{e}_p$ are to be determined by the coupled set of equations obtained by carrying (2.58) and (2.59) in (1.2)–(1.7). To solve these equations, one is led to apply again the multiple-space-scales method by introducing the space scales $\vec{x}_0,\vec{x}_2,\ldots$, and by using for $\vec{F}_p^{(l)}$ and $\vec{e}_p$ expansions in ϵ^2 similar to (2.1). It can thus be shown that this technique allows one to recover the results of this section,[10(a)] and that it provides a theoretical basis for previous works on the propagation of modulated waves in plasmas.[11-13]

III. CONTINUOUS WAVES IN WEAKLY COLLISIONAL MEDIA

We consider now in this section the propagation of a continuous tranverse electromagnetic wave in a weakly collisional plasma, such as $\bar{\nu}/\omega \simeq \epsilon^2$. As seen in the Introduction, we have again a weakly dissipative medium, in which $K_{0I}/K_{0R} \simeq \epsilon^2$ if no assumption is made on the order of magnitude of ω_{pe}^2/ω^2. Moreover, as previously shown,[10] such a system is in a stationary state depending only on the even space scales $\vec{x}_0,\vec{x}_2,\ldots$, where $\vec{x}_2$ is the absorption length. It thus follows that all the ϵ^2 expansions (2.1) and (2.2) for the matter and the electric field can be used again.

With these remarks, it is easy to obtain the fundamental equations of the weakly collisional case by referring to the corresponding ones in the collisional case. For the field equations, Eq. (2.3) and (2.4) have to be modified if no assumption is made on the ratio ω_{pe}^2/ω^2; in this case, the source terms are no longer of order ϵ^2, so that one has now the following sequence of equations:

$$\mathcal{L}^{(0)}(\vec{e}_{(0)}) = -\frac{\omega_{pe}^2}{\omega^2}\frac{\partial \vec{J}_{(1)}}{\partial \tau_0}, \tag{3.1a}$$

$$\mathcal{L}^{(2)}(e_{(0)}, \vec{e}_{(2)}) = -\frac{\omega_{pe}^2}{\omega^2}\frac{\partial \vec{J}_{(3)}}{\partial \tau_0}, \tag{3.1b}$$

$$\mathcal{L}^{(4)}(\vec{e}_{(0)}, \vec{e}_{(2)}, \vec{e}_{(4)}) = -\frac{\omega_{pe}^2}{\omega^2}\frac{\partial \vec{J}_{(5)}}{\partial \tau_0}, \tag{3.1c}$$

and, for the Poisson equations,

$$\rho_{(0)} = 0 \quad \text{whence } n_{(0)} = N, \tag{3.2a}$$

$$\vec{\nabla}_{x_0}\cdot\vec{e}_{(0)} = (\omega_{pe}^2/\omega^2)(n_{(2)}/N), \tag{3.2b}$$

$$\vec{\nabla}_{x_0}\cdot\vec{e}_{(2)} + \vec{\nabla}_{x_2}\cdot\vec{e}_{(0)} = (\omega_{pe}^2/\omega^2)(n_{(4)}/N), \tag{3.2c}$$

while the continuity equations (2.5) remain unchanged.

For the kinetic equations, we now have to take into account the condition $\bar{\nu}/\omega \simeq \epsilon^2$; thus, one finds again sequences of equations similar to (2.6)–(2.8), but with the collisional terms carried over to the next-order equations. For example, we have Eqs. (2.6b), (2.7a), and (2.8) with no collisional terms, while the collisional contributions to (2.6c) and (2.7b) are now $I(F_{(0)}^{\{0\}}) + C_{ee}(F_{(0)}^{\{0\}})$ and $-\nu_1'\vec{F}_{(1)}^{\{1\}}$, respectively.

In order to solve this set of equations, we now have to apply again the multiple-space-scales formalism. As the successive steps of these calculations are very similar to those of the collisional case, we merely give in the following the main results of the method with emphasis on the features peculiar to the weakly collisional case.

A. Zero-order equations

As $F_{(0)}^{\{0\}}$ is independent of τ_0 from (2.6a), the decompositions (2.9) and (2.11) can be used again, so that we still have the two equations (2.12), but without collisional terms. It follows from them two consequences: (i) as in the collisional case, one has $\vec{e}_{(0)}'' = 0$, because $F_{(0)}^{\{0\}}$ is also independent of $\vec{x}_0$ when the plasma is assumed homogeneous in the absence of the wave; (ii) one has for the source term of (3.1a)

$$\frac{\partial \vec{J}_{(1)}}{\partial \tau_0} = \vec{e}_{(0)\perp}', \tag{3.3}$$

so that $\vec{e}_{(0)\perp}'$ now satisfies the usual equation of transverse waves in cold plasmas, whose solutions can be written

$$\vec{e}_{(0)\perp}' = \vec{e}_{(0)\perp}'^{0}(\vec{x}_2)e^{i\psi} + \vec{e}_{(0)\perp}'^{0*}(\vec{x}_2)e^{-i\psi}, \tag{3.4}$$

in which ψ is defined as φ by the relation (2.10b), but with a wave vector $\vec{K}_0$ which is derived in this case from the familiar dispersion relation

$$K_0^2 = (\lambda_0^2\omega_1^2/c^2)(1-\omega_{pe}^2/\omega_1^2). \tag{3.5}$$

B. Order-ϵ^2 equations

We now have to compute the source term of the field equation (3.1b), which is drawn from a kinetic equation identical with (2.7b), but with $-\nu_1'\vec{F}_{(1)}^{\{1\}}$ in place of $-(\nu_1/\omega)\vec{F}_{(3)}^{\{1\}}$. As decompositions of the type (2.27)–(2.29) can still be used for the computation of $F_{(2)}^{\{0\}}$ and $\vec{F}_{(2)}^{\{2\}}$, this equation can be split into two parts, one which is stationary and reads

$$-\nu_1'\vec{F}_{(1)}^{\{1\}\prime\prime} = \vec{\nabla}_{x_2}F_{(0)}^{\{0\}} + \frac{\vec{e}_{(2)}''}{w}\frac{\partial F_{(0)}^{\{0\}}}{\partial w}, \tag{3.6}$$

the other which is periodic in τ_0 and allows one to compute $\vec{F}_{(3)}^{\{1\}\prime}$. As one must have $\vec{j}_{(1)}'' = 0$ on account of the continuity equation (2.5b), one derives from (3.6) the expression of the stationary longitudinal field in terms of $\vec{\nabla}_{x_2}F_{(0)}^{\{0\}}$:

$$\vec{e}_{(2)}''\left(\int_0^\infty \frac{w^3}{\nu_1'}\frac{\partial F_{(0)}^{\{0\}}}{\partial w}dw\right) = -\vec{\nabla}_{x_2}\int_0^\infty \frac{w^4}{\nu_1'}F_{(0)}^{\{0\}}dw\,. \tag{3.7}$$

On the other hand, by computing $\vec{F}_{(3)}^{\{1\}\prime}$ and taking into account the condition $n_{(2)} = 0$ drawn from (3.2b) (transversality of $\vec{e}_{(0)}$), one finds for the source term of (3.1b)

$$\frac{\partial \vec{J}_{(3)}}{\partial \tau_0} = \vec{e}_{(2)}' + i\frac{\omega}{\omega_1}\bar{\nu}_1'(\vec{e}_{(0)}'^{0}e^{i\psi} - \text{c.c.}) + \frac{\omega^2}{\omega_1^2}\frac{K_0^2}{3}\frac{T_e'}{T}\vec{e}_{(0)\perp}' - i\frac{\omega^2}{\omega_1^2}\vec{K}_0(\vec{e}_{(0)\perp}'^{02}e^{2i\psi} - \text{c.c}) \tag{3.8}$$

in which T_e' is the electronic temperature defined by (2.56) and $\bar{\nu}_1'$ is given by

$$\bar{\nu}_1' = -\frac{4\pi}{3N}\int_0^\infty \nu_1' w^3 \frac{\partial F_{(0)}^{\{0\}}}{\partial w}dw, \tag{3.9}$$

and where the longitudinal term in $e^{2i\psi}$ proceeds from the Lorentz force and corresponds to the contribution σ_2^L in the collisional case [cf. (2.41a)]. It is seen that there is no term in $e^{3i\psi}$ in (3.8), because the first odd harmonic contributions are of collisional nature and occur only in the order ϵ^4 equations in the weakly collisional case. It follows that, although of order ϵ^2, the first odd harmonic of the electric field is determined only by the order ϵ^4 field equation, as in the collisional case.

