Localization of high-power laser pulses in plasmas

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This paper considers the nonlinear propagation of an intense laser pulse in plasmas. The equation for the wave envelope is derived accounting for the nonlinearities originating from the electron-mass variation as well as the self-interaction of the laser field. It is shown that the wave equation and the equations governing the slow plasma response can be represented in terms of the energy integral of a classical particle. The analysis of the potential reveals the existence of the localized solutions. The criteria for the latter to occur are obtained. A small-amplitude limit is also discussed.

I. INTRODUCTION

According to linear theory, an electromagnetic wave cannot propagate beyond the cutoff point in a plasma. However, plasma nonlinearities can lead to a downshift[1] of the wave frequency. As a result, the wave can still propagate into the overdense region. The wave energy can thus reach the main body of the plasma, leading to anomalous wave absorption. This phenomenon is important in laser-induced fusion, ionosphere modification by radar, and the interaction of pulsar radiation with its plasma environment.

When the intensity of the laser light is relatively weak, the self-interaction (pondermotive force) nonlinearity can cause wave filamentation.[2] In particular, the gradient in the time-averaged field intensity resulting from the filamentation forces the plasma out of the region of high field strength. Refraction of the radiation into the region of depressed density allows the wave to tunnel through the cutoff point. Localized filaments were first investigated by Gurovich *et al.*[3] and Karpman,[4] and later by Valeo[5] and others.[6]

On the other hand, in the presence of strong laser radiation (e.g., 10^{15}–10^{16} W/cm^2 for a Nd glass laser with $\lambda = 1.06$ μm or 10^{16} W/cm^2 for a CO_2 laser with $\lambda = 10.6$ μm, where λ is the laser wavelength) the oscillatory velocity of the electrons can approach the speed of light and the resulting variation of the electron mass can also cause strong nonlinearity. The latter leads to an instability[7,8] whose growth rate can compete with that of the self-interaction nonlinearity[2]. Computer simulations[9] have verified these findings. Similar to the modulational instability caused by the self-interaction nonlinearity, the evolution of this instability also leads to a solitonlike structure for the wave envelope.[10]

This paper is concerned with the nonlinear propagation of intense circularly polarized electromagnetic waves in an unmagnetized plasma. The wave equation accounting for the nonlinearities arising from both the relativistic mass variation and the self-interaction[11] of the intense laser fields is derived. It is found that both of the nonlinearities contribute to finite-amplitude wave localization.

In Sec. II, we present the basic equations and derive the wave equation. The response of the slow plasma motion to the laser electric field is calculated. Using a modulational representation, we show that the slowly varying complex envelope of the wave electric field obeys a nonlinear Schrödinger equation. Section III considers the stationary solution of the system. The coupled set of equations can be written in the form of the energy integral of a classical particle in a potential well, which is analyzed.[12] The conditions under which wave localization occur are given in Sec. IV. Section V contains a brief discussion of the results of our investigation.

II. FORMULATION

Consider a two-component electron-ion plasma in the presence of a circularly polarized electromagnetic wave in the form

$$\vec{A} = A[\hat{x}\cos\phi + \hat{y}\sin\phi], \tag{1}$$

where $\vec{A}$ is the vector potential, $\phi = \omega_0 t - k_0 z$ is the phasor, ω_0 and k_0 are the wave frequency and wave number, respectively. For linear wave propagation, we have

$$\omega_0^2 = \omega_{pe}^2 + c^2 k_0^2, \tag{2}$$

where $\omega_{pe} = (4\pi n e^2/m_e)^{1/2}$ is the local electron-plasma frequency, and other notations are standard.

