PHYSICAL REVIEW A VOLUME 17, NUMBER 1 JANUARY 1978

Stark broadening in hot, dense, laser-produced plasmas: A two-component, two-temperature formulation*

Richard J. Tighe and C. F. Hooper, Jr.
Physics Department, University of Florida, Gainesville, Florida 32611
(Received 1 August 1977)

Broadened Lyman-α and Lyman-β x-ray profiles from hydrogenic, high-Z radiators immersed in a hot, dense plasma are presented. Particular attention is given to broadened profiles originating in a plasma where there are two ionic species and in which ions and electrons have different temperatures. The applicability of these calculations to early laser-fusion experiments is discussed.

I. INTRODUCTION

Modifications to the formalism needed for the calculation of static-ion electric microfield distribution functions for parameter ranges of interest in the study of laser-produced plasmas were presented recently.[1] These extended microfield calculations incorporated the following properties: (i) the radiating hydrogenic ion may have any net positive charge; (ii) the perturbing ions and electrons are allowed to have different kinetic temperatures; (iii) two species of perturbing ions, of charge z_1 and z_2, may be present in any given proportion.

In the present work, the distribution functions presented in Ref. 1 (hereafter referred to as paper II) are employed in the computation of Stark-broadened x-ray line profiles from hydrogenic ions immersed in a hot dense plasma. Lyman-α and Lyman-β profiles are calculated for different values of the parameters mentioned above.

In Sec. II, we briefly describe the formalism needed for the line-profile calculation. Following this, in Sec. III, we present and discuss our results. Closing remarks, and a critique, are contained in Sec. IV.

II. LINE PROFILE FORMALISM

When ion broadening effects are treated by the quasistatic ion approximation, the intensity distribution for emission of spontaneous electric dipole radiation is given by

$$I(\omega)=\int P(\epsilon)J(\omega,\epsilon)\,d\epsilon\,, \tag{1}$$

where $P(\epsilon)$ is the appropriate distribution function for the static-electric field due to the ions. The calculation of this function is the subject of Paper II. The function $J(\omega,\epsilon)$ contains averaged electron broadening effects calculated for a radiating-hydrogenic ion in a static-electric field ϵ,

$$J(\omega,\epsilon)=-\pi^{-1}\mathrm{Im}\,\mathrm{Tr}_r\{\vec{\mathrm{d}}\cdot[\Delta\omega-e\epsilon R^z/h-H(\omega)]^{-1}\rho_r\vec{\mathrm{d}}\}\,. \tag{2}$$

The notation here follows that of Ref. 2.

When the interaction between the radiator and the perturbing electrons is chosen to be a sum of point-dipole interactions, a second-order perturbation calculation for $H(\omega)$ gives

$$H(\omega)_{\mu\mu'}=\hbar^{-2}\Gamma(\Delta\omega)\sum_{\mu''}\vec{\mathrm{R}}_{\mu\mu''}\cdot\vec{\mathrm{R}}_{\mu''\mu'}\,, \tag{3}$$

where $\Gamma(\Delta\omega)$ is a complex function discussed previously.[2] In the earlier calculations, only Lyman-α transitions were considered, for which the sum in Eq. (3) and the matrix inversion in Eq. (2) were carried out analytically. Here we wish to generalize the previous results to allow calculation of any given Lyman-series line profile. This is made possible by the following analytic form for the sum in Eq. (3),

$$\begin{aligned}A_{\mu\mu'}&=\sum_{\mu''}\vec{\mathrm{R}}_{\mu\mu''}\cdot\vec{\mathrm{R}}_{\mu''\mu'}\\&=\delta_{mm'}(9a_0^2n^2/4Z^2)(-1)^{1+m-n-(q+q')/2}\\&\quad\times\sum_l(2l+1)\begin{pmatrix}\frac{n-1}{2}&\frac{n-1}{2}&l\\\frac{m-q}{2}&\frac{m+q}{2}&-m\end{pmatrix}\begin{pmatrix}\frac{n-1}{2}&\frac{n-1}{2}&l\\\frac{m-q'}{2}&\frac{m+q'}{2}&-m\end{pmatrix}(n^2-l^2+l-1)\,.\end{aligned} \tag{4}$$

In this expression, n, q, m, etc., are the parabolic quantum numbers. Also, a_0 and Z are the Bohr radius and the nuclear charge of the radiator, respectively. The 3-j symbols in this expression are computed by FORTRAN subroutines found in Ref. 3. With the above expressions, the matrix inversion may be carried out numerically to obtain $J(\omega, \epsilon)$.

