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Polarization measurements of He I singlet transitions following beam–tilted-foil excitation

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The Stokes parameters of light emitted following beam-foil excitation have been measured as a function of foil tilt angle from 0° to 70° for the $2s\,^1S$-$3p\,^1P^o$, $2s\,^1S$-$4p\,^1P^o$, $2p\,^1P^o$-$3d\,^1D$, and $2p\,^1P^o$-$4d\,^1D$ transitions in He I. The precision of the measurements has been established using the $2p\,^1P^o$-$4s\,^1S$ transition. A new method is described for analyzing beam–tilted-foil data. The results are compared with those forecast by the theory suggested by Band, and good agreement is found for the transitions from $^1P^o$ terms but somewhat poorer agreement is found for the transitions from ^1D terms. In particular, the M/I polarization is shown to become negative for a foil tilt angle of around 60° in the case of the $^1P^o$ terms and 45° for the ^1D terms, while the total polarization fraction remains nearly constant at all tilt angles observed for the $^1P^o$ terms but shows a pronounced minimum at about 45° for the ^1D terms. A simple procedure is also outlined for the accurate determination of the phase of a retarder plate at any required wavelength.

I. INTRODUCTION

The theory of alignment and orientation as applied to beam-foil experiments has been extensively discussed in the literature.[1-5] It was first demonstrated by Berry *et al.*[6] that tilting the foil permits the observation of elliptically polarized light by destroying the axial symmetry. Though several authors have offered an explanation of this effect,[7-11] too few measurements have been reported to determine which, if any, of the theories gives a good description of the phenomenon. In this report we compare our measurements with one of the more recent and thorough treatments.[7]

We have undertaken a systematic study of the beam-tilted-foil phenomenon among the singlet levels of neutral helium for which neither fine- nor hyperfine-structure quantum beats could complicate the analysis. The only previously published results for these levels are those of Berry and co-workers for the $^1S-{}^1P^o$ transition at 5016 Å,[6,12] and their "less precise" results for the $^1P^o-{}^1D$ transition at 4922 Å.[12] From symmetry considerations one would expect tilted-foil polarization measurements to yield very similar results for levels for which only the principal quantum number changes. We have therefore studied two $^1S-{}^1P^o$ transitions (viz. $2s-3p$ at 5016 Å and $2s-4p$ at 3965 Å) and two $^1P^o-{}^1D$ transitions (viz. $2p-3d$ at 6678 Å and $2p-4d$ at 4922 Å). All higher angular momentum states in neutral helium give electric-dipole transitions above 8000 Å and therefore outside the range of our equipment. We also measured the polarization of a $^1P^o-{}^1S$ transition (at 5048 Å) since this should rigorously show no polarization at any tilt angle and hence these results could be used as an indication of our experimental uncertainties.

II. EXPERIMENT

The laboratory used for this experiment is a new facility which has not been previously described. A beam from a universal ion source is accelerated by a 350-kV electrostatic generator and mass analyzed by a 15-kG magnet from Industrial Coils, Inc. The target chamber was designed to permit both lateral motion of the foil along the beam and foil-tilt positions from 0° to 70° in 10° increments. The foil holder can be repositioned to within 0.01 mm after changing foils under vacuum. Three methods of signal normalization are possible with our equipment: time, integrated beam current, or photon counts from a light pipe positioned just behind the foil.

Figure 1 illustrates the beam, foil, and detector geometry used for this experiment. Two fused silica lenses of equal focal length were employed to focus light at 90° to the beam onto the entrance slit of a $\frac{3}{4}$-m Czerny-Turner Spex monochromator.

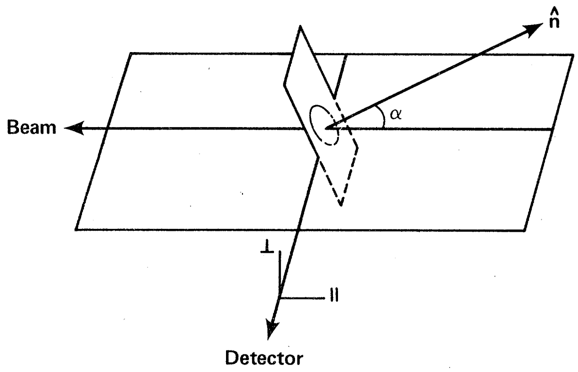

FIG. 1. Beam, foil, and detector geometry.