As the source term (3.8) includes both a longitudinal driving term in $e^{2i\psi}$ and a transverse one in $e^{i\psi}$, the field equation (3.1b) can be split into two equations, one for the longitudinal component $\vec{e}_{(2)\parallel}'$, the other for the transverse component $\vec{e}_{(2)\perp}'$. The former one gives the expression of the first even harmonic $\vec{e}_{(2)\parallel}'$ which is of order ϵ^2 now:

$$\vec{e}'_{(2)\parallel} = i\,\frac{\omega_{pe}^2}{\omega_{pe}^2 - 4\omega_1^2}\,\frac{\omega^2}{\omega_1^2}\,\vec{K}_0(\vec{e}'^{02}_{(0)\perp} e^{2i\psi} - \vec{e}'^{0*2}_{(0)\perp} e^{-2i\psi})\,. \tag{3.10}$$

The latter one is a linear equation in $\vec{e}'_{(2)\perp}$ with a resonating right-hand member in $e^{i\psi}$, and it gets also two equations by applying the multiple-space-scales formalism. The first one shows that $\vec{e}'_{(2)\perp}$ satifies to the usual equation of tranverse waves in cold plasmas, so that it is of the form

$$\vec{e}'_{(2)\perp} = \sum_m ({}^m\vec{e}'^{0}_{(2)\perp}(\vec{x}_2)\, e^{im\psi} + {}^m\vec{e}'^{0*}_{(2)\perp}(\vec{x}_2)\, e^{-im\psi}) \tag{3.11}$$

in which the ${}^m\vec{e}'^{0}_{(2)\perp}(\vec{x}_2)$ are to be determined by the higher-order field equations; the second one allows one to calculate the variation at space scale $\vec{x}_2$ of $\vec{e}'^{0}_{(0)\perp}$ which can be written

$$\vec{e}'^{0}_{(0)\perp}(\vec{x}_2) = \vec{u}_{(0)\perp}\, e^{-\beta(\vec{x}_2) + i\psi_1(\vec{x}_2)}\,, \tag{3.12}$$

in which $\vec{u}_{(0)\perp}$ is determined by the wave amplitude on the frontier and where the absorption coefficient $\beta(\vec{x}_2)$ and the phase shift $\psi_1(\vec{x}_2)$ are defined by

$$\beta(\vec{x}_2) = \frac{K_0}{2}\,\frac{\omega_{pe}^2}{\omega_1^2 - \omega_{pe}^2}\,\frac{\omega}{\omega_1}\int^{x_2} \bar{\nu}'_1\, dx'_2\,, \tag{3.13a}$$

$$\psi_1(\vec{x}_2) = \frac{K_0}{2}\,\frac{\omega_{pe}^2}{\omega_1^2 - \omega_{pe}^2}\,\frac{\omega^2}{\omega_1^2}\,\frac{K_0^2}{3}\int^{x_2} \frac{T'_e}{T}\, dx'_2\,, \tag{3.13b}$$

$\bar{\nu}'_1$ and T'_e being functions of $\vec{x}_2$ through $F^{\{0\}}_{\{0\}}$. (Let us note that we have omitted any dependence on the higher variables $\vec{x}_4, \ldots$, because it can be checked with the higher-order equations that, as in the collisional case, the physical state of such a system is in fact independent of these higher space scales.) The phase shift ψ_1 allows one to determine the order-ϵ^2 correction to the dispersion equation (3.5) of transverse waves in cold plasmas; by (3.13b) and the condition $\eta' = \epsilon^2$, one thus finds for the pertubed wave vector K'_0:

$$K'_0 = K_0\left(1 - \frac{\omega_{pe}^2}{2\omega_1^2}\,\frac{\bar{v}^2}{3c^2}\,\Theta(x_0)\right), \tag{3.14}$$

with $\Theta(x_0) = (1/x_0)\int^{x_0} (T'_e/T)\,dx'_0$. This is an effect due to the finite temperature of the medium, whose existence has been also proved by the kinetic theory of wave propagation in warm plasmas[14]; in the present case, the formula (3.14) takes also into account the inhomogeneity of the plasma induced by the continuous wave.

In conclusion, the first two orders equations allow one, as in the collisional case, to determine the electric field at zero order and to provide the anaytical form of the order-ϵ^2 contributions. However, it has been seen that the longitudinal first even harmonic $\vec{e}'_{(2)\parallel}$ is deduced from (3.1b) and that it is of order ϵ^2 now, contrary to the collisional case; of course, this is due to the fact that no assumption has been made on the ratio ω_{pe}^2/ω^2, so that the order-ϵ^2 field equation has a longitudinal driving term in $e^{2i\psi}$. Moreover, another important difference with the collisional case arises from the fact that the $F^{\{0\}}_{\{0\}}$ equation, and thus the $\vec{x}_2$ dependence of $\vec{e}'^{0}_{(0)}$, can be derived only from the order-ϵ^4 equations, because the collisional term $I(F^{\{0\}}_{\{0\}}) + C_{ee}(F^{\{0\}}_{\{0\}})$ is now carried over into (2.6c).

C. Order-ϵ^4 equations

We first consider the equation corresponding to (2.6c), in which we still have to separate the stationary and oscillating terms. We thus obtain an equation for the oscillating component $F^{\{0\}\prime}_{\{4\}}(\tau_0\vec{x}_0, \ldots)$ and the equation for $F^{\{0\}}_{\{0\}}(\vec{x}_2; w)$. By computing from (2.7b), (3.4), and (3.11) the stationary contributions of $\vec{e}'_{(0)} \cdot \vec{F}^{\{1\}\prime}_{\{3\}} + \vec{e}'_{(2)} \cdot \vec{F}^{\{1\}\prime}_{\{1\}}$, it is seen that all the terms cancel out except the collisional one, so that one obtains finally

$$I(F^{\{0\}}_{\{0\}}) + C_{ee}(F^{\{0\}}_{\{0\}}) = \gamma'\,\frac{w^2}{3}\,\vec{\nabla}_{x_2} \cdot \vec{F}^{\{1\}\prime\prime}_{\{1\}} + \frac{\gamma'}{3w^2}\,\frac{\partial}{\partial w}\times\left(w^3\,\vec{e}''_{(2)} \cdot \vec{F}^{\{1\}\prime\prime}_{\{1\}} - \frac{2\omega^2 u_{(0)}^2\, e^{-2\beta}}{\omega_1^2}\, w^2 \nu'_1\, \frac{\partial F^{\{0\}}_{\{0\}}}{\partial w}\right), \tag{3.15}$$

in which $\beta(\vec{x}_2)$ is defined by (3.13a) and (3.9), and where $\vec{F}^{\{1\}\prime\prime}_{\{1\}}$ and $\vec{e}''_{(2)}$ are given by (3.6) and (3.7); moreover, the normalization condition (2.24b) still holds, by virtue of (3.2a). Equation (3.15), coupled to (3.13a), allows one to determine $F^{\{0\}}_{\{0\}}$ for which we keep only solutions independent of $\vec{x}_0$ for the same reasons as in the collisional case; it is seen that, in the present case, $F^{\{0\}}_{\{0\}}$ depends on the space variable $\vec{x}_2$ not only through the absorption $\beta(\vec{x}_2)$ but also through the longitudinal stationary field $\vec{e}''_{(2)}$ from (3.6) and (3.7). The knowledge of $F^{\{0\}}_{\{0\}}(\vec{x}_2; w)$ is needed to determine the medium properties at space scale $\vec{x}_2$, and particularly the absorption $\beta(\vec{x}_2)$ and the phase-shift $\psi_1(\vec{x}_2)$ of the zero-order electric field as well as the order-ϵ^2 stationary field $\vec{e}''_{(2)}$; as in the collisional case, the plasma inhomogeneity is due to a collisional heating by the continuous wave $\vec{e}'_{(0)\perp}$.

