Decay rate of the $2\,^2S_{1/2}$ state of singly ionized helium

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(Received 1 June 1976; revised manuscript received 25 July 1977)

A precise measurement has been made of the lifetime of He^+ in the metastable $2\,^2S_{1/2}$ state. The result, $\Gamma_{2S} = 525 \pm 5\ \text{sec}^{-1}$, is consistent with the theory of spontaneous two-photon decay and places an upper limit $|\delta| < 2.4 \times 10^{-5}$ on the amplitude of any parity-nonconserving $2P$ admixture.

I. INTRODUCTION

In this paper we report a precise measurement of the lifetime of hydrogenic He^+ in the metastable $2\,^2S_{1/2}$ state. The metastability of the $2S$ state and its closeness in energy to the $2P_{1/2}$ state allow one to test crudely for the presence of terms in the Hamiltonian which do not conserve parity. In the absence of such terms, decay proceeds by two-photon $E1$ and single-photon $M1$ transitions, the former being more probable by 5×10^6.

There have been several experiments to observe the decay of hydrogenic metastables. After a preliminary lifetime measurement[1] on He^+, the two-photon nature of the decay was conclusively established, and the angular correlation of photon pairs was observed.[2] In addition, the gross features of the two-photon spectrum were studied.[3] A summary of this work is found in Ref. 4. In 1972, Prior[5] reported an experiment in which the $2S$ lifetime of He^+ was measured to ±4% by detecting decay photons from an ensemble of excited ions stored in an electromagnetic trap. A beam experiment by Kocher *et al.*[6] confirmed Prior's result. Less precise measurements[7] have been made on H and the hydrogenic ions of Ar, S, F, and O. The present experiment has yielded a value of the decay rate Γ_{2S} in He^+ which is accurate to ±1%.

II. APPARATUS

The apparatus we have used is basically that described by Kocher *et al.*,[6] but substantial modifications, which will be mentioned where appropriate, have increased the precision by a factor of 25. Figure 1 is a diagram of the apparatus. Ions, produced by the electron bombardment of helium gas, are focused electrostatically to form a beam which is injected into a drift tube. Decay of the $2S$ ions is observed by means of a traveling detector.

The electron bombardment source is of the type described by Dworetsky *et al.*[8] with an additional grid inserted close to the cathode (Phillips impregnated type B). When this grid is used to give a space-charge-limited electron current of about 15 mA, we obtain a more stable ion beam than is possible with temperature-limited cathode emission. The emitted electrons, accelerated to 200 eV, ionize and excite He atoms within the anode cage of the source where the density is of order $10^{14}\ \text{cm}^{-3}$. A series of electrostatic lenses collimates the beam as it passes through a differential pumping chamber and enters the drift tube. Once the beam is inside the drift tube, collimation is maintained over its 8-m length by an axial magnetic field of about 70 G. The beam energy is chosen to be 6 or 12 eV, and in each case the energy spread is

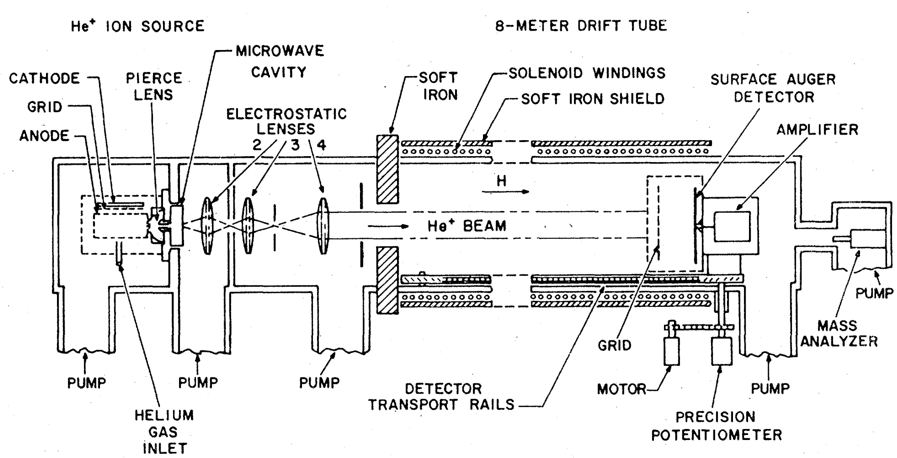

FIG. 1. A schematic diagram of the apparatus.

about ±0.14 eV. The detected beam intensity is ~1×10^{11} ions sec^{-1} of which about 0.8% are in the 2S state.

