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Mechanism of the $(e, 2e)$ reaction with atoms

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The $(e, 2e)$ reaction is studied under kinematic conditions designed to test different reaction theories. The problem is reduced to a calculation involving distorted waves for the electron-ion interactions and a two-body operator for the electron-electron interaction. The amplitude factorizes into an *ee* factor and a distorted-wave factor if the eikonal approximation is made for the distorted waves. The factorization is tested by choosing kinematics so as to keep the distorted-wave factor constant while varying the *ee* factor. The value of the average eikonal potential is established in coplanar symmetric kinematics. The target is helium. At incident energies of 400 and 800 eV, the approximation is good for recoil momenta less than about 1 a.u., provided the effective two-body operator is the t matrix. The v matrix is ruled out. There are strong indications that the impulse approximation itself breaks down for small relative momenta between the outgoing electrons.

I. INTRODUCTION

In common with all theoretical problems for realistic quantum systems, the $(e, 2e)$ reaction with atoms must be reduced to a problem involving a few degrees of freedom. If we consider a reaction leaving the residual ion in its ground state, the simplest problem that has sufficient features of the reaction is a quasi-three-body problem, in which two electrons interact with each other through the Coulomb potential and with the ion through optical-model potentials in which unobserved channels are treated by polarization and absorption terms. It is possible to go part of the way towards obtaining the optical-model potentials from first principles and to make phenomenological approximations for the remaining features that enable elastic scattering in each two-body subsystem to be well described.[1]

The resulting three-body problem cannot be solved. Further approximations must be made, whose nature requires close interaction between experiment and theory for their development. Possible approximations are outlined in Sec. III. They involve some form of impulse approximation in which the interaction connecting initial and final distorted waves (computed in the optical-model potentials appropriate to the relevant two-body subsystem) is treated as a two-body operator.

At present it is still impossible to compute the resulting integral unless the two-body operator is local, for example, the Coulomb potential (distorted-wave Born approximation). However for certain kinematic regions great success has been achieved[2] by representing the distorted waves as plane waves in an average potential, which may be complex. This is called the eikonal approximation. It has the mathematical property that the $(e, 2e)$ amplitude factorizes into a two-body matrix element and a distorted-wave transform of the bound-state wave function of the struck electron.

It is possible to partially restore more realistic distorted waves by including them in the distorted-wave transform but retaining the factorization approximation.[3] This approximation is good in some kinematic regions (e.g., noncoplanar symmetric) but bad in others (e.g., coplanar symmetric). Where it is good, it differs only in high-momentum details from the eikonal approximation.

In the present work we retain the eikonal approximation for consistency in obtaining a factorized approximation. We have a good idea of its range of validity and the values of the average potential from the coplanar symmetric reaction.[2] To study the reaction mechanism it is helpful to isolate certain features. Here we concentrate on the two factors of the factorized approximation. In the noncoplanar symmetric reaction the *ee* factor (antisymmetrized square of the *ee* t-matrix element) is essentially constant as the variable azimuth ϕ is changed to vary the recoil momentum q. This geometry is ideal for studying the structure aspect of the reaction.

In the present work the distorted-wave transform (q factor) is kept constant and the polar angles θ_A, θ_B of the two detectors and the azimuth ϕ are varied so as to vary the *ee* factor (see Fig. 1). The energy E_A and E_B are kept equal. In this way it is possible to study the question of what is the appropriate two-body operator as well as the validity of the factorized approximation. In view of the difficulty of applying an optical model for molecules or solids, it is of great practical importance to understand the range of validity of the factorized (eikonal) approximation in the simpler case of atoms. If a working reaction theory can

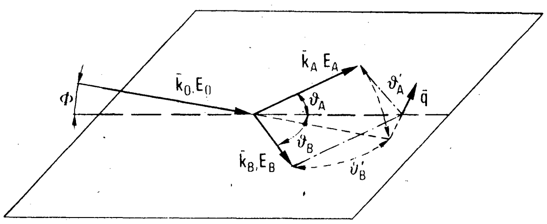

FIG. 1. Schematic diagram of the kinematics.

be found, it completes the means of extracting structure information from the reaction, in the absence of a more fundamental understanding of the mechanism.