We now consider the ϵ^4 order field equation (3.1c), in which the source term $\partial\vec{J}_{(5)}/\partial\tau_0$ must be derived from the equation relative to $\vec{F}^{\{1\}}_{\{5\}}$. According to the expansions (2.1a) and (2.1b), one finds

$$\frac{\partial \vec{F}^{(1)}_{(5)}}{\partial \tau_0} = -\nu_1' \vec{F}^{(1)}_{(3)} - (\vec{\nabla}_{x_0} F^{(0)}_{(4)} + \vec{\nabla}_{x_2} F^{(0)}_{(2)} + \vec{\nabla}_{x_4} F^{(0)}_{(0)}) - \left(\frac{\vec{e}'_{(0)}}{w}\frac{\partial F^{(0)}_{(4)}}{\partial w} + \frac{\vec{e}_{(2)}}{w}\frac{\partial F^{(0)}_{(2)}}{\partial w} + \frac{\vec{e}_{(4)}}{w}\frac{\partial F^{(0)}_{(0)}}{\partial w}\right)$$
$$- \frac{2}{5}\left(w^2(\vec{\nabla}_{x_0}\cdot \vec{F}^{(2)}_{(4)} + \vec{\nabla}_{x_2}\cdot \vec{F}^{(2)}_{(2)}) + \frac{1}{w^4}\frac{\partial}{\partial w}[w^5(\vec{e}'_{(0)}\cdot \vec{F}^{(2)}_{(4)} + \vec{e}_{(2)}\cdot \vec{F}^{(2)}_{(2)})]\right)$$
$$+ \left(\int^{\tau_0}(\vec{\nabla}_{x_0}\times\vec{e}'_{(2)\perp} + \vec{\nabla}_{x_2}\times\vec{e}'_{(0)\perp})d\tau_0'\right)\times \vec{F}^{(1)}_{(1)} + \left(\int^{\tau_0}(\vec{\nabla}_{x_0}\times\vec{e}'_{(0)\perp})d\tau_0'\right)\times\vec{F}^{(1)}_{(3)}, \tag{3.16}$$

where $\vec{F}^{(2)}_{(4)}$ is determined by an equation which is deduced from (1.3) and includes the collisional term $-\nu_2' \vec{F}^{(2)}_{(2)}$. Taking (3.2) into account, the computation of $\partial\vec{J}_{(5)}/\partial\tau_0$ gives two kinds of driving source terms: (i) the longitudinal ones which involve the even harmonics $e^{2i\psi}$ and $e^{4i\psi}$; (ii) the transverse ones in which occur various contributions in $e^{i\psi}$ and $e^{3i\psi}$. So, it is seen that the field equation (3.1c) splits into two equations, one for the oscillating longitudinal component $\vec{e}'_{(4)\parallel}$ which includes even contributions in $e^{2i\psi}$ and $e^{4i\psi}$, the other for the oscillating tranverse field $\vec{e}'_{(4)\perp}$ in which occur resonating terms in $e^{i\psi}$ and $e^{3i\psi}$. By canceling these secularities, one thus obtains two equations for determining the order-ϵ^2 transverse field $\vec{e}'_{(2)\perp}$ which is the sum of two terms similar to (3.11) with $m=1$ and $m=3$.

By putting, as in (2.43), $\vec{e}'_{(2)\perp} = {}^1\vec{e}'_{(2)\perp} + {}^3\vec{e}'_{(2)\perp}$, one then derives from (3.1c) and (3.16) two equations for ${}^1\vec{e}'_{(2)\perp}$ and ${}^3\vec{e}'_{(2)\perp}$, similar to (2.48) and (2.49). As in the collisional case, the equation relative to ${}^1\vec{e}'_{(2)\perp}$ allows to check that $\partial u_{(0)}/\partial x_4 = 0$ and, therefore, that the electric field and the plasma state are independent of $\vec{x}_4$ and other higher-order space scales.

As to the equation relative to ${}^3\vec{e}'_{(2)\perp}$, it is derived from

$$2\frac{c^2}{\lambda_0^2}\frac{\partial}{\partial x_2}\frac{\partial\, {}^3\vec{e}_{(2)\perp}}{\partial x_0} = -6i\frac{c^2K_0}{\lambda_0^2}\left(\frac{\partial\, {}^3\vec{e}'^{0}_{(2)\perp}}{\partial x_2}e^{3i\psi} - \frac{\partial\, {}^3\vec{e}'^{0*}_{(2)\perp}}{\partial x_2}e^{-3i\psi}\right) = \omega_{pe}^2\frac{\partial\, {}^3\vec{J}_{(5)}}{\partial\tau_0}, \tag{3.17}$$

in which (3.11) has been taken into account and where $\partial\, {}^3\vec{J}_{(5)}/\partial\tau_0$ is to be drawn from (3.16) and from the order-ϵ^2 quantities. After tedious calculations, one finally obtains

$$\frac{\partial\, {}^3\vec{e}'^{0}_{(2)\perp}}{\partial x_2} + \left(\frac{K_0}{2}\frac{\omega_{pe}^2}{\omega_1^2-\omega_{pe}^2}\frac{\bar{\nu}_1'\omega}{9\omega_1} - i\frac{K_0}{2}\frac{\omega^2}{\omega_1^2}\frac{\omega_{pe}^2}{\omega_1^2-\omega_{pe}^2}\frac{K_0^2}{3}\left(\frac{T_e'}{3T}\right)\right){}^3\vec{e}'^{0}_{(2)\perp}$$
$$= \left[\frac{K_0}{6}\frac{\omega_{pe}^2}{\omega_1^2-\omega_{pe}^2}\frac{\omega^3}{10\omega_1^3}\left\langle\frac{1}{w^4}\frac{\partial}{\partial w}\left(w^4\frac{\partial\nu_1'}{\partial w}\right)\right\rangle + iK_0\frac{\omega_{pe}^2}{\omega_1^2-\omega_{pe}^2}\frac{\omega^4K_0^2}{\omega_1^2(\omega_{pe}^2-4\omega_1^2)}\right]\vec{u}_{(0)}u_{(0)}^2 e^{-3\theta}e^{3i\psi} \tag{3.18}$$

in which has been introduced the mean collisional quantity

$$\left\langle\frac{1}{w^4}\frac{\partial}{\partial w}\left(w^4\frac{\partial\nu_1'}{\partial w}\right)\right\rangle$$

already used in a previous work[15] and defined by

$$\left\langle\frac{1}{w^4}\frac{\partial}{\partial w}\left(w^4\frac{\partial\nu_1'}{\partial w}\right)\right\rangle = -\frac{4\pi}{3N}\int_0^\infty\frac{1}{w}\frac{\partial}{\partial w}\left(w^4\frac{\partial\nu_1'}{\partial w}\right)\frac{\partial F^{(0)}_{(0)}}{\partial w}dw. \tag{3.19}$$

The equation (3.18) is the weakly collisional version of (2.52); it allows one to determine the amplitude variation at space scale $\vec{x}_2$ of the first odd harmonic. Note that its driving term includes a noncollisional contribution, owing to the fact that the source term of the field equation is now given by $\partial\vec{J}_{(5)}/\partial\tau_0$ in place of $\partial\vec{J}_{(3)}/\partial\tau_0$ in the collisional case; in this latter case, the corresponding terms would occur in the order-ϵ^6 field equation. It can also be seen that all the other terms of (3.18) exactly correspond to those of (2.52), if we take into account the smallness of the ratio $\bar{\nu}/\omega$; especially, this is the case of the collisional driving contribution (3.19) which can be derived from the nonlinear conductivity σ_3 of the collisional case. It then follows that the first odd (transverse) harmonic is, as in the collisional case, well determined by the knowledge of $F^{(0)}_{(0)}$, which occurs through $\bar{\nu}_1'$, T_e', and

$$\left\langle\frac{1}{w^4}\frac{\partial}{\partial w}\left(w^4\frac{\partial\nu_1'}{\partial w}\right)\right\rangle,$$

but that it does *not* vanish when $\nu_1 = cte$.