The two-lens configuration ensures parallel light passes through our polarimeter which consists of a quartz zero-order retarder (quarter wave at 5500 Å) followed by a calcite prism polarizer. Either of these may be rotated by a stepping motor with reproducibility of axis position of 0.1°. The entrance and exit slits of the monochromator were tilted to remain parallel to the foil. The entire spectrometer rests on a movable table which may be adjusted for proper focus at any measurable wavelength.

Photon counting was accomplished using an EMI 6256S photomultiplier below 5500 Å and a cooled EMI 9658R above 5500 Å. Ortec electronics were used to amplify and discriminate the signal which then passed to one of four inputs on a multiscaling front-end of a Tracer Northern TN-11 system for data acquisition and control. This system is configured around a PDP 11/05 minicomputer and offers stepping motor control, floppy disk data, and program storage, and direct interface with the central campus computer for data transmission and analysis.

As many experimental parameters as possible were held constant for all of the measurements. Nominally 5-μg/cm^2 carbon foils were used at a beam energy of 160 keV and beam current of 7 ± 2 μA (18 μA/cm^2). The 0.5-mm spectrometer slits were employed throughout and the weakest signal used gave over 250 counts per second with a signal to noise ratio of 100/1.

The quarter-wave retarder was rotated by the computer-controlled stepping motor in 10° increments for a sweep of 360°. Signal was collected using optical normalization for about 5 sec at each position, thus taking 3 or 4 min for one rotation. Five to ten such sweeps were summed to form one data set. In this way only the most current sweep was discarded when a foil broke. Each final data set thus comprised 36 channels with not fewer than 5000 counts per channel.

III. DATA ANALYSIS

The Stokes parameters, I, M, C, and S, can be defined with respect to the reference axis of Fig. 1 as

$$\begin{aligned} I&=I_{\parallel}+I_{\perp},\\ M&=I_{\parallel}-I_{\perp},\\ C&=I_{45^\circ}-I_{135^\circ},\\ S&=I_{\mathrm{rh}}-I_{\mathrm{lh}}. \end{aligned} \tag{1}$$

They can be completely determined by rotating a quarter-wave retarder before a fixed polarizer,[13] though only the ratios M/I, C/I, and S/I are of interest. This method has the advantage that instrumental polarization of the spectrometer or detector is eliminated.

The general expression for the intensity of light $\mathcal{I}$, after traversing a retarder of phase δ oriented with its fast axis making an angle β to the reference axis, and a polarizer with its transmission axis set at angle γ to the same reference axis is given by[14]

$$\begin{aligned} \mathcal{I}(\gamma,\beta,\delta)=\tfrac{1}{2}\{I+M[&\cos2(\gamma-\beta)\cos2\beta-\sin2(\gamma-\beta)\sin2\beta\cos\delta]\\ &+C[\cos2(\gamma-\beta)\sin2\beta+\sin2(\gamma-\beta)\cos2\beta\cos\delta]+S[\sin2(\gamma-\beta)\sin\delta]\}\,. \end{aligned} \tag{2}$$

Written this way it is obvious that the expression is linear in the Stokes parameters and a straight forward regression[15] suffices to determine these parameters provided γ,β, and δ are accurately known. It is not difficult to devise methods for determining γ and β, the polarizer axis and quarter-wave axis, to about 1°. We have devised a simple, self-consistent, technique to measure δ which is insensitive to the other experimental parameters.

We first introduce a second polarizer P_0 before the retarder with its transmission axis set at 90° to the reference axis (the angle is not critical). We then set the polarizer in the polarimeter (**P**) to 0° and acquire a data set by rotating the retarder as described previously. P is then set to 90° and a second data set is acquired. Since P_0 has not been changed, the relative Stokes parameters must be the same for each data set. We then vary the input value of δ in the program that analyzes the data until the two sets of relative Stokes parameters agree. Actually M/I is the only relevant parameter if performed as outlined above.