II. EXPERIMENTAL APPARATUS

A schematic diagram of the apparatus is shown in Fig. 2. The main difference between this and the earlier version described in Ref. 2 is that an out-of-plane angular variation ϕ of the electron gun is now possible. This allows for measurements both in coplanar and out-of-plane geometry and makes possible a simultaneous θ and ϕ variation. The apparatus mainly consists of a stainless-steel cylindrical chamber (60 cm high and 130 cm in diameter) in which the basic components, an electron gun and two independently rotatable electron spectrometers, are mounted on the bottom flange. The chamber is pumped down to $\simeq 2\times 10^{7}$ Torr by an 8000 liter/sec mercury diffusion pump and two baffles refrigerated with water and liquid nitrogen, respectively.

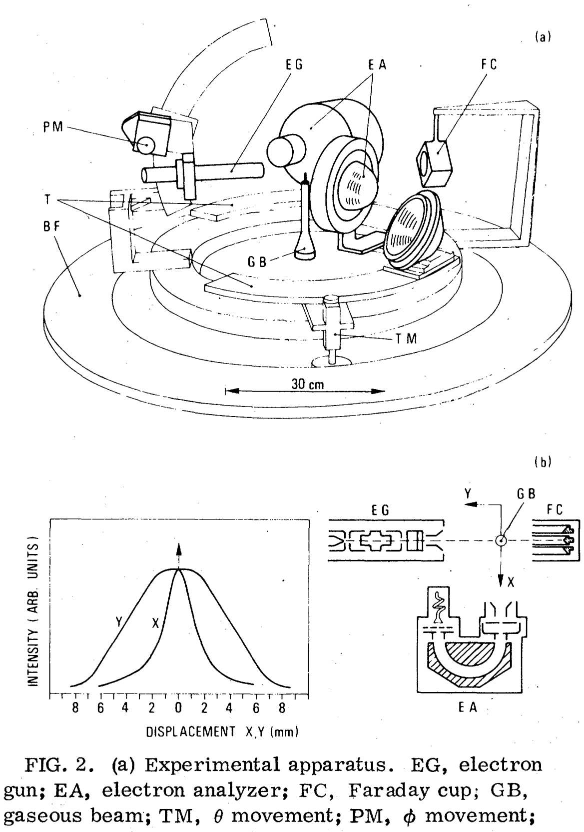

FIG. 2. (a) Experimental apparatus. EG, electron gun; EA, electron analyzer; FC, Faraday cup; GB, gaseous beam; TM, θ movement; PM, ϕ movement; T, turntables; BF, bottom flange. (b) Comparison between the gaseous beam profile (X) at the scattering point and the electron analyzer field of view (Y).

When the gaseous jet is sent into the chamber the pressure rises to about 10^{-6} Torr. A He cryopump, sitting on the top flange in front of the jet, is sometimes used, giving an extra pumping speed. Three Helmholtz coils, placed perpendicularly to each other outside the chamber, reduce earth's magnetic field to less than 5 mG in the interaction region. The spectrometers are further shielded by Mumetal foils.

The electron gun provides a quite well-collimated electron beam ($\Delta\theta$ about $\pm 1.10^{-2}$ rad) in the energy range 150 – 4000 eV. In the same energy range the beam spot at the target has dimensions of the order of 1 mm. The maximum available beam intensity increases, increasing the energy of the beam, from about 10 to about 200 μA. Intensity and position of the electron beam are continuously tested during the measurement by five small Faraday cages arranged together to give information on the total current, direction and profile of the electron beam. The incident angle ϕ can be varied from $-10°$ up to $+30°$ with a precision of $\pm 0.1°$.

The gaseous beam is obtained by allowing the gas to effuse through a Bendix multichannel array whose thickness is 0.25 mm. Each channel is 10-μm i.d. and the active area is about 50% of the total. The multichannel is sealed on a hypodermic needle 0.6-mm i.d. placed about 2 mm below the electron-beam path. This way, in the electron-gaseous beam crossing point the gas density is about 20 times larger than the background gas density, providing a suitable scattering target. Target density can be as high as 2×10^{14} mole/cm^3. It is determined by measuring the incoming gas flow and the effective size of the gas beam. Figure 2(b) shows a measured convolution of the electron and gas beams, obtained by scanning the target with the electron beam, and compares it with the acceptance of the detection system. It can be seen that the crossing region is smaller than the field of view of each detector ($\simeq$10 mm) so that any correction on target dimension has to be done when the angle of the detectors is varied.