In conclusion, the previous results allow one to determine in the weakly collisional case the electric field in the plasma and the electronic distribution function at order ϵ^2, which are, respectively,

$$\vec{e} = [(\vec{e}'^{0}_{(0)\perp} + \epsilon^2\, {}^1\vec{e}'^{0}_{(2)\perp})e^{i\psi} + \text{c.c.}] + \epsilon^2({}^2\vec{e}'^{0}_{(2)\parallel}e^{2i\psi} + \text{c.c.})$$
$$+ \epsilon^2({}^3\vec{e}'^{0}_{(2)\perp}e^{3i\psi} + \text{c.c.}) + \epsilon^2\vec{e}''_{(2)\parallel}, \tag{3.20}$$

$$F_e = F^{(0)}_{(0)} + \epsilon\vec{w}\cdot[\vec{F}^{(1)\prime}_{(1)} + \vec{F}^{(1)\prime\prime}_{(1)} + \epsilon^2(\vec{F}^{(1)\prime}_{(3)} + \vec{F}^{(1)\prime\prime}_{(3)})]$$
$$+ \epsilon^2[F^{(0)\prime}_{(2)} + F^{(0)\prime\prime}_{(2)} + (\vec{w}\vec{w})^0 : \vec{\vec{F}}^{(2)}_{(2)}]. \tag{3.21}$$

By comparing (3.20) and (3.21) with the corresponding formulas (2.50) and (2.55) of the collisional case, it is seen that one has now two supplementary contributions, $\epsilon^2\vec{e}'_{(2)\parallel}$ for the electric field and $\epsilon\vec{F}^{(1)''}_{(1)}$ for the distribution function. As shown in the calculations, the occurrence of these two terms results from the two main differences between the present weakly collisional case and the collisional one, viz., (i) no assumption has been made on the ratio ω_{pe}^2/ω^2 and (ii) in the collisional terms, the factor $\overline{\nu}/\omega$ is now of order ϵ^2. Let us emphasize also another important consequence of this latter condition: to determine completely the expressions (3.20) and (3.21), one has to use higher-order equations for calculating $F^{(0)''}_{(2)}$ and $\vec{e}'_{(2)\perp}$, since they are deduced, respectively, from the order-ϵ^6 equation relative to $F^{(0)}_{(6)}$ and from the order-ϵ^4 equation relative to $\vec{F}^{(1)}_{(5)}$. This being the case, let us note again that this formalism could be applied, in theory, to any order of approximation.

D. Case $\omega_{pe}^2/\omega^2 \simeq \epsilon^2$ ($\simeq\delta$)

If such a condition is fulfilled, one has to deal with a dissipative medium in which $K_{0I}/K_{0R} \simeq (\omega_{pe}^2/\omega^2)(\overline{\nu}/\omega) \simeq \epsilon^4$, so that the absorption takes place now at the space scale $\vec{x}_4$. For the stationary case, the state of the system depends only on the even space scales $\vec{x}_0, \vec{x}_2, \vec{x}_4 \ldots$, and the ϵ^2 expansions (2.1a) and (2.1b) can be used again.

In this case, the essential results of the multiple-space-scales formalism can be easily forecast owing to the following remarks: (i) the kinetic equations of the weakly collisional case still hold; (ii) the field equations are now those of the collisional case, viz., (2.3a)–(2.3c); (iii) the significant space scale being now $\vec{x}_4$, it can be expected that $F^{(0)}_{(0)}$ will depend on $\vec{x}_4$ (instead of $\vec{x}_2$) and, therefore, that $\vec{x}_4$ is the characteristic length of the medium properties. As one is led owing to these remarks to calculations very similar to those of the previous sections, we only give in the following the main results of the method.

Firstly, it is seen from the zero-order field equation (2.3a) that the electric field $\vec{e}'_{(0)\perp}$ is of the form

$$\vec{e}'_{(0)\perp} = \vec{e}'^{0}_{(0)}(\vec{x}_2, \vec{x}_4)e^{i\varphi} + \text{c.c.} , \tag{3.22}$$

in which φ is defined by (2.10b) and where the $\vec{x}_2$ dependence of $\vec{e}'^{0}_{(0)}$ can be derived from the order-ϵ^2 field equation (2.3b); one thus finds:

$$\vec{e}'^{0}_{(0)}(\vec{x}_2, \vec{x}_4) = \vec{e}'^{0,0}_{(0)}(\vec{x}_4) \exp\left(i\frac{K_0\omega^2}{2\omega_1^2}x_2\right) = \vec{e}'^{0,0}_{(0)}(\vec{x}_4) \exp\left(i\frac{K_0\omega_{pe}^2}{2\omega_1^2}x_0\right) , \tag{3.23}$$

where $\vec{e}'^{0,0}_{(0)}(\vec{x}_4)$ is to be determined by the order-ϵ^4 field equation. Let us remark that the last term of (3.23) is obtained by taking into account the condition $\omega_{pe}^2/\omega^2 = \epsilon^2$; by (2.10b) and (3.22), the wave phase at order ϵ^2 is thus

$$\varphi + \frac{K_0\omega_{pe}^2}{2\omega_1^2}x_0 = \frac{\omega_1}{\omega}\tau_0 - \vec{K}_0\left(1 - \frac{\omega_{pe}^2}{2\omega_1^2}\right)\cdot\vec{x}_0 + \varphi_0 ,$$

in which the factor of $\vec{K}_0$ is nothing other than the two first terms of the expansion of $(1-\omega_{pe}^2/\omega_1^2)^{1/2}$ for small ω_{pe}^2/ω_1^2.

The $\vec{x}_4$ dependence of $\vec{e}'^{0,0}_{(0)}(\vec{x}_4)$ is then derived from the field equation (2.3c) and from the knowledge of $F^{(0)}_{(0)}$ which is obtained from an equation of the type (3.15). But it is easily seen, by (3.23), that this equation does not depend explicitly on $\vec{x}_2$ and that the space variable occurs only at the scale $\vec{x}_4$ through $|\vec{e}'^{0,0}_{(0)}|^2$. Therefore, one must keep only the solutions of (3.15), $F^{(0)}_{(0)}(\vec{x}_4; w)$, which are independent of $\vec{x}_2$; by (3.6) and (3.7), one thus has $\vec{e}''_{(2)} = \vec{F}^{(1)''}_{(1)} = 0$, so that there is no longitudinal stationary field at order ϵ^2. With these conditions, the $F^{(0)}_{(0)}$ equation (3.15) becomes

$$I(F^{(0)}_{(0)}) + C_{ee}F^{(0)}_{(0)} = -\frac{2\gamma'\omega^2|\vec{e}'^{0,0}_{(0)}(\vec{x}_4)|^2}{3\omega_1^2 w^2} \times \frac{\partial}{\partial w}\left(w^2\nu_1'\frac{\partial F^{(0)}_{(0)}}{\partial w}\right) , \tag{3.24}$$

whose solution is, for an imperfectly Lorentzian plasma, a Maxwellian distribution with an electronic temperature $T_e'(\vec{x}_4)$ given by

$$T_e'(\vec{x}_4) = T\left(1 + 4\gamma'\frac{\omega^2}{\omega_1^2}|\vec{e}'^{0,0}_{(0)}(\vec{x}_4)|^2\right) . \tag{3.25}$$

On the other hand, Eq. (2.3c) allows one to obtain for $\vec{e}'^{0,0}_{(0)}(\vec{x}_4)$

$$\vec{e}'^{0,0}_{(0)}(\vec{x}_4) = \vec{u}_{(0)\perp}\exp[-\beta(\vec{x}_4) + i\varphi_1(\vec{x}_4)] , \tag{3.26}$$

where, as previously, $\vec{u}_{(0)\perp}$ is determined by the wave on the plane $x=0$ and where the absorption coefficient $\beta(\vec{x}_4)$ and the phase shift $\varphi_1(\vec{x}_4)$ are given by

$$\beta(\vec{x}_4) = \frac{K_0}{2}\frac{\omega^3}{\omega_1^3}\int^{x_4}\overline{\nu_1'}\,dx_4' = \frac{K_0}{2}\frac{\omega_{pe}^2}{\omega_1^2}\frac{\overline{\nu}}{\omega_1}\int^{x_0}\overline{\nu_1'}\,dx_0' , \tag{3.27}$$

$$\varphi_1(\vec{x}_4) = \frac{K_0\omega^4}{8\omega_1^4}x_4 + \frac{K_0\omega^4}{2\omega_1^4}\frac{K_0^2}{3}\int^{x_4}\frac{T_e'}{T}\,dx_4' = K_0\frac{\omega_{pe}^4}{8\omega_1^4}x_0 + \frac{K_0}{2}\frac{\omega_{pe}^2}{\omega_1^2}\frac{\overline{v}^2}{3c^2}\int^{x_0}\frac{T_e'}{T}\,dx_0' , \tag{3.28}$$

in which $\overline{\nu_1'}(\vec{x}_4)$ is defined by (3.9). Let us note that, by (3.28), the phase shift $\varphi_1(\vec{x}_4)$ includes two contributions, one which is the third term of the expansion of $(1-\omega_{pe}^2/\omega_1^2)^{1/2}$ and the other which is similar to (3.13b) and corresponds to the effect of the inhomogeneity of the electronic temperature induced by the wave.