III. RESULTS

In the first four figures, profiles are given which illustrate the effect of a variation in the parameter, T_R,

$$T_R = kTe/kT_i = \Theta_e/\Theta_i \,. \tag{5}$$

Recall from Paper II that the plasma parameter a $(=r_0/\lambda_D)$ is regarded as fixed by the electron temperature and density. This means that for fixed a, an increase in the parameter T_R is accompanied by a decrease in the ion temperature.

The first two figures demonstrate the behavior of Lyman-α profiles when T_R is varied. Figure 1 shows Lyman α for hydrogenic neon $(Z=+10)$ when perturbing ions have charge $Z_p=+1$. Figure 2 shows Lyman α for hydrogenic aluminum $(Z=+12)$ perturbed by ions of charge $Z_p=+12$. The next two figures show Lyman-β profiles for several values of T_R. In Fig. 3 we have Lyman β for hydrogenic aluminum $(Z=+12, Z_p=+12)$. Figure 4 gives Lyman β for hydrogenic argon $(Z=+17, Z_p=+1)$.

There are three facts which can immediately be noticed from these figures. First, the variations in the line center are primarily due to the dependence of the Doppler broadening on the ion temperature. Second, the structure of the line wings follows the form of the electric microfield functions displayed in Paper II. Finally, in the shoulder (dip) region of Lyman α (Lyman β) there is a significant amount of structure which may have diagnostic potential. Furthermore, it should be mentioned that the two-temperature formalism makes possible the calculation of time-averaged line profiles, which may become necessary in the study of laser-produced plasmas.

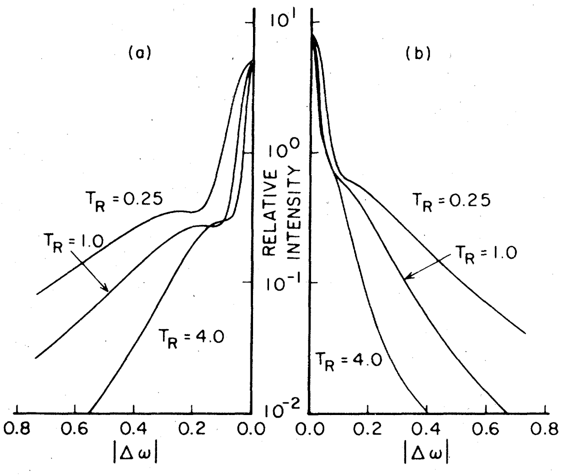

FIG. 2. Lyman-α line profiles for Al XIII. The electron kinetic temperature is (a) 1019.2 eV. (b) 254.8 eV. The electron density is 2.0×10^{23} cm^{-3}. Ion perturbers have a charge of +12.

The final two figures present line profiles computed for various values of the parameter R. This parameter is the ratio of the density of charge-z_2

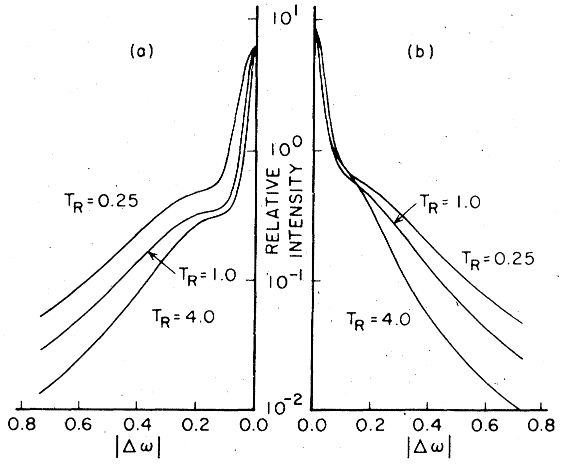

FIG. 1. Lyman-α line profiles for Ne X. The electron kinetic temperature is (a) 1019.2 eV, (b) 254.8 eV. The electron density is 2.0×10^{23} cm^{-3}. Ion perturbers have a charge of +1.