The two twin-electron spectrometers are formed by hemispherical electrostatic selectors having a retarding field at the entrance and a channeltron, as electron detector, at the exit. The retarding field is used to enhance the overall energy resolution of the spectrometer in such a way as to obtain a suitable compromise between energy resolution and accepted solid angle. Retarding ratios as large as 1/300 have been obtained; under these conditions the energy resolution is still Gaussian in shape and as good as full width half maximum (FWHM) $\simeq 5\%$.

Each spectrometer can be rotated independently around the interaction volume in a plane defined by $\phi = 0°$. The angular variation allowed is from 0° to 120°; the precision is ±0.1° and the solid angle accepted is about 3×10^{-4} sr. Calibrations have been tested by measuring the elastic scattering of electrons on noble gases at the energies at which sharp minima are present.[4]

The essential structure of the electronic coincidence circuit, already described,[4] is a conventional one and is capable of a 4 nsec FWHM time resolution.

III. APPROXIMATIONS FOR THE REACTION MECHANISM

We start from the quasi-three-body problem, formulated as if the ee interaction v were of short range. In order to make this approximation as realistic as possible we ensure that high-momentum components of v predominate by keeping $E_A = E_B$. The $(e,2e)$ amplitude is[1]

$$T = \mathfrak{A} \langle \chi_A^{(-)}(\vec{k}_A)\chi_B^{(-)}(\vec{k}_B)|(f|v + v\frac{1}{E^{(-)} - K_1 - K_2 - V_1 - V_2 - v}v|g\rangle|\chi_0^{(+)}(\vec{k}_0)\rangle\rangle . \tag{1}$$

The operator $\mathfrak{A}$ antisymmetrizes the amplitude in the coordinates $\vec{r}_1, \vec{r}_2$ of the two electrons. The distorted waves $\chi_A^{(-)}, \chi_B^{(-)}$ are computed in the optical-model potentials V_1, V_2. The entrance-channel optical-model potential is used for $\chi_0^{(+)}$. We have chosen the post form of the amplitude, since approximations are more easily understood with respect to the final-state distorted waves. The ground state $|g\rangle$ of the target and the final state $|f)$ of the ion are functions of the many-body coordinates of the ion although these coordinates are assumed not to affect the potentials V_1 and V_2 (quasi-three-body approximation).

The first question that can be answered by referring to experiments is whether the operator in Eq. (1) can be approximated by a two-body operator. If so it commutes with the ion wave function $|f)$, so that the amplitude depends on the structure of target and ion only through the overlap $(f|g\rangle$. For closed-shell targets this overlap is expected[1] to be closely proportional to a spectroscopic amplitude $(S_j^{(f)})^{1/2}$. The $(e,2e)$ cross section is proportional to $S_j^{(f)}$, which is the probability that the many-body wave function $|f)$ contains the one-hole configuration, that is the configuration with a hole in the orbital j of the ground-state Hartree-Fock wave function. The spectroscopic factors $S_j^{(f)}$ obey the sum rule

$$\sum_f S_j^{(f)} = 1 , \tag{2}$$

which is verified within experimental error in all cases studied so far.

We are now in a position to ask what two-body operator gives a working theory. In some formulations[5] higher-order terms in v are dropped and the operator is assumed to be the ee potential v (distorted-wave Born approximation). This v-matrix approximation is computationally extremely simple. It gives a six-dimensional integral for Eq. (1) of a type that has been frequently evaluated for coincidence reactions.[6]

However, it is possible to make approximations for the second term in the operator. If V_1 and V_2 are constant, or depend only on the center-of-mass coordinate of the two electrons, then the distorted wave $\chi_A^{(-)}\chi_B^{(-)}$ is an eigenstate of the operator $K_R + V_1 + V_2$, where K_R is the center-of-mass kinetic energy operator of the two electrons. The operator then becomes the ee t matrix computed at the energy[1]

$$p^2 = (\hbar^2/4m)|\vec{k}_A - \vec{k}_B|^2 . \tag{3}$$

Although the approximations leading to this form (impulse approximation) are difficult to evaluate *a priori*, it has the advantage that in the absence of the ion the ee interaction is described completely. It has the computational disadvantage that the resulting nine-dimensional integral is extremely difficult to evaluate because of the difficulty of the coordinate transformation from the relative coordinates (in which the t matrix is expressed) to particle coordinates (in which the distorted waves are expressed).