We now study the nonlinear interaction between the laser light and a warm, collisionless, relativistic electron-ion plasma. The basic equations governing this interaction are

$$\frac{\partial n_j}{\partial t}+\nabla\cdot(n_j\vec{v}_j)=0\,, \tag{3}$$

$$\frac{\partial\vec{p}_j}{\partial t}+\vec{v}_j\cdot\nabla\vec{p}_j=e_j\left(\vec{E}+\frac{\vec{v}_j\times\vec{B}}{c}\right)-\frac{T_j}{n_j}\nabla n_j\,, \tag{4}$$

$$\nabla\cdot\vec{E}=-4\pi e(n_e-n_i)\,, \tag{5}$$

$$\vec{p}_j=m_{j0}\vec{v}_j/(1-v_j^2/c^2)^{1/2}\,, \tag{6}$$

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\,, \tag{7}$$

$$\nabla\times\vec{B}=\sum_j\frac{4\pi}{c}n_j e_j\vec{v}_j+\frac{1}{c}\frac{\partial\vec{E}}{\partial t}\,, \tag{8}$$

$$\nabla\cdot\vec{B}=0\,. \tag{9}$$

We define the scalar and vector potentials ϕ and $\vec{A}$ by the following equations:

$$\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}-\nabla\phi\,, \tag{10}$$

$$\vec{B}=\nabla\times\vec{A}\,, \tag{11}$$

$$\nabla\cdot\vec{A}+\frac{1}{c}\frac{\partial\phi}{\partial t}=0\,. \tag{12}$$

The notations are standard.

It is well known that an electron in the presence of intense laser fields can acquire a relativistic speed. Consequently, a nonlinear current is produced which gives rise to a modification of the linear dispersion relation. Owing to this and other nonlinear effects, the wave can propagate into the overdense region.

In this paper, we are concerned with the nonlinear propagation of a wave pulse, that is, we shall study the localization of wave electric fields due to the various nonlinearities. In particular, we shall take into account the nonlinearities originating from relativistic mass variation, as well as those due to interaction of slow plasma motion with the laser electric field. Both of these appear on a time scale longer than that of the laser field variation.

Taking the curl of (8) and using Eqs. (9)–(12), we readily obtain the wave equation for the laser light

$$\frac{\partial^2\vec{A}}{\partial t^2}-c^2\nabla^2\vec{A}=4\pi c\vec{J}\,, \tag{13}$$

where $\vec{J}\approx -n_e e\vec{v}_e$ is the total current density, and n_e is the slowly varying density produced by the low-frequency electrostatic perturbation ϕ induced by the laser field.

The slowly varying electron density perturbations follow from Eqs. (3), (4), (10), (11). The last three equations can be combined to yield

$$\left(1+\frac{p_e^2}{c^2m_{e0}^2}\right)^{1/2}\frac{\partial\vec{p}_e}{\partial t}+\frac{\vec{p}_e}{m_0}\cdot\nabla\vec{p}_e =\frac{e}{c}\left(1+\frac{p_e^2}{c^2m_{e0}^2}\right)^{1/2}\frac{\partial\vec{A}}{\partial t}-\frac{e}{m_{e0}c}\vec{p}\times(\nabla\times\vec{A}) +\left(1+\frac{p_e^2}{c^2m_{e0}^2}\right)^{1/2}(e\nabla\phi-T_e\nabla\ln n_e) \tag{14}$$

since

$$\vec{v}_e=\frac{\vec{p}_e}{m_{e0}(1+p_e^2/c^2m_{e0}^2)^{1/2}}\,. \tag{15}$$

Equation (14) is satisfied by

$$\vec{p}_e=m_{e0}c\vec{\psi} \tag{16}$$

together with a low-frequency momentum balance,

$$\frac{\beta}{2}\frac{\nabla(\psi^2)}{(1+\psi^2)^{1/2}}=\nabla\phi-\nabla\ln n_e\,, \tag{17}$$

where $\vec{\psi}=e\vec{A}/m_{e0}c^2$, $\beta=c^2/v_{te}^2$, $v_{te}\equiv(T_e/m_{e0})^{1/2}$, and we have normalized ϕ by T_e/e, n by n_0, and the space coordinates by λ_e, $\lambda_e\equiv(T_e/4\pi n_0e^2)^{1/2}$ being the electron Debye length.