It is evident from Eq. (2) that, in the absence of statistical fluctuations, when P_0 and P are crossed, $I-M$ remains constant, while when P_0 and P are aligned, $I+M$ remains constant as the phase δ is scanned in the program. We have observed this to be the case for experimental data within the calculated uncertainties. In this way the crossed polarizer value of M/I is insensitive to changes in δ and yields approximately the correct Stokes parameters for any reasonable value of δ while the aligned value of M/I changes very rapidly with δ. It is thus easy to get a self-consistent value for δ to within 1°. We have measured the phase of our retarder with this method at each of the wavelengths under investigation.

IV. DISCUSSION

Figure 2 shows our results for the $2s\,{}^1S-4p\,{}^1P^o$ transition at 3965 Å. Figure 3 shows the $2p\,{}^1P^o-4d\,{}^1D$ transition at 4922 Å. As was expected from symmetry arguments, similar results were obtained using the corresponding transitions from $n=3$, as is shown in Tables I and II, the agreement being somewhat closer for the ${}^1P^o$ terms. The ${}^1S-{}^1P^o$ results are consistent with the earlier measurements on 5016 Å by Berry *et al.* but the ${}^1P^o-{}^1D$ results are not in good agreement with their "less precise" measurements on 4922 Å.[12]

The smooth curves shown in the figures are from a least-squares fit of the results to the theory of Band.[7] In his paper Band wrote that the Stokes parameters varied with foil-tilt angle as

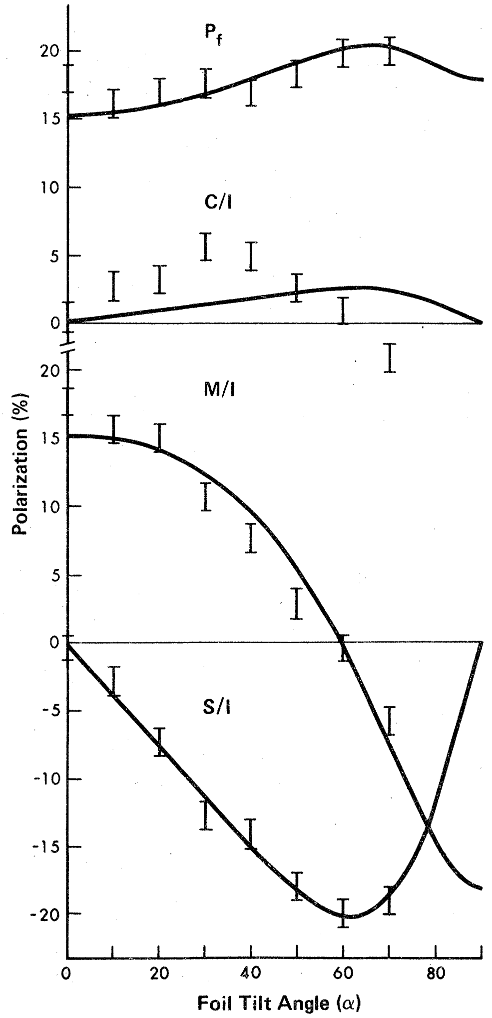

FIG. 2. Stokes parameters vs tilt angle for the 3965-Å $2s\,{}^1S$-$4p\,{}^1P^o$ transition in He I. Smooth curves from Eq. (4) with $A=-3.29$, $B=-1.33$, $D=-0.07$, $E=-0.94$, $F=0.24$.

