Comments and Addenda

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Enhanced gain of a free-electron laser

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The gain of a free-electron laser may be multiplied by a very large factor when a uniform magnetic field B_0 is superimposed on the extant helical field. The effect is due to cyclotron resonance and requires $B_0 = 10.6/l(1-v_\parallel^2/c^2)^{1/2}$ kG, where l is the period of the helical field and $v_\parallel$ is the streaming-electron velocity.

Laser action at infrared wavelengths has recently been demonstrated in experiments in which an electron beam of a linear accelerator is passed through a 2.4-kG helical magnetic field of period $l = 3.2$ cm.[1,2] The infrared wavelength at which wave amplification occurred was given by $\lambda_s = l/2\gamma_\parallel^2$, where l is the period of the static magnetic field pump, $\gamma_\parallel = [1-(v_\parallel/c)^2]^{-1/2}$, and $v_\parallel$ the axial-electron velocity. However, the gain of this process was relatively weak with only a 7% increase in power being achieved in a 5.2-m interaction length in an amplifier experiment[1] at $\lambda_s = 10.7$ μm. Also, only 0.01% of the electron beam energy was converted into electromagnetic radiation in an oscillator experiment[2] employing an optical cavity.

Analyses of the interaction have shown that the process may be viewed as stimulated scattering with the helical magnetic field being equivalent to an incident electromagnetic wave that is coherently scattered and Doppler shifted by the streaming electrons.[3-6] We point out in this comment that it has been shown[7-9] that the growth rate of a stimulated scattering process may be greatly enhanced when a spatially uniform axial magnetic field is employed such that the electron cyclotron frequency is equal to the pump wave frequency in the rest frame of the electron beam. As will be shown below, the gain of the free-electon laser may be increased dramatically by superimposing a large axial uniform magnetic field on the small helical pump field.

The growth rate for magneto-Compton scattering has been calculated using both a quantum-mechanical formalism[9] as well as a classical approach.[10] It is found that the ratio of the spatial gain, associated with the stimulated scattering, with and without the uniform magnetic field B_0 is given by

$$R \equiv G_{B_0\neq 0}/G_{B_0=0} \approx [\omega'/(\omega'-\Omega_0)]^2, \qquad (1)$$

where ω' is the frequency of the pump wave in the rest frame of the beam and $\Omega_0 = |e|B_0/m_0c$ is the electron cyclotron frequency. Although this result was found in the large-cavity regime, i.e., $\lambda_s/L \lesssim v_{th}/c$, where λ_s is the wavelength of the scattered wave, L is the interaction length, and v_{th} is the electron thermal velocity, the ratio in Eq. (1) should also be appropriate in the small-cavity limit. To determine the ratio R, the difference $\Delta\omega' \equiv \omega' - \Omega_0$ must be determined. In the small-cavity limit the frequency spread in the beam frame due to finite-transit-time effects is roughly equal to $2\pi\gamma_\parallel c/L$. The frequency spread in the beam frame due to a spread in the laboratory-frame electron energy $\Delta\gamma m_0c^2$ is given by $2\pi c\Delta\gamma/l$ and can be neglected in the small-cavity limit. By approximating $\Delta\omega'$ by $2\pi\gamma_\parallel c/L$ we find that $R \approx (L/l)^2$. Of course, other effects, such as particle collisions and spatial inhomogeneities in B_0 will further limit the smallest of $\Delta\omega'$, but these effects are considered to be of higher order than the finite-transit-time effects. For a static periodic magnetic pump field the value of the external magnetic field needed to achieve a resonance is given by

$$B_0 = 10.6\gamma_\parallel/l \text{ (kG)}, \qquad (2)$$

where l is in centimeters.

As an illustration, we choose the parameters in

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Ref. 1 where 10.7-μm radiation was amplified, $l = 3.2$ cm, $L = 5.2$ m, and $\gamma_{\parallel} = 38$. Thus, from Eqs. (1) and (2) one can calculate an extremely large enhancement factor in the gain of $R \approx 10^4$ and a resonant background magnetic field of $B_0 = 126$ kG. Suitable superconducting magnets providing a magnetic field of this magnitude are within the state of the art; it may also be possible to use high-field pulsed electromagnets. The enhancement in gain is so large that nonlinearities would quickly modify the interaction process and thus the value $R \approx 10^4$ should be taken only qualitatively to indicate that a large effect may be expected.

Finally we note that for a free-electron laser at longer output wavelengths (as would be of interest for uranium-isotope separation at 16 μm) the magnetic field requirement is reduced. Also, by exploiting resonances at harmonics of the cyclotron frequency rather than at the fundamental, the magnetic field requirement can be further reduced. In general for free-electron lasers operating in the far infrared, increasing gain via magnetic resonance may be a more viable option compared with recycling the electron beam through the interaction region by using a storage ring. A detailed analysis of magneto-Compton scattering, giving particular attention to the nonlinear saturation levels, is presently underway.

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