The coordinate transformation difficulty is removed if the coordinates occur only linearly in the exponents, as they do if V_1 and V_2 are neglected and the distorted waves become plane waves. In this case the amplitude factorizes thus

$$T = \langle \vec{k}'|t_{ee}(p^2)|\vec{k}\rangle\langle\vec{k}_A\vec{k}_B|(f|g\rangle|\vec{k}_0\rangle , \tag{4}$$

where

$$\vec{k} = \tfrac{1}{2}(\vec{k}_0 + \vec{q}), \quad \vec{k}' = \tfrac{1}{2}(\vec{k}_A - \vec{k}_B) .$$

However, it is not necessary to neglect V_1 and V_2 completely, since in the interaction region, at least at sufficiently high energy, the distorted waves are well approximated by plane waves computed in an average constant potential $\bar{V}$. The eikonal approximation has had much success in describing $(e, 2e)$ reactions.[2]

The value $\bar{V} = 20$ eV has given good results for He at several energies[2] both for coplanar-symmetric and noncoplanar-symmetric reactions. The former are extremely sensitive to the value of $\bar{V}$. It is introduced by replacing k_I ($I = 0$, A, or B) everywhere in Eq. (4) by $\vec{\kappa}_I$, where

$$\vec{\kappa}_I^2 = (2m/\hbar^2)(E_I + \bar{V}). \tag{5}$$

Note that the approximation affects both factors in Eq. (4), the ee factor and the q factor.

More realism is achieved by including an imaginary eikonal potential iW, which approximates the imaginary parts of the optical-model potential. It is numerically so small that it does not perceptibly affect the shapes of cross sections as functions of kinematic variables. It produces an attenuation factor[1]

$$\gamma = \exp\left[-\left(\frac{k_0}{E_0} + \frac{k_A}{E_A} + \frac{k_B}{E_B}\right)\overline{W}R\right], \tag{6}$$

where R is a normalization radius chosen so that the magnitude of the distorted wave is 1 on the scattering axis at a distance R before the interaction region. The factor γ can be determined phenomenologically by absolute measurements, and at present it is known[7] only that it is between 0.5 and 1.

The factorization introduces much simplification into the use of the t matrix. Since only the absolute square of a matrix element is used, the difficult phase[8] is avoided. Furthermore only half-off-shell matrix elements[9] are required.

The antisymmetrized ee factor in the v-matrix approximation is

$$f_v = \frac{1}{|\vec{\kappa} - \vec{\kappa}'|^4} + \frac{1}{|\vec{\kappa} + \vec{\kappa}'|^4} - \frac{1}{|\vec{\kappa} - \vec{\kappa}'|^2} \times \frac{1}{|\vec{\kappa} + \vec{\kappa}'|^2} \cos\left(\ln\frac{|\vec{\kappa} + \vec{\kappa}'|^2}{|\vec{\kappa} - \vec{\kappa}'|^2}\right). \tag{7}$$

In the t-matrix approximation it is

$$f_t = C_0^2(\eta)\left[\frac{1}{|\vec{\kappa} - \vec{\kappa}'|^4} + \frac{1}{|\vec{\kappa} + \vec{\kappa}'|^4} - \frac{1}{|\vec{\kappa} - \vec{\kappa}'|^2} \times \frac{1}{|\vec{\kappa} + \vec{\kappa}'|^2} \cos\left(\eta \ln\frac{|\vec{\kappa} + \kappa'|^2}{|\vec{\kappa} - \vec{\kappa}'|^2}\right)\right], \tag{8}$$

where

$$C_0^2(\eta) = 2\pi\eta/[\exp(2\pi\eta) - 1], \tag{9}$$

$$\eta = me^2/2\hbar^2\kappa'. \tag{10}$$

In the present kinematics the only perceptible difference is in the factor $C_0^2(\eta)$.

