Atomic spectral lines when a quark is embedded in the nucleus

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There has been a recent claim for the observation of fractionally charged particles. If this evidence is to be interpreted as due to a quark embedded in a heavy nucleus, a natural follow-up is to look for "quark atoms" through their spectral lines. The recent technique of single-atom detection may be one way of doing this. Such searches will require as inputs knowledge of the spectroscopic transitions in the quark-atom. The presence of the quark causes large shifts in spectral lines which fall in the visible and near-visible regions and a simple technique is presented based on interpolations in isoelectronic series for estimating these shifts quickly and accurately for any atom. Results are also presented for the x-ray transitions in heavy atoms and very accurate results for the various series in quark hydrogen. The techniques and results of this paper may also be relevant for searches in laboratory and astrophysical spectra.

I. INTRODUCTION

There has been renewed interest in experimental searches for fractionally charged particles (quarks, from now on) following the recent claim[1] that charges (here, and throughout this paper, in units of e) of $\pm\frac{1}{3}$ seem to have been observed. In such matters, it is appropriate to look as widely as possible (and in fact, many such searches have been carried out[1]) and consider a variety of possible experiments to look for such particles. This is the motivation for the present paper wherein we examine one possibility, namely, the search for such particles through atomic spectral lines. If the evidence of Ref. 1 is to be interpreted as a quark incorporated into the nucleus of a tungsten atom, one would expect the additional charge on the nucleus to affect the atomic spectrum. Indeed, the effects can be expected to be large since the additional charge is a large fraction of Z_{eff}, the effective charge seen by the outer electrons. Atomic levels scale with Z_{eff}^2 and, therefore, the spectral lines will be shifted to a completely different range of wavelengths and, to that extent, should be easily distinguishable from those in the corresponding ordinary atom. Though it is true that the experimental search will, nevertheless, be very difficult because the fraction of such "quark atoms" will be very small and, therefore, the line intensities very weak, yet recent experimental techniques on the detection of a very small number of atoms (even single atoms[2]!) make it, by no means, impossible. In fact, such a quark search was envisaged[3] with an earlier technique for detecting small abundances (though not as small as the single atoms of Ref. 2). These techniques call, however, for a fairly precise knowledge of the atomic transition energies and it is the purpose of this paper to present a simple procedure for obtaining such energies for quark atoms with accuracies of 1 part in 10^3 (therefore, 1 Å in the visible range) or better. To get this kind of accuracy for the outer transitions in a many-electron atom such as tungsten through *ab initio* theory calls for rather complex and time-consuming computer calculations since these energies are extremely small fractions of the total energy of the atom. Simpler estimates[4] based on considerations of screening constants or Z_{eff} cannot be expected to be accurate to better than a few percent (uncertainties, therefore, of a few hundred angstroms) which is quite inadequate for spectral-line searches in either a laboratory or astrophysical context. The estimation of transition energies in quark atoms through simple but, nevertheless, accurate methods poses an interesting challenge to an atomic physicist and is the main motivation for this paper. We adapt the method of Z expansions[5] or Z extrapolations along isoelectronic series for this purpose and demonstrate that accurate and reliable results can be obtained for quark atoms based on known and standard values of laboratory spectra; the calculations involve only straightforward algebra. As we learned after this work was done, the procedure of extrapolations along isoelectronic series has been suggested before.[6] Only highly stripped ions of O, N, and C with very few electrons were considered in this reference; we demonstrate in this paper that the procedure works well even for neutral heavy atoms and for transitions between excited states of atoms. We also present a sophistication of the technique involving quantum-defect fits which may be expected to lead to higher accuracy. The accent in Ref. 6 was on searches in astrophysical spectra and, therefore, that paper presented both energies and oscillator strengths; a reliable search must include energy

positions, strengths, and multiplet structure before a claim for a quark-atom assignment can be made.