We note that due to the *circular* polarization, ψ^2 does not contain any high-frequency component. This allows us to separate the high- and low-frequency components of (14), as done in obtaining (16) and (17).

The (nonrelativistic) ions are coupled with the electrons via ϕ, so that we have for the ions

$$\frac{\partial n_i}{\partial t}+\nabla\cdot n_i\vec{v}_i=0\,, \tag{18}$$

$$\frac{\partial\vec{v}_i}{\partial t}+\vec{v}_i\cdot\nabla\vec{v}_i=-\nabla\phi\,, \tag{19}$$

and

$$n_e=n_i=n\,, \tag{20}$$

where for simplicity the ions are assumed to be cold. In (18) to (20), and in the following equations, we nondimensionalize t and v_i by ω_{pi}^{-1} and c_s respectively, where $\omega_{pi}\equiv(4\pi n_0e^2/m_i)^{1/2}$ is the ion plasma frequency, and $c_s\equiv(T_e/m_i)^{1/2}$ is the ion acoustic speed. It is of interest to note that the low-frequency equations (17)–(20) yield, within the adiabatic approximation, together with the linear ion response, the relativistic ponderomotive potential[11] $\phi_{\text{pond}}=m_{e0}c^2[(1+\psi^2)^{1/2}-1]/e$.

Equations (13), (14), and (17)–(20) form a coupled set. As is well known,[8,10] the linearized system is modulationally unstable in time. Accordingly, we look for the long-time asymptotic behavior, assuming one-dimensional spatial variation.

Within a modulational representation, we take the amplitude to vary slowly, namely,

$$\vec{\psi} = \tfrac{1}{2}\,\psi(\tau,\zeta)(\hat{x} - i\hat{y})\exp(-i\omega_0 t + ik_0 z) + \text{c.c.}, \tag{21}$$

where $\partial/\partial\tau \ll \omega_0$. Thus, Eq. (13) becomes

$$2i\epsilon\frac{\partial\psi}{\partial t} + i\alpha\frac{\partial\psi}{\partial\zeta} + \beta\frac{\partial^2\psi}{\partial\zeta^2} + \Delta\psi = \frac{n\psi}{(1+|\psi|^2)^{1/2}}, \tag{22}$$

where $\epsilon = (m_e/m_i)^{1/2}(\omega_0/\omega_{p0})$, $\omega_{p0}^2 = 4\pi n_0 e^2/m_{e0}$, $\alpha = 2c^2k_0/\omega_{p0}^2\lambda_e$, and $\Delta = (\omega_0^2 - c^2k_0^2)/\omega_{p0}^2$. The convective term in (22) can be easily transformed out.

III. LOCALIZED SOLUTIONS

We are interested in the stationary solutions of (17)–(20) and (22). Accordingly, we express[12]

$$\psi = \tilde{\psi}(\xi)\exp\{i[\theta(\tau) + \phi(\zeta)]\}, \tag{23}$$

$$n = n(\xi), \tag{24}$$

$$v_i = v(\xi), \tag{25}$$

where $\xi = \zeta - Mt$, and $M = V/c_s$ is the Mach number. In the following, we shall for simplicity drop the tilde on $\tilde{\psi}$.

From Eqs. (22) and (23) it follows that

$$\beta\frac{\partial^2\psi}{\partial\xi^2} + \delta\psi = \frac{n\psi}{(1+\psi^2)^{1/2}}, \tag{26}$$

where

$$\delta = \Delta - 2\epsilon\,\theta_\tau + 2k_0c^2\phi_\zeta/\omega_0 + \epsilon^2M^2/\beta$$

is a nonlinear frequency shift to be determined later, and

$$\phi(\zeta) = \epsilon M\zeta/\beta. \tag{27}$$

We should now find a relation between n and ψ from the equations governing the slow plasma motion. In particular, from (17), one finds

$$\phi = \beta[(1+\psi^2)^{1/2} - 1] + \ln n, \tag{28}$$

where we have assumed the plasma to be unperturbed at infinity, and accordingly used the boundary conditions $n=1$, $\psi = 0$ at $|\xi| \to \infty$.