IV. REACTION MECHANISM IN COPLANAR-SYMMETRIC KINEMATICS

We treat first the coplanar-symmetric kinematics in which the $(e, 2e)$ cross section is very sensitive to the reaction approximation in order to derive a realistic value for the average potential $\bar{V}$ to use in further investigation on the ee factor. The explicit expression of the cross section derived by Eq. (4) is given (in atomic units) by

$$\frac{d^5\sigma}{d\Omega_A d\Omega_B dE} = \frac{4k_A k_B}{k_0} f_\lambda \gamma \rho(\vec{q}), \tag{11}$$

where the f_λ factor becomes f_v or f_t depending on whether the v matrix or the t matrix is used. $\rho(\vec{q})$ is simply

$$\rho(\vec{q}) = |\langle\vec{\kappa}_A\vec{\kappa}_B|(f|g\rangle|\vec{\kappa}_0\rangle|^2, \quad \vec{q} = \vec{\kappa}_0 - (\vec{\kappa}_A + \vec{\kappa}_B), \tag{12}$$

and is the term which carries structure information of the target. In coplanar-symmetric conditions (i.e., equal kinetic energies $E_A = E_B = E$ and equal scattering angles $\theta_A = \theta_B = \theta$ for the electrons emerging in the plane containing the incident beam), both the factors f_λ and $\rho(\vec{q})$ have a large variation over the range of θ for which the cross section is measurable. This gives an overall test of the goodness of the approximations. Moreover in this kinematics, by suitably varying θ, it is possible to scan q values parallel to $\vec{k}_0$ (larger scattering angles) or antiparallel (smaller scattering angles). The $\rho(\vec{q})$ form factor is thus measured twice in the angular-correlation spectrum, while the f_λ factor is continuously varying. This adds a further possibility in testing the cross section.

Data presented in Fig. 3 refer to the ejection of $1s$ electrons of He at an incident energy $E_0 = 400$ eV. Reported curves are obtained by expression (11) in the v-matrix approximation [Fig. 3(a)] and in the t-matrix approximation [Fig. 3(b)] for different values of the distorting potential $\bar{V}$. As the differences of absolute values are within the present accuracy of the measurement[7] and because the interest is mainly in the agreement of the shapes, all the curves have been normalized to 1 at their maximum and $\overline{W}$ has been neglected. The distorting eikonal potential $\bar{V}$ is affecting mainly the q factor, by determining different values of the q momentum at a given scattering angle. This is clearly seen in Fig. 3, where in the abscissa the q values determined by different $\bar{V}$ are reported. Increasing $\bar{V}$, the maximum of the curve is shifted towards larger angles and the width of the curve slightly shrinks, corresponding to scattering events taking place in an attractive potential well, where the total kinetic energy is increased. For $\bar{V} = 20$ eV, by using the f_v factor the experimental maximum can be accounted for, but not the width

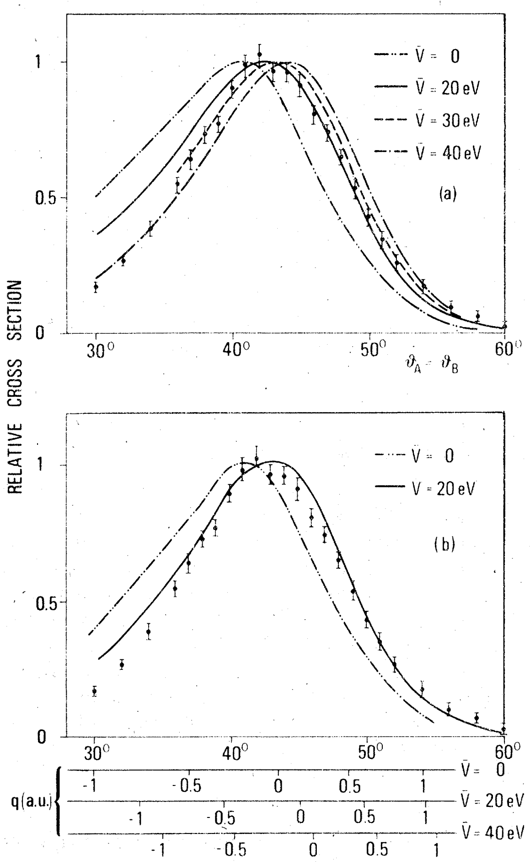

FIG. 3. Relative cross section for the ejection of 1*s* electrons from He measured in coplanar symmetric geometry. Incident energy $E_0 = 400$ eV. Measured coincidence rate is compared with curves computed in eikonal approximation from Eq. (11) with various $\bar{V}$ values, by using He 1*s* Clementi (Ref. 11) wave function. The f_v *ee* factor was used for (a) and the f_t *ee* factor for (b). The correspondence between scattering angle and q value determined from different $\bar{V}$ is also reported.

of the angular distribution [see Fig. 3(a)]. In Fig. 3(b) values $\bar{V}=0$ and $\bar{V}=20$ eV have been used together with the f_t factor. In the latter case a definitely good agreement, better than the previous one with the f_v factor, is obtained with the data. By using the t matrix a depletion in the flux of electrons emerging with lower relative velocity, i.e., at forward scattering angles in the laboratory system, is found.