This paper is in three parts. In Sec. II, we consider the shift in the $K\alpha$ x-ray lines of heavy atoms due to a quark in the nucleus. Section III considers the spectrum of H in detail where the transition energies can be worked out exactly. These calculations serve to point out that the main effect is due to the charge of the quark and that the effect of the mass is very small in comparison. In all the rest of the paper, therefore, where we deal with heavier atoms and the effect due to the quark mass is even less, and smaller than the accuracy of our results, the aspect of mass is completely ignored. Section IV is the main part of the paper wherein we deal with outer transitions in heavy atoms and estimate through isoelectronic fits the values of these transitions for quark atoms. We consider as representative examples of a light atom and a heavy one, He and Hg, respectively. The former may also be of interest astrophysically because of its large abundance (along with H). The latter may be of some interest because it is a heavy atom like tungsten (there are some speculations[7] that quarks may migrate to heavy nuclei) and the mercury lamp is, of course, a standard laboratory apparatus so that the spectrum can be readily studied. We also note that relativistic effects have been neglected for simplicity, particularly because the accuracy of our results do not warrant their inclusion. In Secs. II and III, relativistic corrections may be expected to alter our quoted results past the fifth significant figure. When such accuracies are desired, the procedure in these sections can be easily adapted to include relativistic effects.

II. SHIFT IN X-RAY WAVELENGTHS

With a quark in the nucleus of a heavy atom like tungsten ($Z=74$) the $K\alpha$ and other x-ray lines will be shifted. The additional charge is about 0.5 to 1% of the total and we may expect the same order-of-magnitude effect on the wavelengths. Simple screening constants σ_{nl} for the $1s$ and $2p$ levels between which the transition takes place suffice to get reliable estimates of the shifts. The energies of the $1s$ and $2p$ levels are written according to "Moseley's law,"[8]

$$E_{nl} = -(Z-\sigma_{nl})^2 I_H/n^2, \quad I_H = 13.6 \text{ eV}, \qquad (1)$$

so that the wavelength of the transition is given by

$$\lambda = (911.29 \text{ Å})[(Z-\sigma_{1s})^2 - \tfrac{1}{4}(Z-\sigma_{2p})^2]^{-1}. \qquad (2)$$

With a quark, of say charge $+\frac{1}{3}$ in the nucleus, Z in (2) is replaced by $Z+\frac{1}{3}$ and the shift in wavelength can be written, after trivial algebra, as

$$\Delta\lambda = (\lambda^2/911.29)\,[\tfrac{1}{2}Z + \tfrac{1}{12} + \tfrac{2}{3}(\tfrac{1}{4}\sigma_{2p} - \sigma_{1s})]. \qquad (3)$$

There are various sources of screening constants. One due to Slater[9] leads to a value for $\frac{1}{4}\sigma_{2p} - \sigma_{1s}$ of 0.7375 and one due to Burns[10] to 0.9875. Table I demonstrates that the alternative prescriptions lead to essentially the same values for $\Delta\lambda$ to three significant figures. As expected, the entries show a shift of about 1%, easily observable with the kind of resolutions attainable in the x-ray region. However, experimental searches for such shifted lines will probably be impractical because the very feeble lines will have to be picked up over a bremsstrahlung background.

TABLE I. $K\alpha$ x-ray wavelength shifts, $\Delta\lambda$, with a quark of charge $+\frac{1}{3}$ in the nucleus. λ_0 is the unshifted line and the two values of $\Delta\lambda$ are from two different prescriptions for screening constants. All wavelengths are in angstroms.

Atom	Z	λ_0 (Å)	$\Delta\lambda$ Slater (Å)	$\Delta\lambda$ Burns (Å)
Yb	70	0.236 655	0.002 186	0.002 197
Lu	71	0.229 298	0.002 081	0.002 091
Hf	72	0.222 227	0.001 982	0.001 991
Ta	73	0.215 497	0.001 889	0.001 898
W	74	0.209 010	0.001 801	0.001 809
Rh	75	0.202 781	0.001 718	0.001 726
Os	76	0.196 794	0.001 639	0.001 646
Ir	77	0.191 047	0.001 565	0.001 572
Pt	78	0.185 511	0.001 495	0.001 501
Au	79	0.180 195	0.001 428	0.001 434
Hg	80	0.175 068	0.001 365	0.001 370
Pb	82	0.165 376	0.001 248	0.001 253

III. SPECTRA OF QUARK HYDROGEN

With a quark in the nucleus of the H atom, the spectral-line positions are modified in two ways: the change in the charge on the nucleus and the change in the effective mass of the electron.[11] The latter modification is a very small one. Relative to R_H (= 109 678.758 cm^{-1}), two extreme assumptions about the quark mass which would embrace most quark models, namely, $m_q \simeq m_p$ and $m_q \simeq 39\,m_p$, lead, respectively, to $R_1 = 1.000\,272\,44 R_H$ and $R_2 = 1.000\,531\,4 R_H$. The line positions for these two alternatives differ by about 1–2 Å as seen in Tables II–IV, which give a sample of results[12] for various series in H for different quark charges. We have chosen to display those cases where the lines fall in or near the visible region. The shifts are, of course, large, with the Paschen and Brackett series, which normally lie in the infrared, now shifted into

TABLE II. Balmer series in H. λ_0 is the value for ordinary hydrogen and the other entries are values with different quarks in the nucleus. λ_1 and λ_2 in each case represent two different assumptions for the quark mass, $m_q/m_p = 1$ and 39, respectively. All wavelengths are in vacuum and in angstroms.