The detector consists of a stainless-steel plate, 15 cm in diameter. As He^+ ions approach its surface, secondary electrons are released from the detector in an Auger-type process,[9] and a bias potential of −25 V ensures that they are accelerated away from the plate. Thus the observed current is the sum of the incoming ion current and the outgoing current due to secondary electrons. A metastable ion, having 40 eV of internal energy with respect to the 1S ion, gives a larger secondary-electron yield (≈0.5 for 2S and 0.2 for 1S), and this difference is used in the following way to distinguish the two states in the beam. Ions leaving the source pass through a 14-GHz microwave cavity, driven on resonance by a klystron which is switched on and off at 19 Hz. When the klystron is on, the 2S and $2P_{1/2}$ states are strongly coupled by the microwave field, and total quenching of the metastable population results. Thus there is an ac current at the detector proportional to the metastable population and to the difference in secondary-electron yields mentioned above. This current is about 4×10^{-11} A, peak to peak, while the dc signal is 1.6×10^{-8} A. At the same time, the entire beam is periodically pulsed off by modulating the potential of the anode cage in the source. These pulses, which are 0.5-msec wide and have a 67-Hz repetition rate, provide a means of measuring the flight time of the beam and will therefore be called timing pulses. The total current at the detector is amplified by a Varactor bridge operational amplifier (Analog Devices type 310K with a 10^8-Ω feedback resistor) mounted with the detector on a continuously movable platform. The output voltage of this amplifier is the sum of three parts: a dc total beam signal V_0, a 19-Hz square-wave metastable signal V_m, and the 67-Hz timing pulses.

Each pulse applied to the source starts a clock (Hewlett-Packard model 5245L electronic timer). The corresponding timing pulse appears at the detector after a delay determined by the time of flight of the beam, and a timing single-channel analyzer (Ortec type 420A) is used to stop the clock at the peak of the timing pulse. As the detector is moved along the drift tube, a Nova 1220 minicomputer reads the clock every 0.7 sec and stores each reading as a point on the abscissa of a graph. Corresponding ordinate points are obtained simultaneously as follows. The metastable signal V_m is phase sensitively detected using a lock-in amplifier (Princeton Applied Research model HR-8) with a 1-sec output time constant. The dc signal V_0 is also smoothed with a 1-sec time constant. A logarithmic-ratio amplifier (utilizing Philbrick-Nexus module type 4358) generates a voltage proportional to $\ln(V_m/V_0)$ which is digitized and read by the computer as the ordinate signal. When the ratio is taken in this way, the metastable signal is normalized to compensate for slow changes in beam intensity due to instability in the source or geometric losses along the drift tube. Thus the exponential decay of the metastable beam is measured as a linear plot, the slope of which is equal to the decay rate. After each detector sweep along the drift tube, the computer fits the stored points to a straight line to determine the decay rate.

III. RESULTS

Figure 2 shows a typical decay curve, photographed from a storage oscilloscope which displays the data read by the computer. In this run the beam energy was 6 eV and the uncorrected decay rate was 536.5 sec^{-1}. Individual points fit the best straight line with a standard deviation $\delta[\ln(V_m/V_0)] = 8\times10^{-3}$, to be compared with a total change in $\ln(V_m/V_0)$ of 0.23. In this sense the signal-to-noise ratio is 29:1. When the detector is stationary, we find that signal fluctuations are largely due to statistical noise in the beam, but when a sweep is made, there are systematic fluctuations of comparable size which are evident in Fig. 2. These oscillations occur because the secondary-electron yield is not constant over the detector surface, and since the ions travel helically along the drift tube, there is a cyclic variation of detection efficiency during a sweep. The phase of these oscillations was randomly varied before each detector sweep by making small arbitrary adjustments to the potential on one of the beam focusing lenses. Thus any associated error appears in the observed spread of our experimental results.

