PHYSICAL REVIEW A VOLUME 17, NUMBER 1 JANUARY 1978

# Comments and Addenda

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## Comment on "Theory of resonance fluorescence"

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(Received 14 July 1977)

We comment on the work of Kimble and Mandel with respect to Markovian approximations and the Lamb shift. The importance of the quantum properties of the free electromagnetic field in spontaneous-emission calculations is discussed.

A contribution to the understanding of resonance fluorescence[1] is due to Kimble and Mandel[2] (KM), who have performed very interesting calculations of the fluorescence spectra. In the comment, we shall discuss the quantum-electrodynamic (QED) details of their work which are important for a correct Heisenberg-picture QED calculation, but do not imply any significant errors with respect to their primary goal, an understanding of resonance fluorescence. In particular, we will show that their separation of the field into creation and destruction parts leads to an incorrect expression for the Lamb shift and explains their not needing to explicitly make a Markovian approximation. We will also show that the quantum properties of the free electromagnetic field play an essential role in spontaneous-emission calculations.[3] The basic sections of Ref. 2(b) to which we will continually refer are Sec. II, "Derivation of the equations of motion," and Sec. III, "Spontaneous emission." The equations and claims which appear in detail in Ref. 2(b) were first published in Ref. 2(a).

The solution for the ± components of the vector potential along the direction of the atomic dipole moment evaluated at the atom, (21) of KM, is

$$\vec{\mu}\cdot\hat{A}_{\pm}(0,t)=(\hbar/\omega_0)(\beta\mp i\gamma)\,\hat{b}(t)+\vec{\mu}\cdot A_{\pm}^{\text{free}}(0,t)\,. \quad (1)$$

Since the transverse current, (9) of KM, is distributed over all space, (1) shows a Markovian approximation is made in solving the wave equation for the ± parts of the vector potential. In addition, it is well-known that the long-time dynamics of the atomic operators is nonexponential and proportional to $1/t^2$ (Ref. 4). As an equal-time equation, (1) can only lead to exponential dynamics. The long-time dynamics proportional to $1/t^2$ are intimately related to the logarithmic singularity[4,5] of the Bethe nonrelativistic Lamb-shift formula. By looking at $\gamma$, the parameter identified with the Lamb shift by KM, Eq. (18) of KM,

$$\gamma\equiv\frac{1}{4\pi\epsilon_0}\,\frac{2\mu^2\omega_0^2}{3\hbar^2c^2l}\,, \quad (2)$$

we observe that as $l\to 0$, $\gamma$ is linearly divergent, not logarithmically divergent. The absence of the logarithmic divergence in $\gamma$ is closely tied together with the effective Markovian approximation made by KM.

Ackerhalt, Eberly, and Knight[6] (AEK) made a calculation analogous to the one appearing in Sec. II of KM. They include a special form of retardation convergence. In addition, a field-mode decomposition and a Markovian approximation are explicitly used. The important point is that they solve for the creation and destruction parts of the vector potential by simply equating positive and negative frequency parts of the field and the atomic current, in the same way as KM. The shift which appears in their calculations, essentially $\gamma$, is simply twice the mass-renormalization part of the energy-level shift and is linearly divergent. The logarithmic Lamb-shift formula does not appear. The error in the calculation is pointed out by AEK: the correct equations of motion for the creation and destruction parts of the field can only be derived using the Hamiltonian and the canonical commutation relations. Otherwise, the separation is ill-defined and not unique. In conclusion AEK show that a correct separation of the creation and destruction parts of the vector potential lead to a proper logarithmically divergent shift in agreement with standard stationary-state perturbation theory.

In a subsequent paper by Ackerhalt and Eberly[7] (AE) the wave equation for the creation (+) and destruction (−) parts of the vector potential is derived directly from the Hamiltonian using the canonical commutation relations

$$\Box \vec{A}_{\pm}(\vec{r},t) = -\frac{2\pi}{c}\vec{J}(\vec{r},t) \pm \frac{i}{\pi c^2}\int d^3r' \frac{d\vec{J}(\vec{r}',t)/dt}{|\vec{r}-\vec{r}'|^2} . \tag{3}$$

The second term on the right side of (3) is missing in the work of KM. The absence of this term is totally responsible for the Markovian character of their work. If this term is included in their calculation, then the correct frequency shift and the correct non-Markovian time evolution result.