Transition	λ_0 (Å)	$+\frac{2}{3}$ λ_1 (Å)	$+\frac{2}{3}$ λ_2 (Å)	$+\frac{1}{3}$ λ_1 (Å)	$+\frac{1}{3}$ λ_2 (Å)	$-\frac{1}{3}$ λ_1 (Å)	$-\frac{1}{3}$ λ_2 (Å)
2-3 α	6564.63	2362.62	2362.01	3691.60	3690.64	14 766.4	14 762.6
2-4 β	4862.69	1750.09	1749.64	2734.52	2733.81	10 938.10	10 935.2
2-5 γ	4341.69	1562.58	1562.18	2441.53	2440.90	9 766.13	9 763.60
2-6 δ	4102.89	1476.64	1476.26	2307.25	2306.65	9 228.99	9 226.60
2-7 ϵ	3971.19	1429.24	1428.87	2233.19	2232.61	8 932.75	8 930.44
2-8	3890.15	1400.07	1399.71	2187.61	2187.05	8 750.45	8 748.19
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
2-16	3704.90	1333.40	1333.06	2083.44	2082.90	8333.76	8 331.61
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
2-∞ limit	3647.01	1312.57	1312.23	2050.89	2050.36	8203.55	8 201.42

the visible range for quarks of charge $+\frac{2}{3}$ and $+\frac{1}{3}$. A curious aspect of some of the sequence of numbers in Tables III and IV is that they look like a red-shifted Balmer sequence. This is quantitatively so to a surprisingly high accuracy as shown by the numbers in column 5, which are values for the red-shift parameter used in astrophysics,[13]

$$z = \lambda/\lambda_0 - 1. \tag{4}$$

The z values rise along the series and then fall towards the higher members with a broad region of very gradual variation so that for four or five consecutive lines, the z values remain essentially equal to three-figure accuracy. This curious behavior arises as follows. Consider Table III for instance. In comparing with the Balmer series, one is dealing with

$$1 + z = [Z^2(2^{-2} - m^{-2})]/[(Z + \tfrac{1}{3})^2 (3^{-2} - n^{-2})]. \tag{5}$$

The ratio $[1-(2/m)^2]/[1-(3/n)^2]$ is equal to unity for $m=4$, and $n=6$, but then becomes slightly larger than 1 as m and n increase by steps of 1.

TABLE III. Similar to Table II for the Paschen series. The last column gives the red-shift parameter for the line (λ_1) relative to the ordinary Balmer line of the symbol indicated alongside.

Transition	λ_0 (Å)	$+\frac{2}{3}$ λ_1 (Å)	$+\frac{2}{3}$ λ_2 (Å)	$+\frac{1}{3}$ λ_1 (Å)	$+\frac{1}{3}$ λ_2 (Å)	z
3-4	18 756.1	6750.35	6748.60	10 547.4	10 544.7	
3-5	12 821.5	4614.50	4613.30	7 210.15	7 208.29	
3-6	10 941.0	3937.70	3936.68	6 152.66	6 151.07	β 0.265
3-7	10 052.1	3617.76	3616.83	5 652.76	5 651.29	γ 0.302
3-8	9 548.55	3436.54	3435.65	5 369.59	5 368.21	δ 0.309
3-9	9 231.51	3322.44	3321.58	5 191.31	5 189.96	ϵ 0.307
3-10	9 017.34	3245.36	3244.52	5 070.87	5 069.56	0.304
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
3-20	8 394.66	3021.25	3020.47	4 720.71	4 719.49	
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
3-∞	8 205.78	2953.28	2952.51	4 614.50	4 613.30	0.265

TABLE IV. Similar to Table III for the Bracket series.