From the analysis of the results we can deduce that (i) an eikonal potential $\bar{V}=20$ eV approximating the optical potential is good enough to treat distortion effects in the case of He, at least for incident energy as large as 400 eV; (ii) within the factorized approximation the use of the t matrix has to be definitely preferred in describing the scattering.

V. INVESTIGATION OF THE *ee* FACTOR IN THE EIKONAL APPROXIMATION

The eikonal approximation has had so much success so far in enabling structure information to be extracted from $(e, 2e)$ cross sections that it is worthwhile to make a thorough investigation of its validity. In order to remove the complication of q dependence from the investigation, we have chosen kinematics such that the q factor in the eikonal approximation, Eq. (4), is constant.[10] This means that in choosing the kinematic variables for experiments the value of $\bar{V}$ must be taken into account. We consider that previous work[2] and the data reported in Sec. IV have established the value $\bar{V}=20$ eV for He.

The sensitivity of the approximation to $\bar{V}$ is further investigated by reducing it to zero and performing further experiments under plane-wave kinematic conditions. Two types of kinematics are used, coplanar and noncoplanar. The condition $E_A = E_B$ is always maintained. In the coplanar experiments the angles θ_A and θ_B are varied in such a way that the q factor always retains the value 1 a.u. In the noncoplanar experiment θ_A and θ_B are both equal to θ. The angle ϕ made by the incident beam with the AB plane is varied, and θ is also varied so as to maintain the q factor at a chosen value q_0. In the absence of sufficiently accurate absolute cross section, the experimental cross section is normalized to the theoretical point at the largest angle $\theta_A + \theta_B$. In this region cross sections are least sensitive to $\bar{V}$.

Coplanar experiments have been performed at 800- and 400-eV incident energies so as to test the validity of the two eikonal models, Eqs. (7) and (8), versus the incident energy. With increasing incident energy, even the more crude f_v factor becomes reasonable in describing the scattering process.

Data taken at $E_0 = 800$ eV are reported in Fig. 4 and compared with f_v and f_t *ee* factors calculated in the eikonal approximation. From this comparison it is clear that although the f_t factor is fairly good in accounting for the experimental data, even the f_v factor cannot be ruled out and should be considered a reasonable model in describing the *ee* factor when the incident energy is larger than 800 eV. To complete the information coming from the 800-eV experiment, Fig. 4 shows, together with the two eikonal models, the product of the *ee* factor and the q factor both computed on the basis of undistorted plane waves ($\bar{V}=0$). In contrast with eikonal models this simpler one can be ruled out, even at incident energies as large as 800 eV.

Lowering the incident energy, the measurements

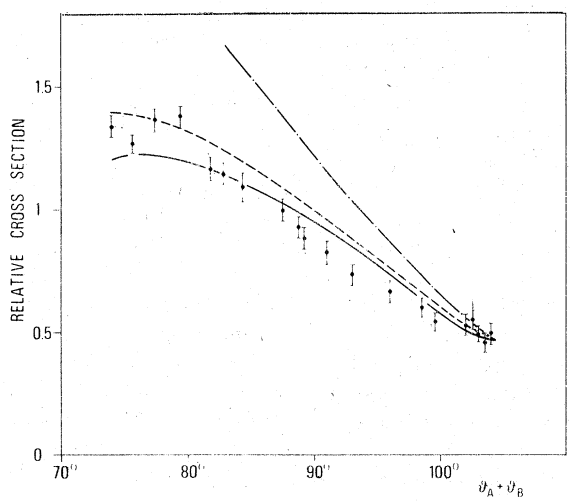

FIG. 4. Relative cross section for the ejection of 1*s* electrons from He in coplanar θ variation so as to keep $q_0 = 1.0$ a.u. Incident energy $E_0 = 800$ eV. On the abscissa the angle $\theta_A + \theta_B$ is reported. Data are compared with curves computed in the $\vec{V} = 0$ eikonal approximation (plane wave) times the f_v *ee* factor (dashed-dotted line), and the $\overline{V} = 20$ eV eikonal approximation with the f_v (dashed line) or the f_t (solid line) *ee* factor.