Equation (2.3c) allows also one to determine the longitudinal first even harmonic, ${}^2\vec{e}'_{(4)\parallel}$, which is now of order ϵ^4 and is given by

$$^2\vec{e}'_{(4)\parallel}=-i\frac{\omega^4}{4\omega_1^4}\vec{K}_0(\vec{e}'^{02}_{(0)\perp}e^{2i\varphi}-\vec{e}'^{0*2}_{(0)\perp}e^{-2i\varphi}) . \quad (3.29)$$

Moreover, the field equations (2.3b) and (2.3c) allow to show that the order-ϵ^2 transverse field $\vec{e}'_{(2)\perp}$ is of the form

$$\vec{e}'_{(2)\perp}=\sum_m[{}^m\vec{e}'^0_{(2)\perp}(\vec{x}_2,\vec{x}_4)e^{im\varphi}+{}^m\vec{e}'^{0*}_{(2)\perp}(\vec{x}_2,\vec{x}_4)e^{-im\varphi}] \quad (3.30)$$

with

$$^m\vec{e}'^0_{(2)\perp}(\vec{x}_2,\vec{x}_4)={}^m\vec{e}'^{0,0}_{(2)\perp}(\vec{x}_4)\exp\left(i\frac{K_0\omega^2}{2m\omega_1^2}x_2\right) . \quad (3.31)$$

Owing to these results, the first odd harmonic and the second even harmonic are then derived from the order-ϵ^6 field equation:

$$\mathcal{L}^{(6)}(\vec{e}'_{(0)},\vec{e}'_{(2)},\vec{e}'_{(4)},\vec{e}'_{(6)})\equiv\frac{\partial^2\vec{e}'_{(6)}}{\partial\tau_0^2}-\frac{c^2}{\lambda_0^2\omega^2}\frac{\partial^2\vec{e}'_{(6)\perp}}{\partial x_0^2}-\frac{c^2}{\lambda_0^2\omega^2}\left(2\frac{\partial}{\partial x_2}\frac{\partial\vec{e}'_{(4)\perp}}{\partial x_0}+2\frac{\partial}{\partial x_4}\frac{\partial\vec{e}'_{(2)\perp}}{\partial x_0}+\frac{\partial^2\vec{e}'_{(2)\perp}}{\partial x_2^2}+2\frac{\partial}{\partial x_4}\frac{\partial\vec{e}'_{(0)\perp}}{\partial x_2}+2\frac{\partial}{\partial x_6}\frac{\partial\vec{e}'_{(0)\perp}}{\partial x_0}\right)=-\frac{\partial\vec{J}'_{(5)}}{\partial\tau_0} , \quad (3.32)$$

in which $\partial\vec{J}'_{(5)}/\partial\tau_0$ is to be derived from (3.16). One is thus led to results similar to those of the previous paragraph (where ω_{pe}^2/ω^2 was not assumed small), but in which the role of $\vec{x}_2$ is now played by $\vec{x}_4$. Especially, one gets the variation at space scale $\vec{x}_4$ of the first odd harmonic,

$$^3\vec{e}'^{0,0}_{(2)\perp}(\vec{x}_4)\exp\left(3i\varphi+i\frac{K_0\omega^2}{6\omega_1^2}x_2\right) ,$$

which is transverse and of order ϵ^2 as previously; likewise, one also obtains (i) the expression of the second even harmonic, ${}^4\vec{e}'^0_{(6)\parallel}e^{4i\varphi}$, which is longitudinal and of order ϵ^6, (ii) the order-ϵ^6 contribution, ${}^2\vec{e}'^0_{(6)}e^{2i\varphi}$, to the first even harmonic, and (iii) two equations which determine, respectively, the $\vec{x}_2$ and $\vec{x}_4$ dependence of the two transverse components $\vec{e}'_{(4)\perp}$ and ${}^1\vec{e}'^0_{(2)}(\vec{x}_4)$.

IV. CONCLUSIONS AND OTHER POSSIBLE APPLICATIONS

The previous methods provide a coherent scheme of approximations to solve completely the equations describing the nonlinear propagation of continuous transverse waves in weakly dissipative warm plasmas. Exact kinetic expressions are thus obtained up to order ϵ^2 for the electronic distribution function and for the electric field inside the plasma, in the collisional and weakly collisional cases.

The essential point is that, in any case, the zero-order distribution function $F^{(0)}_{(0)}$, which occurs in all these expressions, can be calculated by means of two coupled equations; these ones allow to determine, for any type of e-n interaction law, the space variation of the kinetic state of the medium and of the various components of the electric field. It is thus shown that the medium, assumed homogeneous in the absence of waves, becomes inhomogeneous at the zero-order approximation, and that this inhomogeneity is linked with a thermoeffect produced by the waves traveling in the plasma. The knowledge of $F^{(0)}_{(0)}$ allows one to give a kinetic description of the role played by the collisions in the nonlinear behavior of the medium; especially, the order of magnitude of each harmonic can thus be determined, as well as the space scale of its amplitude variation for which kinetic formulas have been obtained.

Although the previous results have been set up in the case $\alpha'\simeq\eta'\simeq\delta$, with $\bar{\nu}/\omega\simeq 1$ or δ, these perturbation techniques can be applied to other physical situations for which one has to define different ordering schemes of the significant parameters. Let us discuss briefly some of them.

(a) Firstly, the previous methods allow one to study the so-called "intermediary case," in which $\bar{\nu}/\omega\simeq\delta^{1/2}=\epsilon$, while the ordering of the other parameters remains unchanged. As the wave absorption is then of order ϵ, the charateristic space scale is $\vec{x}_1$; so, one must introduce the odd space scales $\vec{x}_1,\vec{x}_3,\ldots$, and replace the ϵ^2 expansions (2.1) by the expansions in ϵ defined in I by (5.1) and (5.2). These taken into account, the calculations are similar to those of Sec. III. Their results have been given in detail in Ref. 10(a). Let us note only that one thus finds, for the absorption $\beta(\vec{x}_1)$ of the zero-order field and for the $F^{(0)}_{(0)}$ equation, expressions which are identical, respectively, to (3.13a) and (3.15), but which depend on the space variable $\vec{x}_1$ in place of $\vec{x}_2$.

(b) As a particular case of the previous results, one can also consider physical situations such as $\alpha' \simeq \eta' \ll \delta$, which occur when weak fields are propagating in rather cold plasmas. In this case, the collisional term of the $F^{(0)}$ equation (of order δ) is dominant compared to the nonlinear heating term (of order α'), so that the significant small parameter is then $\gamma' = \alpha'/\delta \ll 1$. It follows that $F_{(0)}^{(0)}$ is a Maxwellian distribution and that one is dealing with a system weakly perturbed around the equilibrium state; it can then easily be checked that there is no need to introduce explicitly the various space scales, because the zero-order electric field does not occur in the determination of $F_{(0)}^{(0)}$.

(c) On the other hand, our methods can also be adapted to the inverse situations where $\alpha' \gg \delta$. In this case, it is the nonlinear heating term due to the field which becomes dominant compared to the collisional contribution. It is then easily seen that, if a stationary state is reached, the thermal velocity $\bar{v}$ is no longer the good characteristic velocity, but that the latter must now be defined in terms of the field amplitude by setting $\bar{v}_\Gamma = \Gamma_0/\omega\delta^{1/2}$. One is thus led to introduce the reduced velocity $\vec{w}' = \vec{v}/\bar{v}_\Gamma = \vec{w}/\gamma'^{1/2}$, with $\gamma' \gg 1$, so that the parameters $\alpha'^{1/2}$ and $\eta'^{1/2}$ have to be replaced, respectively, by $\alpha_\Gamma'^{1/2} = \Gamma_0/\bar{v}_\Gamma\omega = \delta^{1/2}$ and $\eta_\Gamma'^{1/2} = \bar{v}_\Gamma/\lambda_0\omega = \eta'^{1/2}\bar{v}_\Gamma/\bar{v} = \eta'^{1/2}\gamma'^{1/2}$. Then, with this new reduced set of variables, our general perturbation techniques can again be applied for various ordering schemes of the parameters γ', η_Γ', and $\alpha_\Gamma' = \delta$. For example, one can consider physical situations such that $\eta_\Gamma' \simeq \alpha_\Gamma' = \delta$, with $\gamma' \simeq \delta^{-1}$, which correspond to strong electric fields (satisfying to $\alpha' \simeq 1$) propagating into rather cold plasmas (such as $\eta' \simeq \delta^2$); in this case, one is led to use again expansions in $\epsilon^2 \simeq \delta(=\alpha_\Gamma')$, similar to those of (2.1), which get for the $F_{(0)}^{(0)}$ distribution function an expression of the Druyvestein type.