Transition	λ_0 (Å)	λ_1 (Å)	λ_2 (Å)	z
4-5	40 522.4	14 584.1	14 580.3	
4-6	26 258.5	9 450.49	9448.04	
4-7	21661.1	7 795.86	7793.84	
4-8	19450.7	7 000.36	6 998.55	
4-9	18 179.0	6 542.64	6 540.95	β 0.345
4-10	17366.7	6 250.32	6 248.70	γ 0.440
4-11	16814.7	6 051.65	6 050.08	δ 0.475
4-12	16411.6	5 906.55	5 905.03	ϵ 0.487
4-13	16 113.6	5 799.32	5797.82	0.491
4-14	15884.8	5 716.96	5715.48	0.490
4-15	15704.8	5 652.20	5650.74	0.488
4-16	15560.6	5 600.29	5598.84	0.485
•	•	•	•	
•	•	•	•	
•	•	•	•	•
4-30	14852.1	5 345.30	5343.91	•
•	•	•	•	•
•	•	•	•	
•	•	•	•	
4-∞ limit	14588.1	5 250.27	5248.91	0.440

But, again in the limit of $m,n\to\infty$, the ratio becomes unity. This result is interesting insofar as it points to another situation (to add to the Doppler and gravitational ones) that could simulate a red-shifted ordinary hydrogenic series for even a *whole group* of lines. Note, of course, that this behavior is independent of what quark charge occurs in Eq. (5). In particular, it holds even for the case when no quarks are involved; that is, the ordinary Paschen series itself, apart from its lowest members, looks like a red-shifted Balmer series with a slow variation of z along the series. We hasten to add that it is unlikely that a Paschen series will be misidentified as a red-shifted Balmer series. The only reason for recording the observation here is the curious aspect of the slow variation of z along the series, an aspect that we have not seen noted elsewhere. The slow variation, which we have traced to the behavior of a simple algebraic ratio, has the effect of simulating a red-shift (which is not there) for a whole sequence of spectral lines.

IV. SHIFT IN SPECTRAL LINES OF HEAVY ATOMS

Unlike the inner transitions considered in Sec. II, the transitions of the outer electrons in a heavy atom cannot be handled through the concept of screening constants if high accuracy is desired. Such considerations of effective charges can be used at best as only a rough estimate and by no means capable of yielding four-figure accuracy. Yet, for the purposes of searching through lists of unidentified lines in laboratory or astrophysical spectra as possibly due to quark atoms, such high accuracy is a must since there are often several dozen lines in any 100-Å interval.[14] Also, for searches through single-atom detection or some similar technique, once again an accurate (to, say, 1 Å in the visible domain) estimate of a transition in the quark atom is necessary since these experiments usually rely on using tunable lasers tuned to excite selectively such atoms. *Ab initio* atomic calculations are usually not capable of giving outer transitions in a general heavy atom to high accuracy since they involve handling all electrons and the last electron, which is of interest here, contributes a negligibly small amount to the total energy of the atom. Hence, a transition which has been obtained as a very small difference of two large numbers cannot be expected to be of high accuracy. In any case, our goal of getting accurate values for the transition energies can be achieved through *ab initio* numerical calculations only at a considerable expense. If it is desired to generate such numbers simply and readily for a large variety of atoms and as the need arises because of experimental convenience, we must look elsewhere. This section presents such a technique. Since the presence of the quark has only the effect of changing the nuclear charge, the isoelectronic series of the atom of interest should provide a handle on the dependence of such outer transitions on the nuclear charge. Fitting known values of isoelectronic series to an expansion in Z provides, therefore, an immediate procedure to read off the corresponding values for the quark atom. There is a built-in index of reliability because often many members of the isoelectronic series are known, not all need be used in determining the Z expansion so that the others can serve as a check on how good the fit is. We demonstrate through different examples that the procedure works very well and is capable of yielding accurate values. We focus throughout on obtaining accurate energies because it is this information that is required for experiments of the type discussed in Refs. 2 and 3. We note that for the purposes of assigning unknown lines in astrophysical spectra, not only energy positions but strengths and multiplet structure are needed. The Z-expansion technique can also be used for obtaining oscillator strengths.[6]