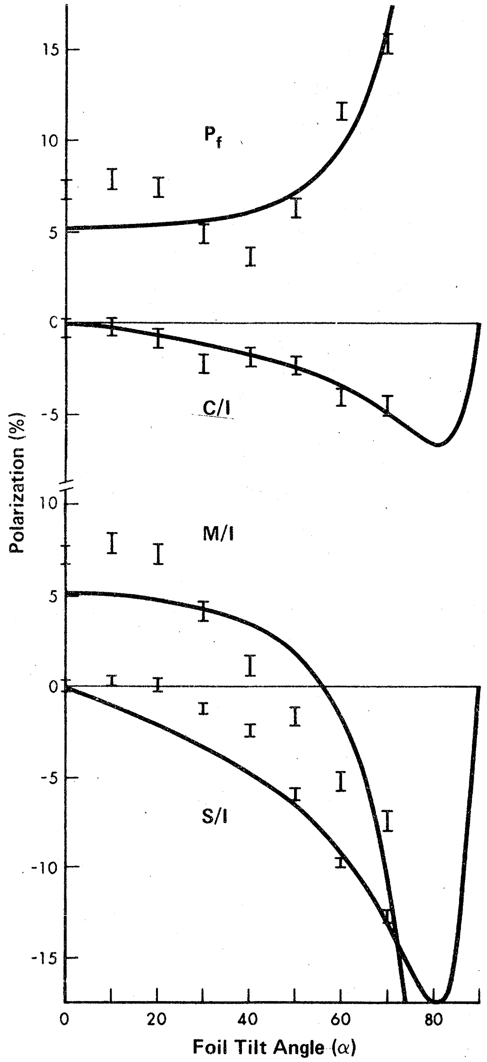

FIG. 3. Stokes parameters vs tilt angle for the 4922-Å $2p\,{}^1P^o$-$4d\,{}^1D$ transition in He I. Smooth curves from Eq. (4) with $A=-17.39$, $B=-0.45$, $D=0.19$, $E=-1.35$, $F=0.41$.

TABLE I. Polarization[a] of He I $^1P^o$ transitions.

Foil-tilt angle α (degrees)	$2s\,^1S$-$3p\,^1P^o$ 5016 Å				$2s\,^1S$-$4p\,^1P^o$ 3965 Å			
	M/I	C/I	S/I	P_f	M/I	C/I	S/I	P_f
0	18.7 ± 1.1	−1.3 ± 1.1	−0.5 ± 0.7	18.7 ± 1.1	17.9 ± 0.8	0.5 ± 0.8	−0.3 ± 0.9	17.9 ± 0.8
10	19.9	2.7	−3.8	20.4 ± 1.1	15.7	2.8	−2.8	16.2 ± 0.8
20	15.9	4.6	−6.2	17.7 ± 1.1	15.0	3.2	−7.2	17.0 ± 0.8
30	13.9	5.5	−9.6	17.8 ± 1.0	10.8	5.7	−12.7	17.6 ± 0.8
40	11.1	4.2	−12.8	17.5 ± 0.9	7.6	5.1	−14.2	16.9 ± 0.9
50	6.1	5.0	−16.7	18.4 ± 0.8	2.9	2.8	−17.9	18.3 ± 0.9
60	−1.8	6.7	−20.6	21.7 ± 0.8	−0.4	1.0	−19.8	19.8 ± 0.9
70	−2.3	2.7	−20.6	20.9 ± 0.7	−5.7	−2.4	−19.1	20.0 ± 0.9

[a] Values in percent. Uncertainties constant throughout column unless otherwise noted.

$$S/I=(c\tan\alpha)/Q(\alpha)\,,$$
$$C/I=(2d\tan\alpha)/Q(\alpha)\,, \tag{3}$$
$$M/I=-[e+(f+d\cos 2\alpha)/\cos^2\alpha]/Q(\alpha)\,,$$

with $Q(\alpha)=1+a/\cos^2\alpha+b\cos 2\alpha/\cos^2\alpha$. The appearance of six parameters, a through f, is erroneous as only five of them are independent. Indeed, Band suspected as much when he analyzed the matrix of second derivatives of χ^2 with respect to his six parameters.

Let $A=(2b+1)/c$, $B=(a-b)/c$, $D=d/c$, $E=-(2d+e)/c$, $F=(d-f)/c$; Eqs. (3) then become

$$S/I=(\sin\alpha\cos\alpha)/(A\cos^2\alpha+B),$$
$$C/I=(D\sin 2\alpha)/(A\cos^2\alpha+B), \tag{4}$$
$$M/I=(E\cos^2\alpha+F)/(A\cos^2\alpha+B)\,.$$

We have inverted the S/I results to determine A and B, then held these parameters fixed while determining D, E, and F from the C/I and M/I results.