Finally, as it has been already pointed out,[9,10,16(a)] these techniques can still be applied to study (i) the nonlinear interaction of several transverse waves in weakly dissipative plasmas and, particularly, the generation of the various harmonics and frequency mixing terms; (ii) the nonlinear distortion of a frequency or amplitude-modulated wave propagating in such a medium. We end this paper with a short outline of the method for an amplitude modulated electric field.

Let us consider a collisional plasma in the sense of Sec. II and let us assume that the field-amplitude on the plane $x=0$ is of the form

$$E(t) = E_0\{[1+\tfrac{1}{2}\mu_0(e^{i\Omega t}+e^{-i\Omega t})]e^{i\omega_1 t} + \text{c.c.}\} ,$$

with $\mu_0 \ll 1$ and $\Omega/\omega_1 \ll 1$. Owing to this latter condition, the periodic phenomenon is now characterized by the two time scales $t_\omega \simeq 1/\omega$ and $T_\Omega \simeq 1/\Omega$, with $t_\omega/T_\Omega \ll 1$, so that the stationary state of the system depends in turn on these two time scales. According to our perturbation technique, one is thus led to include this new parameter in the ordering scheme; if, for example, $t_\omega/T_\Omega \simeq \delta(=\epsilon^2)$, the modulation takes place at the time scale τ_2. In this case, all the zero-order quantities of Sec. II become functions of τ_2; then, it is easily seen that the zero-order distribution function $F_{(0)}^{(0)}(\tau_2, \vec{x}_2; w)$ and the amplitude $\vec{e}_{(0)}'^{0}(\tau_2, \vec{x}_2)$ of the zero-order electric field $\vec{e}_{(0)}'$ [of the form (2.10a)] are determined by the two coupled equations

$$\frac{\partial F_{(0)}^{(0)}}{\partial \tau_2} = I(F_{(0)}^{(0)}) + C_{ee}(F_{(0)}^{(0)}) - \frac{2\gamma'\omega^2|\vec{e}_{(0)}'^{0}|^2}{3w^2} \times \frac{\partial}{\partial w}\left(\frac{w^2\nu_1'}{\nu_1^2+\omega_1^2}\frac{\partial F_{(0)}^{(0)}}{\partial w}\right), \tag{4.1}$$

$$2\frac{\omega_1}{\omega}\frac{\partial \vec{e}_{(0)}'^{0}}{\partial \tau_2} + 2K_0\frac{c^2}{\lambda_0^2\omega^2}\frac{\partial \vec{e}_{(0)}'^{0}}{\partial x_2} = -\omega_1\sigma(\omega_1)\vec{e}_{(0)}'^{0} , \tag{4.2}$$

where $\sigma(\omega_1)$ is defined in terms of $F_{(0)}^{(0)}$ from (2.14). As one is dealing with a stationary problem, the solutions of the system (4.1) and (4.2) can be sought under the form of the time Fourier series:

$$\vec{e}_{(0)}'^{0} = \sum_{n=-\infty}^{+\infty} \vec{e}_{(0),n}'^{0}(\vec{x}_2)e^{in\tau_2} ,$$
$$F_{(0)}^{(0)} = \sum_{n=-\infty}^{+\infty} F_{(0),n}^{(0)}(\vec{x}_2; w)e^{in\tau_2} , \tag{4.3}$$

with the usual reality conditions. By substituting the expansions (4.3) into (4.1) and (4.2), one finds for the Fourier component $\vec{e}_{(0),n}'^{0}$ the equation

$$in\frac{\omega_1}{\omega}\vec{e}_{(0),n}'^{0} + \frac{1}{K_0}\frac{\omega_1^2}{\omega^2}\frac{\partial \vec{e}_{(0),n}'^{0}}{\partial x_2} = -\tfrac{1}{2}\omega_1[\sigma(\omega_1)\vec{e}_{(0)}'^{0}]_n \tag{4.4}$$

where the bracket $[\;]_n$ stands for the nth Fourier component of $\sigma\vec{e}_{(0)}'^{0}$ which is to be derived from (4.1) and (2.14). Owing to the nonlinearity of this term and of $|\vec{e}_{(0)}'^{0}|^2\partial F_{(0)}^{(0)}/\partial w$, all the frequencies of the spectrum occur in the right-hand side of (4.4), so that the Fourier components $\vec{e}_{(0),n}'^{0}$ and $F_{(0),n}^{(0)}$ are determined by an infinite set of coupled nonlinear equations; so, in the general case, the resolution of this system is an intractable problem. But, if one takes into account the smallness of the modulation depth μ_0, one can expand the solutions of the system (4.1)–(4.4) according to the powers of μ_0. It is then possible to uncouple the previous set of nonlinear equations and to solve it step by step; the calculations are easy up to order μ_0^2 and the results allow one to describe at this approximation the nonlinear distortion of the wave modulation. Particularly, it can thus be shown that the solution, at first order in μ_0, includes only the Fourier components $F^{(0)}_{(0),0}$, $\vec{e}'^{0}_{(0),0}$, of order 1, and $F^{(0)}_{(0),1}$, $\vec{e}'^{0}_{(0),1}$ of order μ_0: $F^{(0)}_{(0),0}$ and $\vec{e}'^{0}_{(0),0}$ are determined by a system of two equations identical to (2.18) and (2.25), while $F^{(0)}_{(0),1}$ and $\vec{e}'^{0}_{(0),1}$ satisfy an analogous system of more general equations, one of which (for $F^{(0)}_{(0),1}$) being of the same type as (A10).[16(b)] Obviously, this method, set up for the ordering scheme $\alpha' \simeq \eta' \simeq \delta$ and $\gamma' \simeq 0(1)$, can be also applied to the case $\alpha' \simeq \eta' \ll \delta$ and $\gamma' \ll 1$; then, as mentioned above, there is no need to introduce the various space scales so that one finds, as a particular case, the more simple results which have been obtained in previous works.[11-13]

APPENDIX

We give here the essential steps and definitions which are required for the calculation of the various conductivities introduced in Sec. II.

(a) *Calculation of* $\overleftrightarrow{F}^{(2)}_{(2)}$ *and* $\vec{F}^{(1)}_{(3)}$. They are obtained by integrating the kinetic equations (2.8a) and (2.7b). One thus gets

$$\overleftrightarrow{F}^{(2)}_{(2)} = -e^{-(\nu_2/\omega)\tau_0}\overleftrightarrow{F}^{(2)}_{(2)}(0) - e^{-(\nu_2/\omega)\tau_0}\int^{\tau_0} e^{(\nu_2/\omega)\tau_0'}[\vec{\nabla}_{x_0}\vec{F}^{(1)}_{(1)}]^0 d\tau_0' - \frac{e^{-(\nu_2/\omega)\tau_0}}{w}\int^{\tau_0} e^{(\nu_2/\omega)\tau_0'}\left[\vec{e}'_{(0)}\frac{\partial \vec{F}^{(1)}_{(1)}}{\partial w}\right]^0 d\tau_0', \tag{A1}$$

in which the first term on the right-hand side is the contribution of the initial anisotropy and cancels out when $\tau_0 \to \infty$, and