We made 82 such sweeps using a 6-eV ion beam

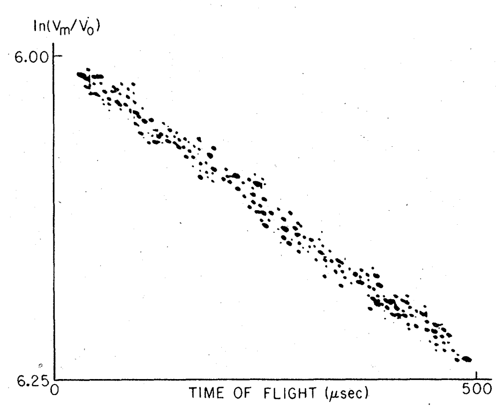

FIG. 2. A typical experimental decay curve.

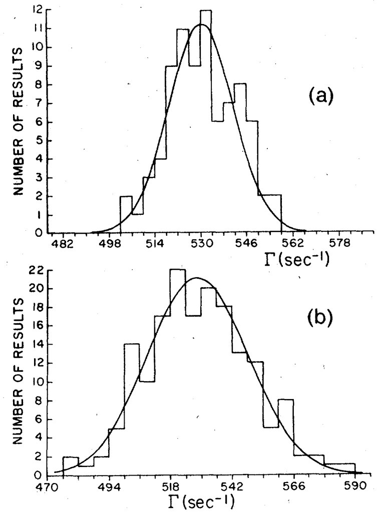

FIG. 3. (a) Raw data obtained with a 6-eV beam. The curve is a Gaussian probability distribution having the same mean and standard deviation as those calculated from our results. (b) The corresponding graphs for 12-eV data.

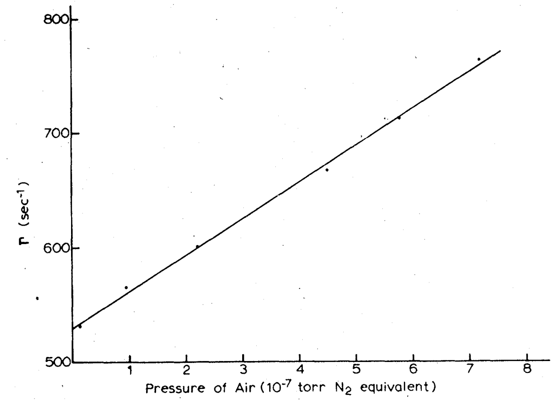

FIG. 4. A graph of apparent decay rate vs pressure of air in the drift tube. The line is a least-squares fit.

and 172 sweeps using a 12-eV beam. For the 6-eV beam we find the mean and standard deviation of one measurement to be $\Gamma = 530.1 \pm 11.3$, while the 12-eV beam gives $\Gamma = 528.7 \pm 19.7$. Figures 3(a) and 3(b) show histograms of our raw results, together with Gaussian distributions having the same mean and standard deviation. The good agreement between our results and the Gaussian curves suggests that a statistical treatment of our results is appropriate. Table I gives the mean and standard deviation of the mean for each set of data and also summarizes a number of small systematic error corrections which we now discuss.

The metastable beam current has a dc component which contributes to V_0 and which decays with the metastable beam. Hence the measured, normalized decay rate is too small, as discussed in Appendix A. The appropriate correction is to add $\Gamma \overline{V}_m / V_0$ to the measured value of Γ, where $\overline{V}_m$ is the average value of V_m over the sweep and V_0 is the dc signal. This correction is shown in Table I.