Equations (18)–(20) and (28) give

$$v = (n-1)M/n \tag{29}$$

and

$$(1+\psi^2)^{1/2} = -\frac{M^2}{2\beta}\left(\frac{1}{n^2} - 1\right) - \frac{\ln n}{\beta} + 1, \tag{30}$$

where again the boundary conditions at infinity have been imposed.

We are interested in localized solutions in which the electric field has a maximum at the location where the density perturbation is maximum. Therefore, we let $\psi = \psi_m$ and $n = N$ at $\xi = 0$. Equation (30) then gives a relation between ψ_m and N. Differentiating Eq. (30) we have

$$\psi\frac{\partial\psi}{\partial\xi} = -\frac{1}{\beta n}\left(1 - \frac{M^2}{n^2}\right)(1+\psi^2)^{1/2}\frac{\partial n}{\partial\xi}. \tag{31}$$

Multiplying Eq. (26) by $\partial\psi/\partial\xi$ and using (31) we get

$$\frac{\beta}{2}\frac{\partial}{\partial\xi}\left(\frac{\partial\psi}{\partial\xi}\right)^2 + \frac{\delta}{2}\frac{\partial}{\partial\xi}\psi^2 = -\frac{1}{\beta}\left(1 - \frac{M^2}{n^2}\right)\frac{\partial n}{\partial\xi}. \tag{32}$$

Integrating the above equation once and using the boundary conditions at infinity ($n=1$, $\psi=0$, $\partial\psi/\partial\xi = 0$), one obtains

$$\frac{\beta}{2}\left(\frac{\partial\psi}{\partial\xi}\right)^2 + \frac{\delta}{2}\psi^2 = \frac{1}{\beta}\left(1 - n + M^2 - \frac{M^2}{n}\right). \tag{33}$$

Using the boundary conditions at $\xi = 0$, we find from (33) the nonlinear frequency shift, namely,

$$\delta = 2(1 - N + M^2 - M^2/N)/\beta\psi_m^2, \tag{34}$$

where the maximum electric field ψ_m and the minimum density N are related by

$$(1+\psi_m^2)^{1/2} = -\frac{M^2}{2\beta}\left(\frac{1}{N^2} - 1\right) - \frac{\ln N}{\beta} + 1. \tag{35}$$

From (31) and (24), one finally obtains

$$\frac{1}{2}\left(\frac{\partial n}{\partial\xi}\right)^2 + V(n;\delta,M) = 0, \tag{36}$$

where the potential energy V is given by

$$V = \frac{[n - 1 + M^2/n - M^2 + \frac{1}{2}\delta\beta\psi^2]n^2\psi^2}{(1 - M^2/n^2)^2(1+\psi^2)}, \tag{37}$$

and $\psi^2 = \psi^2(n)$ is given by (30).

IV. ANALYSIS

In Sec. III, we have derived from the coupled set of fluid and wave equations a conservation law in the form of the energy integral of a classical particle. Within this picture, Eq. (36) describes the motion of a particle in the potential well. Localized solutions for n exist provided $V < 0$ between two points $n = N$ and $n = 1$. Solitary wave solutions exist if the "velocity" $\partial n/\partial\xi$ approaches zero infinitely slowly at the latter point, so that a "particle" with zero total energy starting from $n = 1$ ($V = 0$) takes infinite time to return.

