Energy spectrum of stopping negative mesons and the concentration dependence of capture fractions

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The physics that determines the shape of the energy spectrum of stopping negative mesons is discussed. We note that a white energy spectrum, for *arrival probability* $F(E)$, not *flux* $P(E)$, follows for a large class of models. The importance of distinguishing between $F(E)$ and $P(E)$ is stressed: $P(E)$ reflects the energy (E) dependence of the total inelastic and capture cross sections, while $F(E)$ is independent of these. The $F(E) \propto E$ behavior found in some models is due to meson capture at positive energies, i.e., capture behind a positive centrifugal barrier. We explore the possibility of a concentration effect on capture ratios in binary mixtures, and conclude that, as long as the energy losses in meson-atom collisions are not much smaller than the barrier height, such concentration effects are negligibly small.

The atomic capture of negative mesons is a topic of lively interest, especially in view of the fact that the observed chemical effects in this capture remain unexplained.[1,2] One interesting question, which has even been addressed experimentally,[3] is whether there is a concentration dependence of the fraction captured on a given element in an alloy or mixture of gases (after dividing out the obvious dependence on the number of atoms). Such a dependence would presumably come from a variation in the shape of the energy spectrum (at very low energies) of the stopping mesons as the composition of the target is varied. In the present Comment, we discuss the physics that determines the shape of the energy spectrum and hence the likelihood of observing a concentration dependence of capture fractions.

In a recent Comment, Haff and Vogel[4] criticized (wrongly) a paper of Daniel[5] for using a "white" energy spectrum in his model of negative-muon capture, instead of the

$$P(E) \propto E \tag{1}$$

behavior found by Vogel *et al.*[6] in their model of muon capture. In his reply to this criticism, Daniel[7] pointed out (rightly) that the white energy spectrum follows from his model[5] in which negative muons lose the same energy ΔE in any collision with a particular atomic species. It is important to realize here that Daniel[5] and Haff and Vogel[4] are referring to *two different functions*, either of which can be used to describe the slowing down and capture of negative mesons. Thus Haff and Vogel[4] employ the *flux* $P(E)$, while Daniel[5] evidently has in mind the *arrival probability* $F(E)$ when he writes of a white energy spectrum. Of course $F(E)$ being constant does *not* imply that $P(E)$ is constant, since $P(E)$ acquires an E dependence from that of the total inelastic cross section (see below and Ref. 6). We shall now show that indeed the white energy spectrum [for $F(E)$!] holds for a much wider class of models, those for which (i) the *shape* of the distribution of energy losses ϵ is independent of the incident energy E, and (ii) capture requires final energy $\leqslant 0$.

The basic quantity, from which both the slowing down and the capture must for consistency be calculated, is the *differential energy-loss cross section* $(d\sigma(E,\epsilon)/d\epsilon)$. If the target material is made up of several species, each with atomic fraction a_i, then this differential energy-loss cross section is the average

$$\left(\frac{d\sigma}{d\epsilon}\right)_{\text{av}} = \sum_i a_i \frac{d\sigma_i}{d\epsilon}\,. \tag{2}$$

The transport (in energy) problem is most easily formulated in terms of the arrival probability (density) $F(E)$; i.e., the *number* of mesons arriving in an energy interval dE is $F(E)dE$. $F(E)$ obviously satisfies

$$F(E) = \int_0^\infty B^{\text{free}}(E+\epsilon,\epsilon)\,F(E+\epsilon)\,d\epsilon\,, \tag{3}$$

where the branching ratio (density) for free-free collisions B^{free} is given by

$$B^{\text{free}}(E,\epsilon) = \sum_i a_i B_i^{\text{free}}(E,\epsilon)\,, \tag{4}$$

and

$$B_i^{\text{free}}(E,\epsilon) = \Theta(E-\epsilon)\,\frac{d\sigma_i(E,\epsilon)}{d\epsilon}\Big/\sigma(E) \tag{5}$$

[$\Theta(x)$ is the unit step function]. Here the *average total inelastic cross section* $\sigma(E)$ is

$$\sigma(E) = \sum_i a_i \int_0^\infty \frac{d\sigma_i}{d\epsilon}\,d\epsilon\,, \tag{6}$$

where it is to be understood that the $\int d\epsilon$ specifically excludes any (Dirac δ function) $\delta(\epsilon)$ from elastic scattering. The $\Theta(E-\epsilon)$ in Eq. (5) expresses condition (ii) above. The total number of mesons captured on species i is

$$N_i = \int_0^\infty c_i(E)\, dE\,, \tag{7}$$

with the capture probability $c_i(E)$ being given by

$$c_i(E) = \int_0^\infty a_i B_i^{\text{capt}}(E,\epsilon) F(E)\, d\epsilon\,, \tag{8}$$

and the branching ratio (density) for capture on i by

$$B_i^{\text{capt}}(E,\epsilon) = \Theta(\epsilon - E)\, \frac{d\sigma_i(E,\epsilon)}{d\epsilon} \Big/ \sigma(E)\,. \tag{9}$$

The integral equation for $F(E)$ [Eq. (3)] has the same physical content as the analogous one for flux $P(E)$ [see Eq. (3.24) of Ref. 6], but is much easier to use since it avoids the irrelevant dependence on E that enters through $\sigma(E)$. $F(E)$ is related to $P(E)$ by

$$F(E) = T\sigma(E) P(E)\,, \tag{10}$$

where T is an arbitrarily chosen time.