$$\begin{aligned}\vec{F}^{(1)}_{(3)} = \vec{F}^{(1)\prime}_{(3)} + \vec{F}^{(1)\prime\prime}_{(3)} = &-\frac{\omega}{\nu_1}\vec{\nabla}_{x_2}F^{(0)}_{(0)} - \frac{\omega}{\nu_1 w}\vec{e}''_{(2)}\frac{\partial F^{(0)}_{(0)}}{\partial w}\\ &- e^{-(\nu_1/\omega)\tau_0}\vec{\nabla}_{x_0}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}F^{(0)\prime}_{(2)}d\tau_0' - \frac{e^{-(\nu_1/\omega)\tau_0}}{w}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}\vec{e}'_{(0)}\frac{\partial F^{(0)\prime}_{(2)}}{\partial w}d\tau_0'\\ &- \frac{\omega}{w}\frac{\partial F^{(0)\prime\prime}_{(2)}}{\partial w}\left(\frac{\vec{e}'^{0}_{(0)}e^{i\varphi}}{\nu_1 + i\omega_1} + \frac{\vec{e}'^{0*}_{(0)}e^{-i\varphi}}{\nu_1 - i\omega_1}\right) - \frac{e^{-(\nu_1/\omega)\tau_0}}{w}\frac{\partial F^{(0)}_{(0)}}{\partial w}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}\vec{e}'_{(2)}d\tau_0'\\ &- \frac{2}{5}\left[w^2\vec{\nabla}_{x_0}\cdot e^{-(\nu_1/\omega)\tau_0}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}\overleftrightarrow{F}^{(2)}_{(2)}d\tau_0' + \frac{e^{-(\nu_1/\omega)\tau_0}}{w^4}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}\frac{\partial}{\partial w}(w^5\vec{e}'_{(0)}\cdot\overleftrightarrow{F}^{(2)}_{(2)})d\tau_0'\right]\\ &- \frac{\omega^2\vec{K}_0}{\omega_1 w}\frac{\partial F^{(0)}_{(0)}}{\partial w}e^{-(\nu_1/\omega)\tau_0}\int^{\tau_0} e^{(\nu_1/\omega)\tau_0'}\left[\frac{\vec{e}'^{02}_{(0)}e^{2i\varphi}}{\nu_1 + i\omega_1} + \frac{\vec{e}'^{0*2}_{(0)}e^{-2i\varphi}}{\nu_1 - i\omega_1} + \frac{2\nu_1\vec{e}'^{0}_{(0)}\cdot\vec{e}'^{0*}_{(0)}}{\nu_1^2 + \omega_1^2}\right]d\tau_0'.\end{aligned} \tag{A2}$$

Owing to (2.10a)–(2.12a) and (2.19), one first derives from (A1) the expression of $\overleftrightarrow{F}^{(2)}_{(2)}$ given by (2.31). By carrying this result, as well as (2.29), into (A2), it is then seen that $\vec{F}^{(1)\prime}_{(3)}$ is to be split into three terms according to (2.33) and (2.34); thus, by separating in (A2) the terms dependent and independent of τ_0, one finds the expressions (2.32) and (2.35) for $\vec{F}^{(1)\prime\prime}_{(3)}$ and $\vec{F}^{(1)\prime}_{(3)}$, in which have been introduced the following quantities:

$$A_1(F^{(0)}_{(0)}) \equiv \frac{i}{6\omega_1 w}\frac{\partial O_1(F^{(0)}_{(0)})}{\partial w} - \frac{4}{15w^4}\frac{\partial}{\partial w}\left[w^4\left(\frac{O_1'(F^{(0)}_{(0)})}{\nu_2 + 2i\omega_1} + \frac{O'(F^{(0)}_{(0)})}{\nu_2}\right)\right], \tag{A3}$$

$$A_1'(F^{(0)}_{(0)}) \equiv \frac{w}{(\nu_1 + i\omega_1)(\nu_2 + i\omega_1)}\frac{\partial F^{(0)}_{(0)}}{\partial w} \tag{A4}$$

$$A_2(F^{(0)}_{(0)}) \equiv \frac{1}{3\omega_1}O_1(F^{(0)}_{(0)}) - \frac{4i}{15}\frac{wO_1'(F^{(0)}_{(0)})}{(\nu_2 + 2i\omega_1)} + \frac{i}{5w^4}\frac{\partial}{\partial w}\left(\frac{w^4}{(\nu_1 + i\omega_1)(\nu_2 + i\omega_1)}\frac{\partial F^{(0)}_{(0)}}{\partial w}\right) - \frac{1}{\omega_1 w(\nu_1 + i\omega_1)}\frac{\partial F^{(0)}_{(0)}}{\partial w}, \tag{A5}$$

$$A_3(F^{(0)}_{(0)}) \equiv \frac{i}{6\omega_1 w}\frac{\partial O_1(F^{(0)}_{(0)})}{\partial w} - \frac{4}{15w^4}\frac{\partial}{\partial w}\left(\frac{w^4 O_1'(F^{(0)}_{(0)})}{\nu_2 + 2i\omega_1}\right), \tag{A6}$$

with

$$O_1(F^{(0)}_{(0)}) \equiv \frac{1}{w^2}\frac{\partial}{\partial w}\left(\frac{w^2}{\nu_1 + i\omega_1}\frac{\partial F^{(0)}_{(0)}}{\partial w}\right) \tag{A7a}$$

$$O_1' \equiv \frac{\partial}{\partial w}\left[\frac{1}{w(\nu_1 + i\omega_1)}\frac{\partial}{\partial w}\cdots\right], \tag{A7b}$$

$$O' \equiv \frac{\partial}{\partial w}\left[\frac{2\nu_1}{w(\nu_1^2 + \omega_1^2)}\frac{\partial}{\partial w}\cdots\right]. \tag{A7c}$$

These formulas allow one to determine completely the higher-order conductivities of Sec. II defined by (2.39)–(2.41) and to specify the nature of their various contributions. Particularly, let us note that the conductivity $\sigma_2(\omega_1)$ is split into three terms according to (2.41a) in which it must be written from (A5)

$$\sigma_2' = \frac{1}{3\omega_1}\left(\frac{4\pi}{3N}\int_0^\infty \frac{w^4 O_1(F^{(0)}_{(0)})}{\nu_1 + 2i\omega_1}dw\right) - \frac{4i}{15}\left(\frac{4\pi}{3N}\int_0^\infty \frac{w^5 O_1'(F^{(0)}_{(0)})}{(\nu_1 + 2i\omega_1)(\nu_2 + 2i\omega_1)}dw\right), \tag{A8a}$$

$$\sigma_2^L = -\frac{1}{\omega_1}\left(\frac{4\pi}{3N}\int_0^\infty \frac{w^3}{(\nu_1+i\omega_1)(\nu_1+2i\omega_1)}\frac{\partial F_{(0)}^{(0)}}{\partial w}dw\right), \tag{A8b}$$

$$\sigma_2'' = \frac{i}{5}\left(\frac{4\pi}{3N}\int_0^\infty \frac{1}{\nu_1+2i\omega_1} \times \frac{\partial}{\partial w}\left[\frac{w^4}{(\nu_1+i\omega_1)(\nu_2+i\omega_1)}\frac{\partial F_{(0)}^{(0)}}{\partial w}\right]dw\right). \tag{A8c}$$

(b) *Calculation of* $F_{(2)}^{(0)''}$. To complete the determination of the kinetic plasma state at this approximation, it remains for one to compute $F_{(2)}^{(0)''}$ which occurs in the definition of the conductivity $\sigma_{(2)}(\omega_1)$ given by (2.40).

According to our multiple-space-scales method, the equation satisfied by $F_{(2)}^{(0)''}$ is deduced from the order-ϵ^4 equation (2.6c) by canceling in it the terms independent of τ_0. As $\vec{F}_{(1)}^{(1)''}=0$ and owing to the transversality of the waves, one thus gets

$$I(F_{(2)}^{(0)''})+C_{ee}(F_{(2)}^{(0)''}) = \frac{\gamma'\omega}{3\overline{\nu}w^2}\frac{\partial}{\partial w}\left[w^3(\vec{e}_{(0)}'\cdot\vec{F}_{(3)}^{(1)\prime}+\vec{e}_{(2)}'\cdot\vec{F}_{(1)}^{(1)\prime})\right]'' , \tag{A9}$$

where the double prime means that only the terms independent of τ_0 have to be kept.