A second correction involves extrapolating our result to zero-background gas pressure in the following way. A quadrupole mass spectrometer (Electronic Associates, Inc. model Q1110A) has been used to identify the residual gas species as N_2, O_2, CO_2, and He. When a small air leak is opened, the relative pressures of N_2, O_2, and CO_2 are unchanged; presumably these residual gases are the result of small leaks rather than outgassing. Residual He originates entirely from the source gas. By interrupting its flow, we find that under normal running conditions the mean partial pressures in the drift tube are typically 1.5×10^{-8} Torr of air and 1.3×10^{-9} Torr of He as measured on ionization gauges (Veeco RG 75N) calibrated for N_2. We have measured the apparent decay rate as a function of pressure up to 10^{-6} Torr N_2, using both He and air. A typical set of results for air with a 6-eV beam is given in Fig. 4. Hence the result of each run can be linearly extrapolated to zero pressure. Table I shows how the mean decay rate is affected after each run has been individu-

TABLE I. Summary of results for 6- and 12-eV beam energies.

	6-eV beam (sec^{-1})	12-eV beam (sec^{-1})
Average uncorrected decay rates	530.1 ± 1.3	528.7 ± 1.5
Systematic error corrections:		
dc component of metastable beam	$+0.7 \pm 0.1$	$+0.8 \pm 0.1$
Residual air	-4.5 ± 0.5	-6.1 ± 0.7
Residual helium	$+0.9 \pm 0.2$	$+1.3 \pm 0.3$
Velocity spread	-1.7 ± 0.2	-0.5 ± 0.1
Stark quenching	-0.2 ± 0.1	-0.2 ± 0.1
Corrected decay rates	525.3 ± 1.4	524.0 ± 1.7
Γ_{2S}: combined result	524.8 ± 1.1 sec^{-1}	

ally corrected in this way. The residual gas corrections account not only for collisional deexcitation of the $2S$ beam, but for all collisional mechanisms which attenuate the $1S$ and $2S$ ions differently and are therefore not corrected by the signal normalization.

Next we consider the distribution of axial velocities in the beam. Figure 5 shows the axial kinetic-energy distribution of 6-eV metastable ions, measured by applying a variable retarding potential to the detector grid. A similar plot has been made for the ground-state beam. The peak energies differ by less than ±0.01 eV, and hence the most probable flight time for metastable ions is within ±0.08% of that for the ground state. Thus, although the timing pulses are largely generated by the ground-state beam, they effectively measure the most probable flight time of $2S$ ions. This was also found to be so for the 12-eV beam. Since the metastable signal V_m is a convolution of the exponential decay with the whole velocity distribution, a systematic error arises in our measured result as discussed in Appendix B. Numerical integration of the measured velocity distributions yields the corrections given in Table I.

Lastly we make a very small correction for quenching of the metastable ions by the motional electric field. We have measured the decay rates as a function of magnetic field up to 250 G. Our results show that the quenching rate at 70 G, the magnetic field used in our experiment, is 0.2 $\pm 0.1\ \mathrm{sec}^{-1}$ for both the 6- and the 12-eV beam.

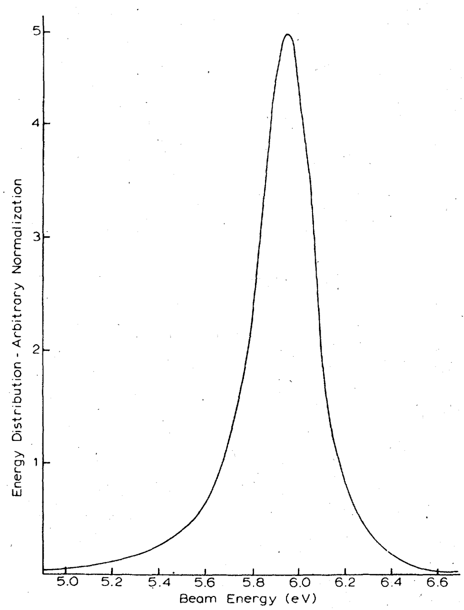

FIG. 5. Energy distribution of the metastable beam. The energy in question is that associated with the axial velocity component.

This is consistent with a detailed analysis of the quenching given elsewhere.[10]

The other possible systematic errors which we have considered are found to be negligibly small. Some of the more important ones are discussed here.