We now derive the conditions under which the potential (37) has a form required for localized solutions as discussed above. First, the physical requirement $\psi^2 > 0$ yields for an expansion $n = 1 + \delta n$, $\delta n \ll 1$ the result

$$\delta n(1 - M^2) < 0, \tag{38}$$

that is, subsonic solitons could only exist for density dips and supersonic solitons are accompanied by density humps. Second, $V(n) < 0$ demands

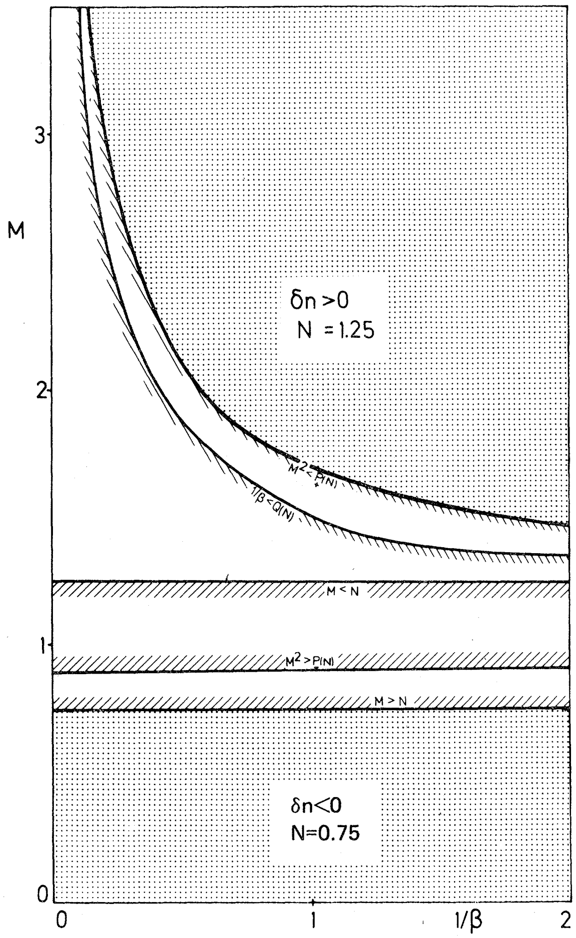

FIG. 1. Regions of existence of solitary wave solutions (dotted area) in the (β^{-1}, M) plane for two typical values of density minimum $(N=0.75)$ and maximum $(N=1.25)$, respectively. For $N<1$, i.e., density dips, solitons exist below the line $M=N$; the line $M^2=P_-(N)$ represents the condition (44b) whereas the condition (41) is here not restrictive at all. For $N>1$, i.e., density humps, condition (44a) represented by the $M^2=P_+(N)$ line is most restrictive; condition (41) is shown by the $1/\beta=Q(N)$ line, and the $M=N$ line is not very significant here.

$$\left(\frac{M^2}{n}+n-1-M^2\right)+\frac{\delta}{2}\beta\psi^2<0, \tag{39}$$

which together with (38) yields $\delta<1$, or

$$\psi_m^2>-2\beta^{-1}[N-2-M^2(N^{-1}-1)]. \tag{40}$$

Eliminating ψ_m, we can also write (40) in the form

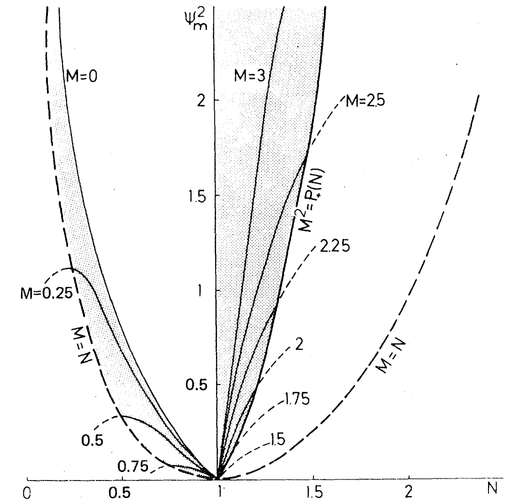

FIG. 2. Relation between maximum amplitude ψ_m and the corresponding density (maximum or minimum) N for $\beta=2$ and different Mach numbers M. According to Eq. (35), solitons only exist in the dotted area. For $N<1$, the region of existence is bordered by the $M=0$ and $M=N$ lines, whereas for $N>1$ the $M^2=P_+(N)$ line representing condition (44a) limits the area where supersonic solitons accompanied by density humps can appear.

$$\beta^{-1}>2(1+b/a)/a\equiv Q(N), \tag{41}$$

where $a=\frac{1}{2}M^2(N^{-2}-1)+\ln N$, and $b=(1-M^2/N)\times(1-N)$. Furthermore, it can readily be shown that $V(n\to 1)\propto-|\delta n|^2$. Thus the potential is properly behaved near $n=1$ if (39) and (40) are satisfied.