Milonni, Ackerhalt, and Smith, Senitzky, and Milonni and Smith[8] showed that the ordering of atom and field operators in the atomic operator equations of motion is unimportant with respect to obtaining the correct final equations of motion. However, the Lamb shift can originate from either the source field or from the free-field parts of the vector potential making an interpretation of the Lamb shift a function of the ordering. Using normal ordering, the Lamb shift results entirely from the second term of the source current in (3). Kimble and Mandel use normal ordering, but they do not include the second term of the source current in (3) obtaining an incorrect expression for the Lamb shift. Since the source-field part of the vector potential operates on the entire Hilbert space and time evolves due to the interaction of both the atomic and field degrees of freedom, the radiation-reaction interpretation of the Lamb shift does not imply that the atomic degrees of freedom are solely responsible for the origin of the Lamb shift. In a one-sided or symmetric ordering, where only the total vector potential $\vec{A}_+ + \vec{A}_-$ appears and the second term of the source current in (3) makes no contribution, the Lamb shift results totally from the free-field part of the vector potential. Since the free-field part of the vector potential operates essentially only in the Hilbert space of the field for all time (it is the identity operator in the atomic Hilbert space), the vacuum-field fluctuation interpretation of the Lamb shift does imply that the field degrees of freedom are the origin of the Lamb shift. Therefore, the quantum properties of the free field either directly or indirectly, through the time evolution of the additional source term in (3), are the origin of the Lamb shift in Heisenberg-picture QED spontaneous-emission calculations. A measurement of the Lamb shift is consequently an excellent test of the important role played by the quantum properties of the field.[9]

## ACKNOWLEDGMENT

I would like to thank J. H. Eberly, P. L. Knight, and P. W. Milonni for their helpful comments and criticisms of the text.

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[1]For a recent review of the present status of resonance fluorescence research, see the proceeding and abstracts of the International Conference on Multiphoton Processes, June 6–9, 1977.

[2](a) H. J. Kimble and L. Mandel, Phys. Rev. Lett. 34, 1485 (1975); (b) H. J. Kimble and L. Mandel, Phys. Rev. A 13, 2123 (1976); (c) H. J. Kimble and L. Mandel, Phys. Rev. A 15, 689 (1977).

[3]It should be emphasized that for KM, a test of the QED nature of a process rests upon whether the quantum properties of the free field play a role. For example, they state "still insofar as Eqs. (32) and (33) are derivable without explicit introduction of the operator character of the free field, it does seem that measurement of spontaneous emission in a two-level transition, under different conditions of excitation, does not represent the best test of the quantum nature of the field. We shall see that the phenomenon of resonance fluorescence offers an opportunity for a more convincing test."

[4]P. L. Knight, Phys. Lett. 61A, 25 (1977), and references therein; P. L. Knight and P. W. Milonni, Phys. Lett. 56A, 275 (1976), and references therein; K. Wódkiewicz and J. H. Eberly, Ann. Phys. (N.Y.) 101, 574 (1976); C. A. Nicolaides and D. R. Beck, Phys. Rev. Lett. 38, 683 (1977); and errata in Phys. Rev. Lett. 38, 1037 (1977).

[5]H. A. Bethe, Phys. Rev. 72, 339 (1947).

[6]J. R. Ackerhalt, J. H. Eberly, and P. L. Knight, in *Coherence and Quantum Optics*, edited by L. Mandel and E. Wolf (Plenum, New York, 1973), p. 635.

[7]J. R. Ackerhalt and J. H. Eberly, Phys. Rev. D 10, 3350 (1974).

[8]I. R. Senitzky, Phys. Rev. Lett. 31, 955 (1973); P. W. Milonni, J. R. Ackerhalt, and W. A. Smith, Phys. Rev. Lett. 31, 958 (1973); P. W. Milonni and W. A. Smith, Phys. Rev. A 11, 814 (1975).

[9]A summary of many of the insights and calculations relevant to this comment have been given by P. W. Milonni, Phys. Rep. 25, 1 (1976).