Any component of the beam which is modulated by the microwaves can, in principle, contribute to the measured decay rate. For example, highly excited ions created in the source may become doubly ionized in the microwave field, thereby contributing to the modulated beam current. However, before reaching the microwave cavity, an ion must pass through the Pierce lens (Fig. 1) in which there is a significant static electric field. A computer solution of Poisson's equation[11] shows that the maximum electric field experienced by an ion in the Pierce lens is at least 348 V cm^{-1}, depending on its trajectory. The threshold for ionization in this field is given by[12] $n^{-4}=2E$ a.u., where n is the principal quantum number at threshold. Hence ions having $n \geqslant 53$ are doubly ionized in the Pierce lens and cannot be modulated by the microwave field. The power in the microwave cavity was adjusted to give a $2S$ quenching probability of 0.95. Since we know the beam energy in the cavity (40 ± 4 eV), the cavity length (5 mm), and the microwave quenching rate near resonance ($2.8\times10^4E^2$), it is easily shown that the microwave field strength is $E=31\pm1$ V cm^{-1}. Thus the threshold for microwave ionization, given by $n^{-4}=4E$ a.u.,[12] is $n=80$. It follows that the modulated part of the beam does not include highly excited states.

Another possibility is that the ground-state beam is slightly modulated in some way so that the metastable signal includes a small constant component; $V_m=A(e^{-\Gamma t}+\delta)$. It is readily shown that the measured decay rate would then be approximately $(1-\delta)\Gamma$ for small δ. In looking for such an effect, we have reduced the energy of the electron beam in the source below the threshold for metastable production; the ground-state beam intensity is essentially unchanged. Under these conditions the lock-in amplifier detects no signal at the level $\delta\approx10^{-3}$. This result gives us some confidence that V_m indeed consists of a sufficiently pure, metastable beam decay signal.

An exactly analogous error occurs if the signal acquires a dc voltage offset in the electronic processing. We have avoided introducing an offset in the lock-in amplifier by zeroing its output every few runs. This is easily accomplished within ±10 mV, which corresponds to $|\delta|\lesssim2\times10^{-3}$, and merely contributes to the observed spread of results. The logarithmic amplifier has been calibrated each day by applying suitable, accurately known voltages to its inputs and reading the output directly into the computer. In this way, we compensate, to first order in δ, for the input offsets which are present (≈ 5 mV). We conclude that electronic offsets are not a source of error.

Nonlinearity of the lock-in amplifier can also introduce a systematic error. Suppose that the amplifier response is quadratic so that $V_{out} = a + bV_{in} + cV_{in}^2$. One can readily show that the measured decay rate due to a metastable signal $V_0 e^{-\Gamma t}$ is then approximately $\Gamma(1 + cV_{in}/b)$. We have measured the linearity of the HR-8 lock-in amplifier on the 2-mV scale and find that $cV_{in}/b = (-3 \pm 4) \times 10^{-4}$ at $V_{in} = 1.4$ mV, a typical running voltage. The corresponding error in Γ is negligible.

As the detector is swept along the drift tube, the flight time of the beam introduces a varying phase angle ϕ between the metastable signal and the reference oscillator in the lock-in amplifier. It is shown in Appendix B that this produces an error in the measured decay rate of $\omega\overline{\phi}$ sec^{-1}, where ω is the angular frequency of the modulation and $\overline{\phi}$ is the mean of ϕ over a sweep. In order to minimize $\overline{\phi}$, the detector is placed at the center of a sweep, and a sensitive phase shifter is used to minimize the signal. In this way we can set $\overline{\phi}$ to $\frac{1}{2}\pi$ with an uncertainty, due to noise on the beam, of $\pm 5 \times 10^{-3}$ rad. An accurate phase shift of $-\frac{1}{2}\pi$ is then introduced digitally, leaving $\overline{\phi} = (0 \pm 5) \times 10^{-3}$ rad or $\omega\overline{\phi} < 0.6$ sec^{-1}. The phase was reset every few runs so that this effect appears in the observed spread of results and not as a systematic error.