Let us now investigate the behavior of $V(n)$ near $n=N$. Because of the choice of δ in (34), the condition $V(N)=0$ is satisfied. Letting $n=N+\delta n$, we obtain, near $n=N$,

$$V(n)\approx\frac{-[\delta(1+\psi_m^2)^{1/2}-N]\psi_m^2}{(1+\psi_m^2)(1-M^2N^{-2})}N\delta n. \tag{42}$$

Thus, $V(n\to N)\propto-|\delta n|$, which is necessary for soliton existence, provided

$$1-\delta(1+\psi_m^2)^{1/2}/N<0. \tag{43}$$

Eliminating ψ_m, we obtain from (43), for $N<3$,

$$M^2>D+E^{1/2}\equiv P_+(N), \tag{44a}$$

and

$$M^2<D-E^{1/2}\equiv P_-(N), \tag{44b}$$

where

$$D\equiv[(N^{-2}-1)(1-N^{-1})\beta^{-1}+2(\beta^{-1}\ln N-1)(N^{-2}-N^{-1})-(N^{-2}-1)(\beta^{-1}\ln N-1)]/[2\beta^{-1}(N^{-2}-N^{-1})(N^{-2}-1)-(N^{-2}-1)^2\beta^{-1}2^{-1}],$$

and

$$E\equiv D^2-[2(1-N^{-1})(\beta^{-1}\ln N-1)-\beta(\beta^{-1}\ln N-1)^2+\beta]/[\beta^{-1}(N^{-2}-N^{-1})(N^{-2}-1)-4^{-1}\beta^{-1}(N^{-2}-1)^2].$$

In Fig. 1, we have given the existence regions of solitary waves using (41) and (44). In Fig. 2, we have presented the ψ_m^2 vs N lines for different values of M.

To conclude this section, we present the small-amplitude limit, for which it is possible to give an analytical solution. Accordingly, we let $n=1+\delta n$ and $N=1+\delta N$, where $\delta n, \delta N \ll 1$. We obtain

$$\delta = -\frac{2\delta N(1-M^2)}{\beta\psi_m^2}\left(1+\frac{\delta N M^2}{1-M^2}\right), \tag{45}$$

$$\psi_m^2 = -(2\delta N/\beta)(1-M^2), \tag{46}$$

$$\psi^2 = \frac{2\delta n}{\beta}(M^2-1)+\frac{\delta n^2}{\beta}(1-3M^2)+\frac{\delta n^2}{\beta^2}(1-M^2)^2. \tag{47}$$

Hence, the potential for small-amplitude limit is given by

$$V(\delta n) = -4\beta^{-1}\{(1-\delta)\delta n^2+\tfrac{1}{2}[\beta^{-1}(1-M^2)+1]\delta n^3\}, \tag{48}$$

where $\psi^2 \ll 1$, $1-\delta = O(\delta n)$, and $M^2-1 > O(\delta n)$ have been assumed. As a result, the usual hyperbolic-secant soliton solution emerges and the results are known in the literature.[3,4,10]

V. DISCUSSION

Recent progress in laser-fusion and pulsar physics calls for a detailed investigation of the interaction of radiation with plasmas. In the existing literature,[4,10] a consistent picture describing the nonlinear evolution of the modulation instabilities associated with such interactions has been worked out only for cases in which effects such as radiation pressure and relativistic mass variation are studied separately. The argument usually given to justify such a treatment is that the time scales for growth and saturation of the instabilities associated with each mechanism can be different. However, when considering the long-time behavior of an interaction such as soliton formation, several mechanisms and the interplay between them should be simultaneously considered.