Consider the resonance transition between the ground state and the $1s2p\,{}^1P_1$ state in the isoelectronic series of He. The corresponding energies are known[15] for ten members of this se-

quence. Any four of these values may be used to fit a four-parameter expression of the form

$$aZ^2 + bZ + c + dZ^4. \tag{6}$$

The above choice is in keeping with the spirit of the $1/Z$ expansion which is a sum of two sets of quantities, one a series that starts with Z^2 and represents the dominant electronic energy and the second, a series beginning with Z^4, that represents the spin-orbit energy. For the present application to He, this latter may be expected to be small as, indeed, it turns out to be but our choice in (6) is dictated by the later applications we have in mind to heavy atoms. For most of the applications in this paper we restrict ourselves to four parameters because often there is data only for that many members of an isoelectronic series. But when, as in the present case, more numbers are available we can improve the accuracy of such four-parameter fits by selecting data points which are somewhat separated and span the range of available data rather than choosing them in sequence. For the He sequence, we use the values for He, Li^+, C^{4+}, and F^{7+}, starting with He considered as $Z=1$. Here again, a small comment is in order. With an eye to later applications, counting the first member of the series as $Z=1$ rather than the value appropriate to its nuclear charge is convenient and causes only a slight error in the values we obtain for the quark atoms of interest, an error well within the accuracy we can expect for our results. In fact, were we to include in (6) the next term in the expansion, that is, eZ^3, as we will for higher accuracy later on, then there will be no error involved; the arbitrariness in the starting value of Z is merely absorbed into redefining the constants a, b, etc.

Fitting the values for He, Li^+, C^{4+}, and F^{7+} to (6), we get (all numbers in cm^{-1}) $a=82\,440.8$, $b=83\,359.0$, $c=5328.98$, and $d=0.363\,261$. These in turn lead to the following transition energies, respectively, for Be^{2+}, B^{3+}, N^{5+}, O^{6+}, Mg^{10+}, and Al^{11+}, where the numbers in parentheses are the known[15] experimental values: 997 403 (997 466), 1 657 910 (1 658 020), 3 473 820 (3 473 790), 4 629 310 (4 629 200), 10 902 930 (10 907 300), and 12 884 650 (12 891 900). The comparison demonstrates the goodness of the fit and we can use the same coefficients to get with confidence the transition energies in the quark-He atom. For quark charges $+\frac{2}{3}$, $+\frac{1}{3}$, and $-\frac{1}{3}$, we have, respectively,

$$373\,266,\ 263\,037,\ \text{and}\ 97\,542. \tag{7}$$

An alternative expression to (6), namely,

$$aZ^2 + bZ + c + dZ^{-1}, \tag{8}$$

which focuses on the electronic energy alone was also used and leads to $a=82\,504.4$, $b=82\,941.3$, $c=6148.49$, and $d=-464.974$. For the atoms Be^{2+}, B^{3+}, N^{5+}, O^{6+}, Mg^{10+}, and Al^{11+}, we now obtain 997 357, 1 657 870, 3 473 880, 4 629 380, 10 901 490, and 12 882 030, respectively. For quark He, we get in place of (7) the values

$$373\,284,\ 263\,063,\ \text{and}\ 97\,414\ . \tag{9}$$

The expression in (6) seems to do somewhat better than (8) and the values in (7) for the quark charges $+\frac{2}{3}$ and $+\frac{1}{3}$ may be expected to be somewhat more accurate. In any case, it is clear that we have nearly five-figure accuracy. Note also that both in (6) and (8), d turns out to be small, pointing to the quadratic form being the essential part of the $1/Z$ expansion. This is just what one would expect from arguments about screening which lead to expressions of the form $(Z-s)^2$ for atomic energies with s a screening constant. Finally, to get more accuracy, we have also used an expression which goes a step further than (6), namely,

$$aZ^2 + bZ + c + dZ^4 + eZ^3\ . \tag{10}$$

The above expression can be written in a completely equivalent but alternative form: $a'Z^2 + b'Z + c' + d(Z-\sigma)^4$. Fitting the values for He, Li^+, C^{4+}, N^{5+}, and F^{7+}, we get $a=82\,506.70$, $b=83\,240.3$, $c=5394.03$, $d=1.176\,45$, and $e=-13.0111$. These lead to 997 419, 1 657 930, 4 629 260, 10 904 250, and 12 887 150 for Be^{2+}, B^{3+}, O^{6+}, Mg^{10+}, and Al^{11+}, respectively, and for quark He ($+\frac{2}{3}$, $+\frac{1}{3}$, and $-\frac{1}{3}$, respectively),

$$373\,262,\ 263\,032,\ \text{and}\ 97\,554. \tag{11}$$

These are improvements on (7). Further numerical improvement may be achieved by a least-squares fitting of all the available data rather than fitting five of them as we have done. However, in most of the cases considered in this paper, there are only four or five data points available and we have, therefore, chosen not to go further than this.