As an example, we give in the following the calculations for the imperfectly Lorentzian case. By (2.10a), (2.19), (2.33), and (2.35a) and by omitting the term $C_{ee}(F_{(2)}^{(0)''})$, one obtains for the $F_{(2)}^{(0)''}$ equation after some algebra

$$\begin{aligned}\mathcal{G}(F_{(2)}^{(0)''}) &\equiv I(F_{(2)}^{(0)''})+\frac{2\gamma'\omega^2u_{(0)}^2e^{-2\beta(\vec{x}_2)}}{3w^2}\frac{\partial}{\partial w}\left(\frac{w^2\nu_1'}{\nu_1^2+\omega_1^2}\frac{\partial F_{(2)}^{(0)''}}{\partial w}\right)\\ &= -\frac{2\gamma'\omega^2}{3w^2}\frac{\partial}{\partial w}\left(\frac{\nu_1'w^2}{\nu_1^2+\omega_1^2}\frac{\partial F_{(0)}^{(0)}}{\partial w}\right)\left({}^1\vec{e}_{(2)}^{\prime 0}\cdot\vec{e}_{(0)}^{\prime 0*}+{}^1\vec{e}_{(2)}^{\prime 0*}\cdot\vec{e}_{(0)}^{\prime 0}\right)\\ &\quad+\frac{\omega^4\gamma'u_{(0)}^2e^{-2\beta(\vec{x}_2)}}{3w^2\overline{\nu}}\frac{\partial}{\partial w}\left[\frac{w^3}{\nu_1+i\omega_1}\left(A_1(F_{(0)}^{(0)})\vec{e}_{(0)}^{\prime 0}\cdot\vec{e}_{(0)}^{\prime 0*}+A_1'(F_{(0)}^{(0)})\frac{K_0^2}{5}\right)+\text{c.c.}\right],\end{aligned} \tag{A10}$$

where the operator $\mathcal{G}(F_{(2)}^{(0)''})$ is such that it admits $F_{(0)}^{(0)}$ [defined by (2.26)] as solution of $\mathcal{G}(F)=0$. This equation allows one to determine $F_{(2)}^{(0)''}$ which must satisfy a normalization condition which is deduced from the Poisson equation (2.4b); by virtue of the transversality of the wave, one must have $n_{(2)}''=0$, whence

$$\int_0^\infty w^2F_{(2)}^{(0)''}dw=0. \tag{A11}$$

By seeking the solutions of (A10) under the form $F_{(2)}^{(0)''}=uF_{(0)}^{(0)}$, one gets easily for $F_{(2)}^{(0)''}$

$$F_{(2)}^{(0)''}=F_{(0)}^{(0)}\left[g_1(F_{(0)}^{(0)})({}^1\vec{e}_{(2)}^{\prime 0}\cdot\vec{e}_{(0)}^{\prime 0*}+{}^1\vec{e}_{(2)}^{\prime 0*}\cdot\vec{e}_0^{\prime 0}) + g_2(F_{(0)}^{(0)})u_{(0)}^2e^{-2\beta}\right], \tag{A12}$$

where $g_1(F_{(0)}^{(0)})$ and $g_2(F_{(0)}^{(0)})$ are given by

$$g_1(F_{(0)}^{(0)}) = 2\int^w\left[h_1(F_{(0)}^{(0)})\Big/w^2\nu_1'\left(1+\frac{4\gamma'\omega^2u_{(0)}^2e^{-2\beta}}{\nu_1^2+\omega_1^2}\right)\right]dw+k_1, \tag{A13}$$

$$g_2(F_{(0)}^{(0)})=g_{(2)}'(F_{(0)}^{(0)})u_{(0)}^2e^{-2\beta(\vec{x}_2)}+g_{(2)}''(F_{(0)}^{(0)})\frac{K_0^2}{5}, \tag{A14}$$

with

$$g_{(2)}'(F_{(0)}^{(0)}) = 2\int^w\left[h_2'(F_{(0)}^{(0)})/w^2\nu_1'\left(1+\frac{4\gamma'\omega^2u_{(0)}^2e^{-2\beta}}{\nu_1^2+\omega_1^2}\right)\right]dw+k_2', \tag{A15a}$$

$$g_{(2)}''(F_{(0)}^{(0)}) = 2\int^w\left[h_2''(F_{(0)}^{(0)})/w^2\nu_1'\left(1+\frac{4\gamma'\omega^2u_{(0)}^2e^{-2\beta}}{\nu_1^2+\omega_1^2}\right)\right]dw+k_2'', \tag{A15b}$$

and

$$h_1(F_{(0)}^{(0)})\equiv -2\gamma'\omega^2\frac{\nu_1'w^2}{\nu_1^2+\omega_1^2}\frac{\partial F_{(0)}^{(0)}}{\partial w}, \tag{A16a}$$

$$h_2'(F_{(0)}^{(0)})\equiv\frac{\gamma'\omega^4}{\overline{\nu}}\frac{w^3}{\nu_1+i\omega_1}A_1(F_{(0)}^{(0)})+\text{c.c.}, \tag{A16b}$$

$$h_2''(F_{(0)}^{(0)})\equiv\frac{\gamma'\omega^4}{\overline{\nu}}\frac{w^3}{\nu_1+i\omega_1}A_1'(F_{(0)}^{(0)})+\text{c.c.} \tag{A16c}$$

The constants k_1, k_2', and k_2'' occurring in (A13), (A15a), and (A15b) have to be determined from the three conditions:

$$\int_0^\infty w^2 F_{(0)}^{(0)} g_1(F_{(0)}^{(0)})dw = \int_0^\infty w^2 F_{(0)}^{(0)} g_2'(F_{(0)}^{(0)})dw = \int_0^\infty w^2 F_{(0)}^{(0)} g_2''(F_{(0)}^{(0)})dw = 0 , \tag{A17}$$

which are derived from (A10).

The expressions (A11)-(A15) determine completely the order-ϵ^2 stationary isotropic distribution function $F_{(2)}^{(0)\prime\prime}$ and allow one to define the conductivities $\sigma^1_{(2)}$, $\sigma^{2\prime}_{(2)}$, and $\sigma^{2\prime\prime}_{(2)}$ of (2.40) by

$$\sigma^1_{(2)} = -\frac{4\pi}{3N}\int_0^\infty \frac{w^3}{\nu_1 + i\omega_1}\frac{\partial(F_{(0)}^{(0)} g_1)}{\partial w}dw ,$$

$$\sigma^{2\prime}_{(2)} = -\frac{4\pi}{3N}\int_0^\infty \frac{w^3}{\nu_1 + i\omega_1}\frac{\partial(F_{(0)}^{(0)} g_2')}{\partial w}dw , \tag{A18}$$

$$\sigma^{2\prime\prime}_{(2)} = -\frac{4\pi}{3N}\int_0^\infty \frac{w^3}{\nu_1 + i\omega_1}\frac{\partial(F_{(0)}^{(0)} g_2'')}{\partial w}dw .$$

[1]V. L. Ginzburg and A. V. Gurevich, Sov. Phys. Usp. 3, 115 (1960); 3, 175 (1960).
[2]P. Rosen, Phys. Fluids 4, 361 (1961).
[3]R. F. Whitmer and E. B. Barrett, Phys. Rev. 121, 661 (1961); 125, 1478 (1962).
[4]E. B. Barrett, R. F. Whitmer, and S. J. Tetenbaum, Phys. Rev. 135, 369 (1964).
[5]Ting-Wei Tang, Phys. Fluids 9, 145 (1966).
[6]M. S. Sodha and P. K. Kaw, Phys. Fluids 8, 1402 (1965); 9, 603 (1966); J. Phys. Soc. Jpn. 21, 134 (1966); 21, 2674 (1966).
[7]I. P. Shkarofsky, Plasma Phys. 10, 169 (1968).
[8]F. G. Bass and Y. G. Gurevich, Sov. Phys. Usp. 14, 113 (1971).
[9]R. Jancel, Phys. Rev. A 12, 1662 (1975).
[10](a) R. Jancel, Laboratorie de Physique Théorique et Mathimatique, Université Paris VII, (1973) (to be published); (b) R. Jancel (unpublished).
[11]D. Basu, Thèse de Doctorat d'Etat (Faculté des Sciences de Paris, 1968) (unpublished).
[12]J. D. Allanche, Thèse de Doctorat d'Etat (Faculté des Sciences de Paris, 1970) (unpublished).
[13]J. D. Allanche, R. Jancel, and T. Kahan, Phys. Rev. A 10, 637 (1974).
[14]See, for example, W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (MIT, Cambridge, Mass., 1963); or T. H. Stix, The Theory of Plasma Waves (McGraw-Hill, New York, 1962).
[15]R. Jancel, Phys. Scr. 13, 373 (1976).
[16]R. Jancel, (a) Acta Phys. Hung. 40, 177 (1976); and (b) unpublished.