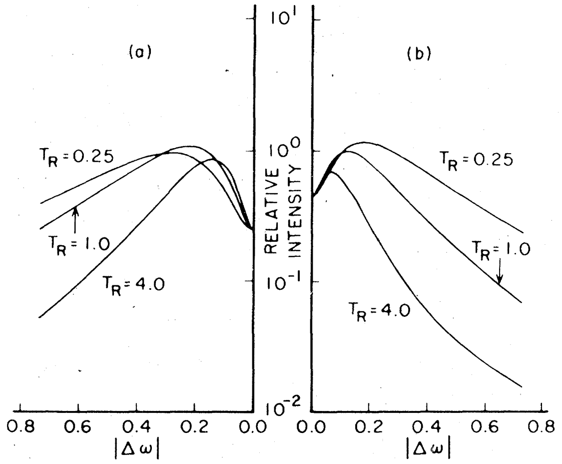

FIG. 3. Lyman-β line profiles for Al XIII. The electron kinetic temperature is (a) 809.1 eV, (b) 202.3 eV. The electron density is 1.0×10^{23} cm^{-3}. Ion perturbers have a charge of +12.

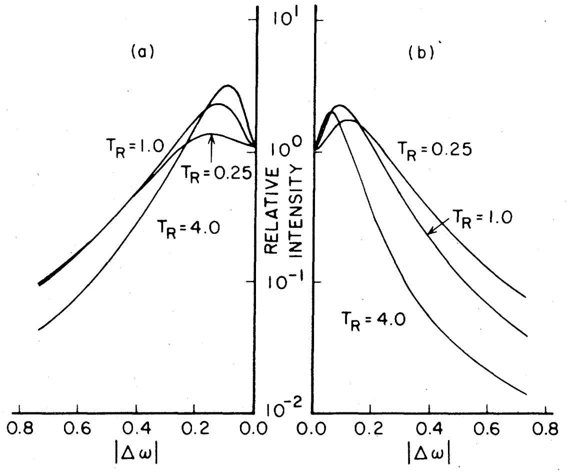

FIG. 4. Lyman-β line profiles for Ar XVIII. The electron kinetic temperature is (a) 809.1 eV, (b) 202.3 eV. The electron density is 1.0×10^{23} cm^{-3}. Ion perturbers have a charge of +1.

species to the density of charge-z_1 species. For the case where $R = 0(\infty)$ all the perturbing ions have charge $z_1(z_2)$. In both of these last two figures the parameter T_R is a set to unity. Only $a = 0.2$ cases are considered here because at higher a values, the microfield functions apparently lose their sensitivity to variations in R. Figure 5 shows Lyman α for hydrogenic neon and argon. Lyman-β profiles are given in Figure 6.

The behavior of these curves reflects the behavior of the ion-microfield distribution functions. As before[2] we see that most of the variation in the profile occurs between the values $R = 0.0$ and $R = 0.1$. This would seem to indicate that for low a values ($a = 0.2, 0.4$), when the concentration of the z_2 species is greater than 10%, the profiles behave largely as if *all* ion perturbers have charge z_2.

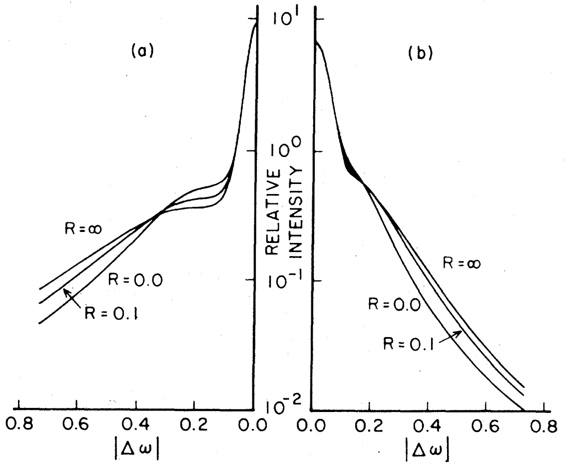

FIG. 5. Lyman-α line profiles for (a) Ne X, (b) Ar XVIII. The electron kinetic temperature is 1019.2 eV. The electron density is 2.0×10^{23} cm^{-3}. Ion perturbers have charges (a) +1 and +9; (b) +1 and +17.