IV. CONCLUSION

It is not obvious how to combine the 6- and 12-eV results. Conceivably there is still some energy-dependent systematic effect which has not been considered, although the two results do agree within the errors. In the absence of any model for possible residual energy dependence, we take as our final result a mean value, $\Gamma_{2S} = 524.8$ sec^{-1}, in which each of the results is weighted inversely by the square of its standard error. Our final random error is ± 1.1 sec^{-1}, obtained by averaging the two random errors in quadrature. Although there is no apparent disagreement between the 6- and 12-eV results, we feel that a reasonable estimate of the maximum systematic error in our final result is ± 4 sec^{-1}. Hence

$$\Gamma_{2S} = 525 \pm 5 \text{ sec}^{-1}. \tag{1}$$

This result is to be compared with the theoretical decay rate,[13] $\Gamma_{theor} = 526.51 \pm 0.02$ sec^{-1}. Note that our result has been obtained on the assumption that the decay signal V_m is a pure exponential. Some observations of the $M1$ transitions, $2\,^3S_1$-$1\,^1S_0$, in heliumlike ions have yielded nonexponential decay curves.[14] Lin and Armstrong[15] have recently proposed that this behavior can be explained by contamination of the heliumlike beam with lithiumlike ions in which one of the electrons is in a high Rydberg state and the other two electrons are in the $(1s2s)\,^3S_1$ state. They performed a detailed calculation that accounts for the nonexponential decay of heliumlike atoms. The analogous situation in the present experiment would consist of contamination of the hydrogenic beam with helium atoms in which one electron is in the $2S$ state and the other electron is in a high Rydberg state. Such atoms would, of course, be neutral and would not be focused by the lens system and would not be trapped by the magnetic field. Thus we are confident that this effect cannot be of importance to the present experiment. We have taken great care to ensure that V_m is truly proportional to the metastable beam intensity. These considerations lead us to assume that the decay was indeed exponential, and to interpret our result as a confirmation of the theory of two-photon spontaneous decay.

A 3-σ random deviation from our experimental result plus the largest plausible systematic error (4 sec^{-1}) sets an upper limit on the true decay rate of 532.1 sec^{-1}. This limits the possible $E1$ decay rate Γ_a to 5.6 sec^{-1}, assuming the two-photon rate Γ_{theor} to be correct. If we write the wave function of the metastable state as $\psi = \psi_{2S} + \delta\psi_{2P}$, the limit on Γ_a places a general restriction on possible values of the parity mixing amplitude

$$|\delta| < 2.4 \times 10^{-5}. \tag{2}$$

Most of the possible parity-nonconserving interactions are too small to give rise to observable effects at the level of sensitivity of the present experiment, e.g., neutral current weak interactions,[16-18] higher-order charged current interactions,[19] and those involving an electronic[20] or nuclear[21] electric dipole moment. The current situation is reviewed in Ref. 17. In terms of the pseudocharge interaction,[22,23] our limit on $|\delta|$ implies

$$|\lambda| < 1.9 \times 10^{-5}, \tag{3}$$

where λ is the coupling constant of Feinberg.[23]

ACKNOWLEDGMENT

This work was supported by the NSF under Grant Nos. GP 13479 and MPS 75-14066.

APPENDIX A: CORRECTION FOR DECAY OF THE dc COMPONENT OF THE METASTABLE BEAM

We write the detector output voltage due to a metastable beam current I_m and a ground-state beam current I_g as $I_m\gamma_m$ and $I_g\gamma_g$, respectively. Here γ_m and γ_g are proportional to one plus the

Townsend coefficient for metastable-state and ground-state ions, respectively. When the quenching microwaves are turned off, the detector output voltage is easily shown to be

$$V_{\rm off}=I_m e^{-\Gamma t}(\gamma_m-\gamma_g)+(I_g+I_m)\gamma_g\,, \qquad \text{(A1)}$$

where t is the time of flight from source to detector, Γ is the metastable decay rate, and I_m is the metastable beam current at the source. When the quenching microwaves are on, the detector output voltage is

$$V_{\rm on}=(I_g+I_m)\gamma_g\,. \qquad \text{(A2)}$$

Hence the metastable signal voltage is given by

$$V_m=\tfrac{1}{2}(V_{\rm off}-V_{\rm on})=\tfrac{1}{2}I_m(\gamma_m-\gamma_g)e^{-\Gamma t}\,, \qquad \text{(A3)}$$

and the dc signal is

$$V_0=\tfrac{1}{2}(V_{\rm on}+V_{\rm off})=V_{\rm on}+V_m\,. \qquad \text{(A4)}$$