The curves for the $^1P^o$ data appear convincing while those of the ^1D may not. Perhaps the most noticeable discrepancy is in the total fractional polarization, $P_f=(M^2+C^2+S^2)^{1/2}/I$, which for the ^1D achieves a pronounced minimum near 45° while no such minimum is predicted theoretically.

Curiously the C/I data gives a better fit in the ^1D case than for the $^1P^o$. However, as the C/I polarization is so low, perhaps the only reliable conclusion which can be drawn is that C/I is positive for the $^1P^o$ terms and negative for the ^1D terms.

As mentioned previously, our errors have been estimated from the scatter of the ^1S results presented in Table III. We quote one standard deviation. Though the fitting procedure which yields the Stokes parameters from the data is a linear least-squares fit, the errors obtained from this fit are one half the values indicated by the C/I and S/I results from the ^1S data. The fitting procedure dictates that the uncertainties in M/I and

TABLE II. Polarization[a] of He I ^1D transitions.

Foil-tilt angle α (degrees)	$2p\,^1P^o$-$3d\,^1D$ 6678 Å				$2p\,^1P^o$-$4d\,^1D$ 4922 Å			
	M/I	C/I	S/I	P_f	M/I	C/I	S/I	P_f
0	6.8 ± 1.0	0.2 ± 1.0	−0.2 ± 0.4	6.8 ± 1.0	7.3 ± 0.5	−0.3 ± 0.5	0.0 ± 0.3	7.3 ± 0.5
10	7.1	−0.8	0.6	7.2 ± 1.0	7.9	−0.2	0.3	7.9 ± 0.5
20	7.6	0.7	1.4	7.8 ± 1.0	7.3	−0.8	0.2	7.4 ± 0.5
30	5.4	−1.2	1.6	5.8 ± 1.0	4.2	−2.2	−1.2	4.9 ± 0.5
40	1.8	−0.9	−0.3	2.0 ± 1.0	1.3	−1.9	−3.0	3.8 ± 0.4
50	−2.5	−1.0	−1.9	3.3 ± 0.9	−1.6	−2.3	−5.8	6.4 ± 0.4
60	−7.1	−1.0	−5.8	9.2 ± 0.8	−5.2	−4.0	−9.7	11.7 ± 0.4
70	−11.0	−2.3	−12.1	16.5 ± 0.7	−7.4	−4.4	−12.7	15.4 ± 0.4

[a] Values in percent. Uncertainties constant throughout column unless otherwise noted.

TABLE III. Polarization[a] of He I $2p\,^1P^o$–$4s\,^1S$ 5048 Å.

Foil-tilt angle α (degrees)	M/I	C/I	S/I
0	0.0 ± 1.1	−0.2 ± 1.1	−0.5 ± 0.7
10	−0.4	−0.2	0.3
20	0.3	−1.7	−0.7
30	−0.7	−1.4	0.1
40	0.3	−1.4	−0.4
50	1.2	0.7	−1.4
60	0.6	−1.3	−0.2
70	−1.1	−0.1	0.3

[a] Values in percent. Uncertainties constant throughout column.

C/I should be equal. Accordingly, the quoted uncertainties in Tables I–III are the computer values multiplied by two. This extra factor may be caused by fluctuations in the beam and/or foil degradation, the effects of which were not completely compensated by the optical normalization and multiple scans. Efforts to improve the normalization are underway.

V. SUMMARY

We have measured the Stokes parameters for transitions from two $^1P^o$ terms and two ^1D terms in He I. The form of their variation with tilt angle was found to be very similar for the two $^1P^o$ terms, and qualitatively similar for the two ^1D terms. However, this behavior of the Stokes parameters for the ^1D terms was found to be significantly different from that observed for the $^1P^o$ terms. In particular, the total polarization fraction, which had remained almost constant for all angles studied for the $^1P^o$ terms, showed a pronounced minimum at around 45° for the ^1D terms. Fits to the results based on the theory by Band failed to reproduce this minimum and were generally less than satisfactory for the ^1D data, in contrast with the rather good agreement found for the $^1P^o$ terms. Finally, it should be stressed that none of our data was able to conclusively demonstrate whether or not S/I becomes small at large tilt angle as would be predicted by the theory. It will be interesting to see if the discord between singlet states of different orbital angular momentum will be found in other spectra.

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