IV. CONCLUSIONS AND CRITIQUE

The treatment of electron broadening effects by a second-order perturbation calculation has a range of validity which covers the center of the line profile and extends roughly out to the electron plasma frequency. For the present cases, the electron plasma frequency falls off scale. This implies that static electron effects, whose main contributions are expected to occur for frequency separations $\Delta\omega > \omega_p$, will not be important in this work. The indication here is that electron broadening widths are small compared to the separation of the levels of the transition and, therefore, may be calculated by a perturbation approach; this follows from the scaling of the electron broadening widths with T and Z, noted previously.[2]

Similarly static ion effects are expected to be important for $\Delta\omega > \omega_{p(\text{ions})}$. Thus the quasistatic ion approximation is valid over most of the line profile, breaking down only at the extreme line center where ion dynamic effects may become important. Current experimental observations of the Lyman-α line of neon X made during pellet-implosion experiments at the Lawrence Livermore Laboratory have been restricted by background conditions, to the very center of the line where ion-dynamic effects are expected to be significant. The problem at line center is probably even more uncertain: In a line-broadening experiment re-

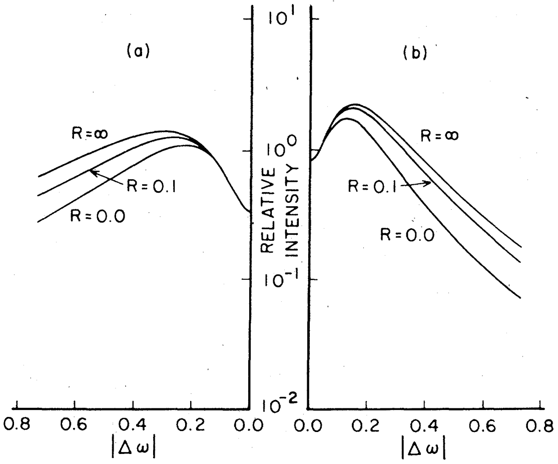

FIG. 6. Lyman-β line profiles for (a) Ne X, (b) Ar XVIII. The electron kinetic temperature is 1019.2 eV. The electron density is 1.0×10^{23} cm^{-3}. Ion perturbers have charges (a) +1 and +9; (b) +1 and +17.

cently conducted by Grützmacher and Wende, in an optically thin plasma, the line center of the Lyman-α line of hydrogen was shown to be much broader than that previously predicted by any theoretical calculation.[4] It would follow from this that the Lyman-α line from all hydrogenic ions would be similarly affected. Professor H. R. Griem has proposed an additional broadening mechanism that might account for this discrepancy.[5] Whether or not his conjecture is correct, it is certainly proper to say that the center-line theory is most uncertain; this is especially true when the broadening plasma is optically thick. Hence, if center-line theory alone is to be used as an accurate compression diagnostic, the known uncertainties must be much reduced.

In view of the difficulties associated with the center-line theory of the Lyman-α line, perhaps the Lyman-β line would be a better choice for diagnostic purposes. Since the part of this line which is important for diagnostic studies occurs away from the line center, the difficulties involved with Lyman-α interpretation may be avoided. In fact, it has been pointed out recently[6] that for certain plasma conditions, the first three Lyman-series lines for hydrogenic neon may be resolved above the background. The higher series members also offer the advantage that they are less susceptible to radiative transfer effects. The choice of Lyman β as a compression diagnostic would appear to avoid many of the present theoretical uncertainties associated with the center of the line profile.

---

*Work supported in part by ERDA and the Northeast Regional Data Center of the State University of Florida.

[1] Richard J. Tighe and C. F. Hooper, Jr., Phys. Rev. A 15, 1773 (1977).

[2] Richard J. Tighe and C. F. Hooper, Jr., Phys. Rev. A 14, 1514 (1976).

[3] C. R. Vidal, J. Cooper, and E. W. Smith, National Bureau of Standards Monograph 116 (Boulder, Colorado, 1970).

[4] K. Grützmacher and B. Wende, Phys. Rev. A 16, 243 (1977).

[5] H. R. Griem, Phys. Rev. A (to be published).

[6] B. Yaakobi (private communication).