An example of a line in the visible region, unlike the resonance transition considered above, is the 5877-Å $2\,^3P_1 - 3\,^3D_1$ line. The known values for He (17 014.7 cm^{-1}), Li^+ (66 972), C^{4+} (402 157), N^{5+} (575 140), and F^{7+} (1 013 050) were fitted to (10) with $a=15\,905.04$, $b=2749.46$, $c=-1562.84$, $d=3.934\,37$, and $e=-80.8994$. As a check, we then have 778 728 cm^{-1} for O^{6+} to be compared with the measured 778 760. For the quark-He atoms we obtain (for $+\frac{2}{3}$, $+\frac{1}{3}$, and $-\frac{1}{3}$, respectively)

46 856.1 (2134.2), 30 199.4 (3311.3),

and 7315.9 (13 670), (12)

where the numbers are λ^{-1} in cm^{-1} and, in parentheses, λ in angstroms. Likewise, for the $2\,^3P_1$-$4\,^3D_1$ 4473-Å line, the known values for He, Li^+, C^{4+}, and N^{5+} were used in (6) to get for quark He

61 719 (1620.2), 39 725 (2517.3),

and 9619.1 (10 396). (13)

It is interesting that the ratios of the entries in (12) and (13) to the corresponding values for the transition in He are very close (to within 1%) to

$$(1+\tfrac{2}{3})^2,\ (1+\tfrac{1}{3})^2,\ \text{and}\ (1-\tfrac{1}{3})^2, \qquad (14)$$

which is reasonable because these are transitions between two states, both of which are excited so that we would expect $Z_{\text{eff}}=1$ to be an accurate description. The additional quark will then result in $Z_{\text{eff}}=\frac{5}{3}$, $\frac{4}{3}$, and $\frac{2}{3}$, respectively. On the other hand, the ratio of the entries in (11) for the resonance transition to the corresponding 171 129.15 cm^{-1} in He depart considerably from the sequence in (14). This is again no surprise because the effective charge for the ground state in He is considerably larger than 1. We also note that for the triplet transitions considered in (12) and (13), the values of d and e in (10) turn out to be larger than for the singlet resonance transition. This is an indication of the slightly stronger spin-orbit effect for triplet states though even here a, b, and c are still the major coefficients, spin-orbit coupling being rather small in He. The relative importance of spin-orbit terms will increase in our later applications to heavy atoms.

Heavy atoms: Hg^+. Consider next some very heavy atom and the shift in spectral lines for the corresponding quark atom. Certain stars show prominent lines of the Hg^+ spectrum[16] and we consider it as an example. The isoelectronic series is Au, Hg^+, Tl^{2+}, Pb^{3+}, and Bi^{4+} with corresponding transition energies for $6s\,^2S_{1/2}$ $\rightarrow 6p\,^2P_{3/2}$ given by 41 174.3, 60 608, 78 970, 97 219, and 115 674. Fitting the first and last two to (6), we have $a=-686.619$, $b=21\,360.5$, $c=20\,491.6$, and $d=8.872\,92$ and, as a check, they lead to 79 112 for Tl^{2+}. For quark Hg^+, we have in turn (again $+\frac{2}{3}$, $+\frac{1}{3}$, $-\frac{1}{3}$, and $-\frac{2}{3}$, respectively)

73 019 (1370), 66 857 (1496), 54 254 (1843)

and 47 780 (2093); (15)

the entries are in cm^{-1} (Å).

As anticipated, d is somewhat larger now but even more interestingly, a is now small so that b and c are the major coefficients in (6). This can be understood with reference to the $1/Z$ theory.[5] In this theory,[17] the coefficient a of the term in Z^2 is given by the sum of $1/n^2$ for each electron (in Rydberg units). For the applications to transitions between states as in our considerations, a is, therefore, the difference of such sums for the two states in question. For the $6s$-$6p$ transition in the Hg^+ sequence, this difference is zero. For the resonance transition in He, on the other hand, we expect $\frac{3}{4}$ Ry $\simeq 82\,000$ cm^{-1}, in good agreement with our values for that case. Similarly for the $2\,^3P$-$3\,^3D$ transition we expect $\frac{5}{36}$ Ry $\simeq 15\,300$ cm^{-1} and for the $2\,^3P$-$4\,^3D$ we expect $\frac{3}{16}$ Ry $\simeq 20\,500$ cm^{-1}, again in very good agreement with the results of our fitting. This aspect, therefore, is a satisfying check of our procedure.