The quantity which we measure is $S=-(\partial/\partial t)\times\ln(V_m/V_0)$. It is readily shown that, to first order in $V_m/V_{\rm on}$, S is given by

$$S\approx\Gamma(1-V_m/V_{\rm on})\approx\Gamma(1-V_m/V_0)\,. \qquad \text{(A5)}$$

Since the measured decay rate is given by the average slope of the decay curve, our result is too small by a fractional amount $\bar{V}_m/V_0$, where $\bar{V}_m$ is the average metastable signal over the sweep and V_0 is the dc signal.

APPENDIX B: ANALYSIS OF EFFECTS DUE TO BEAM VELOCITY DISTRIBUTION AND PHASE ANGLE OF THE SIGNAL

The metastable signal V_m at the output of the lock-in amplifier may be written

$$V_m=k\int_0^\infty f(v)e^{-\Gamma z/v}\cos\left(\theta+\frac{\omega z}{v}\right)dv\,, \qquad \text{(B1)}$$

where v is the axial velocity, $f(v)$ is the axial velocity distribution function, z is the distance from source to detector, ω is the modulation frequency, and θ is the phase setting of the lock-in amplifier. It is convenient to write $v=v_0+u$, where v_0 is the most probable velocity in the beam. For our beam, u/v_0 is small ($\sim10^{-2}$) wherever $f(v)$ is appreciable, and hence we can usefully expand Eq. (B1) to first order in u/v_0, thus

$$V_m\approx ke^{-\Gamma z/v_0}\int_{-v_0}^\infty f(u)\left[1+\frac{\Gamma z}{v_0}\left(\frac{u}{v_0}\right)\right]\cos\phi\,du\,, \qquad \text{(B2)}$$

where $\phi=\theta+\omega z/v$ and $\Gamma z/v_0<1$. With a proper choice of θ, we ensure that ϕ is small, varying between $\pm3\times10^{-2}$ rad over the detector sweep. $\cos\phi$ is then essentially constant over the range of velocities present in the beam, and an adequate approximation for our purposes is

$$V_m\approx ke^{-\Gamma t}\cos\phi\left[\int_{-v_0}^\infty f(u)du+\Gamma t\int_{-v_0}^\infty f(u)\left(\frac{u}{v_0}\right)du\right]\,, \qquad \text{(B3)}$$

where $t=z/v_0$ is the time of flight measured by the timing pulses. The second integral in Eq. (B3) will be called A. Then since $f(u)$ is normalized to unity,

$$V_m=ke^{-\Gamma t}\cos\phi(1+A\Gamma t)\,, \qquad \text{(B4)}$$

in which $A\Gamma t\ll1$. Hence the measured decay rate is given approximately by

$$-\frac{\partial}{\partial t}\ln V_m=\Gamma+\omega\phi-A\Gamma\,. \qquad \text{(B5)}$$

Since ϕ is proportional to t, the best-fitting straight line to our data will yield the result

$$-\left\langle\frac{\partial}{\partial t}\ln V_m\right\rangle=\Gamma+\omega\bar{\phi}-A\Gamma\,, \qquad \text{(B6)}$$

where $\bar{\phi}$ is the average of ϕ over the run.

The term $\omega\bar{\phi}$ in Eq. (B6) is due to phase shift of the metastable signal associated with the time of flight. The term $A\Gamma$ reflects the distribution of time of flight around the measured, most probable value. Using the measured velocity distributions, we find by numerical integration that $A=-(33\pm4)\times10^{-4}$ for the 6-eV beam and $A=-(9\pm2)\times10^{-4}$ for the 12-eV beam.

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