We now want to apply condition (i), that the shape of the energy-loss spectrum is independent of energy. This is true if $d\sigma_i/d\epsilon$ is a *product*:

$$\frac{d\sigma_i(E,\epsilon)}{d\epsilon} = \alpha(E) f_i(\epsilon)\,. \tag{11}$$

Then from Eqs. (5)–(7)

$$B^{\text{free}}(E,\epsilon) = \Theta(E-\epsilon) \sum a_i f_i(\epsilon) \Big/ \sum a_i \int_0^\infty f_i(\epsilon')\, d\epsilon' \tag{12}$$

so that the kernel of Eq. (3), $B^{\text{free}}(E+\epsilon,\epsilon)$, is *independent of* E. This immediately implies a white spectrum, i.e.,

$$F(E) = \text{const}\,. \tag{13}$$

[For, suppose $B^{\text{free}}(E,\epsilon)=0$ for $\epsilon<\Delta$ for some $\Delta>0$. Then if F is constant for $E' \geq E+\Delta$, Eq. (3) implies that $F'(E)=0$, so that $F(E)$ is equal to the same constant. This is true no matter how small Δ is.] Obviously Daniel's model[5] is included as a special case.

Suppose in contrast that the energy-loss spectrum becomes narrower with decreasing E [thus abandoning condition (i)]. Then clearly $F(E)$ will turn up as $E\to 0$. Conversely, if the average energy loss increases as $E\to 0$, $F(E)$ will turn down at low energy. These three modes of behavior are illustrated in Fig. 1. How then can we produce the $F(E)\propto E$ behavior at small E found in Refs. 8 and 9? Only by abandoning condition (ii) and allowing

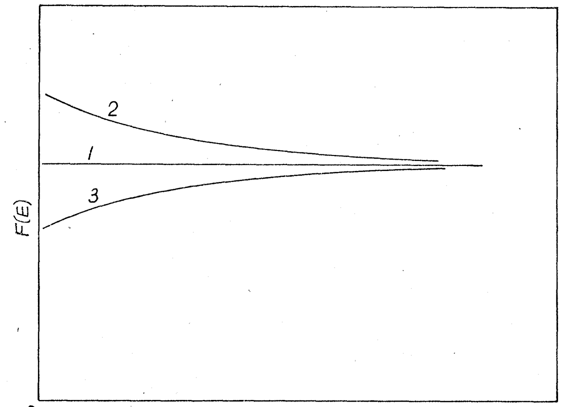

FIG. 1. Behavior of the energy spectrum $F(E)$ when capture takes place only at $E \leq 0$. Curve 1 is for energy losses independent of energy [condition (i)], curve 2 for energy losses decreasing as $E\to 0$, curve 3 for increasing energy losses.

capture at positive energy. That is, the reduction in population represented by the $F(E)\propto E$ behavior results simply from the exhaustion of the free-meson spectrum by captures behind a positive centrifugal barrier.[8] Since the atomic potential assumed by Daniel[5] is too steep to support such a barrier, it will not produce $F(E)\propto E$ behavior even if a distribution of energy losses (which we know is required by quantum mechanics)[9] were used with it.

We now want to investigate the possible magnitude of any concentration dependence of the capture ratios. Suppose first that there is *no* positive-energy capture, conditions (i) and (ii) hold, so that as we have seen $F(E)=\text{const}$. Then from Eqs. (7) and (8) the *reduced capture ratio* is

$$\frac{n_i}{n_j} \equiv \frac{\int\int dE\, d\epsilon\, B_i^{\text{capt}}(E,\epsilon)}{\int\int dE\, d\epsilon\, B_j^{\text{capt}}(E,\epsilon)}\,, \tag{14}$$

which implies *no* concentration dependence. Such concentration dependence requires varying the shape of $F(E)$, and since $F(E)$ changes more drastically as a result of dropping condition (ii) rather than (i), we will concentrate on this class of models.

To proceed, we need to know the energy-loss shape $f_i(\epsilon)$ and the probability for positive-energy capture $p_i(E)$ for each component; as a qualitative guide to these quantities we can use the results of the "fuzzy Fermi-Teller model."[9] The function $p_i(E)$ depends on the height of the barrier as a function of l (orbital angular momentum), the relative probability of scattering as a function of l, and the relative amount of scattering from within and without the barrier. Examination of Fig. 4 of Ref. 9, which gives the distribution in energy of captured muons for $Z = 40$, indicates that the function

$$p_i(E) = \begin{cases} 1, & E \leqslant 0 \\ 1 - \sin\left(\frac{\pi}{2}\frac{E}{E_{Bi}}\right), & 0 < E < E_{Bi} \\ 0, & E \geqslant E_{Bi} \end{cases} \quad (15)$$

has the correct qualitative behavior, and this will suffice for our purposes. The *maximum barrier height* E_{Bi} we take $\lesssim 1$ a.u. (1 a.u. = 27.2 eV). Next, Fig. 3 of Ref. 9 leads us to adopt for $f(\epsilon)$

$$f_i(\epsilon) = \Theta(\epsilon - I_i)/(\epsilon + \beta_i)^n \quad (16)$$

with $n = 2$. The ionization energies I_i can be set to zero for metal alloys, but are nonzero for mixtures of noble gases; the constants β_i are taken ≈ 0.3 a.u.