Hg. The above conclusions hold true for other heavy atoms. Table V capsules the results for several well-known lines in the spectrum of Hg obtained through (6). The entries can be expected to be reliably accurate to 1 Å for the quark charges $+\frac{2}{3}$ and $+\frac{1}{3}$ and about 10 Å for $-\frac{1}{3}$. In all our results obtained through the $1/Z$ fits, we expect our extrapolations to lower nuclear charge (negative quarks) to be less accurate because the $1/Z$ expansion breaks down as $Z\rightarrow 0$. If accuracy is desired here, it will be more appropriate to include terms in $(Z-k)^{-1}$ and obtain k by fitting, but we do not consider such fits here.

Quantum-defect fits. The quantum defect of an energy level is a more sensitive parameter than the energy itself and it is possible to use Z expansions of quantum defects as an alternative way of getting the transitions in quark atoms. This is particularly effective in one-electron systems such as the Hg^+ sequence when complications due to multichannel effects are absent and a simple single-channel quantum-defect fit will work well. For other cases, multichannel quantum-defect fits may be necessary. We consider here only single-channel quantum-defect fits as a sample of the improvements that are possible over the direct fits to the energy considered earlier. We have

TABLE V. Wavelengths in angstroms for quark Hg, for a few of the prominent lines in the spectrum.

λ_0	Quark $\frac{2}{3}$	Quark $\frac{1}{3}$	Quark $-\frac{1}{3}$
		(Å)	
1849.5	1454.9	1624.8	2161
2537.3	2074.9	2279.4	2873
4047.7	2217.1	2877.9	6677
4359.6	2350.9	3070.0	7335
5462.3	2871.0	3783.0	9565

TABLE VI. Ionization potentials, E_I, the energy of the $6p\ ^2P_{3/2}$ state and its effective quantum number n^* in the Hg^+ isoelectronic series. The entry in parentheses for Bi^{4+} and all the entries for quark Hg^+ are obtained from fitting n^* for the first four members of the series to Eq. (8).

Atom	E_I (cm^{-1})	$6p\ ^2P_{3/2}$ (cm^{-1})	n^*
Au	74410	41174.3	1.817083
Hg^+	151280	60608	2.200243
Tl^+	240600	78970	2.471937
Pb^{3+}	341350	97219	2.681796
Bi^{4+}	451700	115674	2.857332
			(2.839744)
$\frac{2}{3}$ Hg^+	209538	72828	2.389162
$\frac{1}{3}$ Hg^+	179756	66716	2.298991
$-\frac{1}{3}$ Hg^+	124152	54421	2.090790
$-\frac{2}{3}$ Hg^+	98461	48007	1.966378

the expression

$$n^* = n - \mu = [Z^2 I_H/(E_I - E_n)]^{1/2}, \tag{16}$$

where I_H is 13.6 eV, μ is the quantum defect, and n^* the effective quantum number of the energy level E_n in a Rydberg series with the ionization limit E_I. Table VI considers the $6s^2S_{1/2} - 6p^2P_{3/2}$ transition of the Hg^+ sequence. The first four members are fitted to (8) to obtain both the E_I and n^* for the quark-Hg^+ atoms and, thereby, a value for the transition energy. The entries for Bi^{4+} serve as a check; the number in parentheses obtained by our fits should be compared with the exact expression. The close similarity of the entries in the second column for quark Hg^+ to the results obtained in (15) by a direct fit to the energy is reassuring, lending confidence in the reliability of both sets of figures. Similar quantum-defect fits to any other sequence can be carried out.

One interesting result in Table VI is that the ionization potentials, just as much as the spectral lines, will be different for the quark atoms as compared to the corresponding ordinary ones. This can be the basis of techniques to enrich the relative amount of quark atoms in a sample by selective ionization of one or the other species followed by electrostatic separation of the ions. Since the change in E_I is large, this can be readily done and may be of interest for experimental quark searches. We note that E_I values for quark Hg are (in electron volts) 16.94, 13.62, 7.4, and 4.5 for $+\frac{2}{3}$, $+\frac{1}{3}$, $-\frac{1}{3}$, and $-\frac{2}{3}$, respectively, in contrast to the value of 10.43 eV for Hg.