The ponderomotive force appears because of the interaction between the high-frequency wave and the background plasma. Because of the large ion mass, the ponderomotive force acts on the electrons, and ultimately also on the ions owing to the resulting ambipolar field. Due to the expulsion of particles from the region of high field intensity, locally the plasma density is reduced, thus trapping the radiation. This argument is essentially unchanged when relativistic effects are included. In fact, only the magnitude of the ponderomotive force is altered[11] because of a different dependence of the field intensity.

The relativistic electron-mass variation by itself also causes trapping of radiation, since, due to the large electron quiver velocity, the electron mass increases, causing the local plasma frequency to decrease. Such a decrease is produced by the ponderomotive force by reducing the local plasma density. It is then expected that both the effects of self-interaction and electron-mass variation can cause the formation of localized wave packets.[10,12]

In this paper, we consider the propagation of an intense circularly polarized electromagnetic wave in a collisionless plasma. We include the effects of relativistic mass variation, the relativistic ponderomotive force, as well as the *fully nonlinear* electron and ion dynamics[12] within the fluid approximation. Our investigation thus extends beyond the existing works[10] in which either the relativistic effects alone are treated, or a weak nonlinearity is assumed.

Our main results are presented in Figs. 1 and 2. These describe the regions of existence as well as the properties of the solitary waves. Thus, supersonic solitons of a given speed can appear only when β^{-1} is sufficiently large, while subsonic solitons can appear for all β^{-1}. Furthermore, from Fig. 2, we see that sub- and supersonic solitons are associated with density depressions and humps, respectively. We note that solitons with speeds $M\approx 1$ are of vanishingly small amplitude.

A new phenomenon occurs when the effects of relativistic mass variation and ponderomotive force are included simultaneously. This is the appearance of supersonic solitary waves with density humps when the relativistic effects are sufficiently strong. Such a situation can occur because the relativistic mass increase dominates over the density increase associated with the density hump, the combined effect still being a net reduction of the local plasma frequency. Quantitatively, one can understand the above discussion by considering the small-amplitude limit as follows: The local plasma frequency, including the ponderomotive force modification of the density $n=1-\beta\psi^2/2(1-M^2)$ and the relativistic mass variation $m=m_0(1+\psi^2)^{1/2}\approx m_0(1+\frac{1}{2}\psi^2)$, is given by $\omega_{pe}^2=[1-\beta\psi^2/2(1-M^2)-\frac{1}{2}\psi^2]\omega_{p0}^2$. Modulational instability and soliton formation is possible if the nonlinear frequency shift $\omega_{pe}^2-\omega_{p0}^2$ is negative, or $q\equiv\beta\psi^2/2(1-M^2)-\frac{1}{2}\psi^2<0$, which clearly allows the appearance of $M^2>1$ solitons as long as $\beta/|1-M^2|<1$. On the other hand, in the nonrelativistic limit, which is realized by letting $\beta\to\infty$ while keeping $\beta\psi^2=E^2/8\pi n_0 T_e$ fixed, the above condition becomes $-\beta\psi^2/2(1-M^2)<0$, so that only $M^2<1$ solitons can appear.

Since $q\psi$ is simply the small-amplitude limit of the nonlinear term in the nonlinear Schrödinger equation (22), we take this opportunity to discuss the time scale associated with the ponderomotive and relativistic effects. The time scales are determined by the growth rates of the modulational instabilities associated with these effects. They are $|E_0|^2$ and $\beta^{-1}|E_0|^2$ for the ponderomotive and relativistic modulations, respectively. Here we took the purely growing mode ($M=0$), and E_0 is the pump electric field amplitude normalized with $(8\pi n_0 T_e)^{1/2}$. Thus when $\beta = c^2/v_{te}^2 \approx 1$, the two effects are equally important.[13]

The application of our investigation to problems of recent interest, such as mode conversion, profile modification, and shock formation, shall be presented in forthcoming publications.

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