The presence of capture at positive energy of course alters the branching ratios; instead of Eqs. (5) and (9) we have

$$B_i^{\text{free}}(E,\epsilon) = [1 - p_i(E-\epsilon)] \frac{d\sigma_i(E,\epsilon)}{d\epsilon} \Big/ \sigma(E) \quad (5')$$

and

$$B_i^{\text{capt}}(E,\epsilon) = p_i(E-\epsilon) \frac{d\sigma_i(E,\epsilon)}{d\epsilon} \Big/ \sigma(E). \quad (9')$$

Calculating the N_i requires solving the integral Eq. (3) for $F(E)$. This is most easily done numerically by forming energy bins and replacing Eqs. (3)–(9) by their discrete analogs; any accuracy can then be achieved by making the mesh fine enough. Thus Eq. (3) becomes

$$F_\alpha = \sum_{\beta > \alpha} B_{\alpha\beta}^{\text{free}} F_\beta \quad (3')$$

with

$$B_{\alpha\beta}^{\text{free}} = \sum_i a_i B_i^{\text{free}}(E_\beta, E_\beta - E_\alpha)\Delta E, \quad (4')$$

etc. We start with a population of one meson distributed uniformly between 9 and 10 a.u., and let this propagate downward in energy. Because of the spread of energy losses embodied in the $f_i(\epsilon)$, the unphysical "bumpy" behavior of $F(E)$ caused by the arbitrary starting distribution quickly smooths out, so that we can be confident that the shape of $F(E)$ at lower energies is independent of the starting distribution. As an example which might at least qualitatively resemble a mixture of noble gases, we take $I_1 = 1$, $I_2 = 0.5$, $E_{B1} = 0.5$, $E_{B2} = 1.0$. The resulting $F(E)$ is shown for $a_1/a_2 = 10$, 1, and 0.1 in Fig. 2. Here we see the $F(E) \propto E$ behavior at low energies produced by the capture at positive energies. But in spite of the significant change of shape of $F(E)$, the ratio n_1/n_2 is virtually unchanged. This is shown in the top line of Table I. This is evidently because the energy losses are not confined to values small compared to the barrier heights.

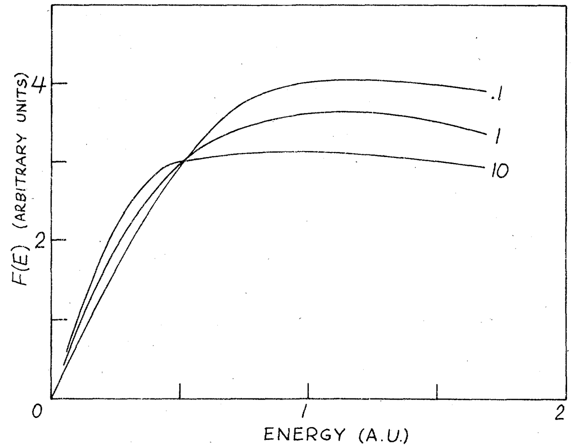

FIG. 2. Computed $F(E)$ curves corresponding to the top line of Table I.

In an effort to generate concentration effects, we put both I_i to zero and reduced β; as a result the *median energy loss* ϵ_m is reduced. Even under these assumptions the concentration effect is quite small, as can be seen from the second row of Table I. Only when we *drastically* reduce the magnitude of the energy losses by increasing the ex-

TABLE I. Reduced capture ratios for different concentrations and parameters.

I_1	I_2	E_{B1}	E_{B2}	β	n	ϵ_m	n_1/n_2 $a_1/a_2 = 10$	1.0	0.1
1.0	0.5	0.5	1.0	0.33	2		0.781	0.789	0.795
0.0	0.0	0.5	1.0	0.05	2	0.05	0.643	0.644	0.650
0.0	0.0	0.5	1.0	0.05	3	0.021	0.350	0.290	0.247
0.0	0.0	0.5	1.0	0.05	4	0.013	0.279	0.163	0.100
0.0	0.0	0.5	1.0	0.05	6	0.007	0.197	0.098	0.041

ponent n of Eq. (16) do we see significant concentration effects; this is shown in the bottom rows of Table I.

Such a drastic reduction in the spread in ϵ [i.e., in the width of $f(\epsilon)$] from the predictions of Ref. 9 seems to us to be unlikely. Thus we believe that concentration effects have not been observed[3] and will not be observed in the near future. This situation makes experiments to look for such effects all the more worthwhile, of course, because a positive result would unambiguously rule out the kind of $f(\epsilon)$ predicted by the fuzzy Fermi-Teller model.[9]

ACKNOWLEDGMENT

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