Finally, we note that Ref. 3 mentions a quark search in sodium. For the $3s$-$3p$ and $3s$-$4p$ transitions in quark Na with a quark of charge $-\frac{1}{3}$, the values of 9500 and 6160 Å, respectively, are quoted. It is not clear how these numbers were arrived at but the following are our results in angstroms for quark charges $+\frac{2}{3}$, $+\frac{1}{3}$, and $-\frac{1}{3}$, respectively, obtained from (10):

$$\begin{array}{lccc} 3s\text{-}3p_{1/2} & 3387 & 4294 & 9512 \\ 3s\text{-}4p_{1/2} & 1603 & 2193 & 6089. \end{array} \tag{17}$$

V. CONCLUSIONS

Quick, simple, and accurate estimates of the position of spectral lines in an atom which has a quark embedded in the nucleus can be readily made. For x-ray transitions of inner electrons, screening theory suffices and can lead to results such as in Sec. II. For the outer transitions, lying in the visible and near-visible regions, a fit of the known transitions in an isoelectronic series to a Z expansion seems to be the most effective way of estimating them rapidly and accurately. Accuracies of 1 part in 10^3 can be achieved with little difficulty and this procedure may be of interest to experimentalists involved in quark searches as a way of generating such numbers for the particular atom of interest to them.

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[1]George S. LaRue, W. M. Fairbank, and A. F. Hebard, Phys. Rev. Lett. 38, 1011 (1977). This paper also contains references to earlier quark searches. See also Roger Bland, D. Bocobo, M. Eubank, and J. Royer, Phys. Rev. Lett. 39, 369 (1977), and a recent review, L. W. Jones, Rev. Mod. Phys. 49, 717 (1977).

[2]G. S. Hurst, M. H. Nayfeh, and J. P. Young, Phys. Rev. A 15, 2283 (1977).

[3]W. M. Fairbank, Jr., T. W. Hänsch, and A. L. Schawlow, J. Opt. Soc. Am. 65, 199 (1975).

[4]One such estimate is by D. Boccaletti, V. DeSabbata, and C. Gualdi [Nuovo Cimento A 45, 513 (1966)].

They consider shifts in the quark-atoms to be given by changes in Z^2, with Z the full nuclear charge. This is too crude, however, to give reliable values. See our discussion following Eq. (14).

[5]See, for example, D. Layzer, Ann. Phys. (N.Y.) 8, 271 (1959), and the references cited in Ref. 6 below.

[6]B. J. Skutnik, Phys. Rev. 181, 2145 (1969); and Phys. Rev. D 2, 635 (1970). The former paper contains comments on an earlier search for quark lines in quasar spectra.

[7]Ya. B. Zel'dovich, L. B. Okun, and S. B. Pikel'ner, Usp. Fiz. Nauk 87, 113 (1965) [Sov. Phys. Usp. 8, 702 (1966)].

[8]H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), Sec. 17B. The right-hand side of Eq. (1) should also include an additive constant called the "outer-screening potential" due to the outer electrons. However, to a good approximation, we can ecpect it to be essentially the same in the quark atom as in the ordinary counterpart and then Eq. (3) remains valid.

[9]J. C. Slater, *Quantum Theory of Matter* (McGraw-Hill, New York, 1951), Appendix 13.

[10]G. Burns, J. Chem. Phys. 41, 1521 (1964).

[11]Any effect due to a change in the nuclear radius is neglected because finite-size effects are proportional to $(R/a_0)^2$, where R is the nuclear and a_0 the Bohr radius, and such corrections are considerably smaller than the accuracy of our results.

[12]An earlier work has considered the Lyman and Balmer lines to be expected from a hydrogenlike atom in which an electron is bound to a quark of charge $+\frac{2}{3}$ [R. A. Leacock, W. I. Beavers, and C. T. Daub, Astrophys. J. 151, 1179 (1968)]. Note that our numbers in the $-\frac{1}{3}$ column in Table II are essentially equivalent to these results except for a small difference in reduced masses. The authors of this paper had conducted a search for such lines in solar spectra with inconclusive results.

[13]See, for instance, E. M. Burbidge, Ann. Rev. Astron. Astrophys. 5, 399 (1967).

[14]See, for example, P. W. Merrill, Astrophys. J. 134, 556 (1961).

[15]Here, and throughout the paper, the experimental values for spectral lines are from C. E. Moore [*Atomic Energy Levels*, U. S. Natl. Bur. Stand. No. 35], (U.S. GPO, Washington, D.C., 1971).

[16]R. E. White, A. H. Vaughan, Jr., G. W. Preston, and J. P. Swings, Astrophys. J. 204, 131 (1976). I thank Dr. Howard Bond for calling my attention to this matter.

[17]See also R. O. Mueller, A. R. P. Rau, and L. Spruch, Phys. Rev. A 8, 1186 (1973).