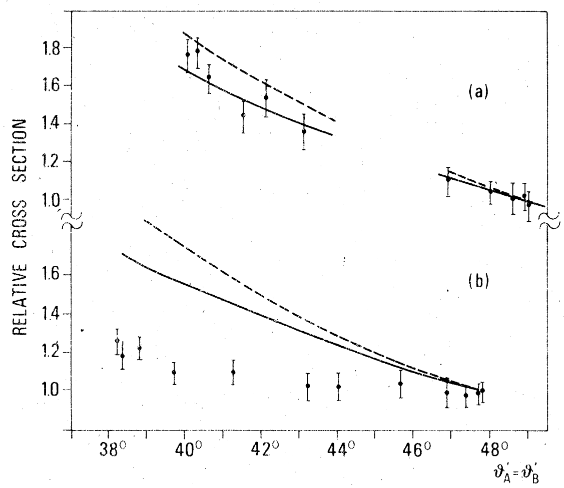

FIG. 6. Relative cross section for the ejection of 1*s* electrons from He in the noncoplanar ϕ variation so as to keep $q_0 = 0.44$ a.u. Incident energy $E_0 = 400$ eV. On the abscissa the scattering angle θ' between the incident and final electrons has been reported. (a) Data taken in eikonal approximation $\overline{V} = 20$ eV kinematic conditions are compared with f_v (dashed line) and f_t (solid line) *ee* factors. (b) Data taken in plane-wave kinematic ($\overline{V} = 0$) conditions are compared with f_v (dashed line) and f_t (straight line) *ee* factors.

are more sensitive to the model applied in evaluating the *ee* factor. Figure 5 shows how measurements taken at 400 eV, in coplanar conditions and with the q factor kept constant, are well accounted for by the f_t *ee* factor, while the f_v factor fails.

However when the scattering angle is small ($\theta_A + \theta_B \leq 70°$, i.e., small momentum transfer) also the f_t factor fails in fitting data, showing that the limits for applicability of the models are given both by incoming energy and momentum transfer.

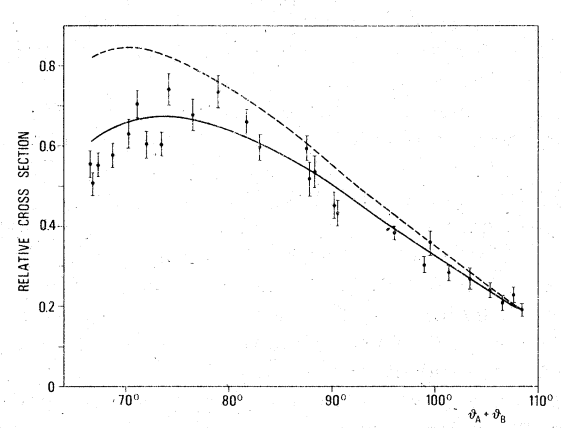

FIG. 5. Relative cross section for the ejection of 1*s* electron from He in coplanar θ variation so as to keep $q_0 = 1.0$ a.u. Incident energy E_0 400 eV. On the abscissa the angle $\theta_A + \theta_B$ is reported. Data are compared with f_v (dashed line) and f_t (solid line) *ee* factors computed in the eikonal approximation $\overline{V} = 20$ eV.

To further clarify this point noncoplanar experiments with θ and ϕ variation were undertaken at incident energy $E_0 = 400$ eV for three different values of the q momentum. In Fig. 6 the measured relative cross sections (normalized to one at the largest scattering angle) for $q_0 = 0.44$ a.u. are shown. In the abscissa the scattering angle $\theta_A' = \theta_B' = \theta'$ between the incident and final electrons is reported. Figure 6(b) refers to data taken under plane-wave kinematic conditions ($\overline{V} = 0$). Data are compared with calculated f_v and f_t *ee* factors. It clearly appears that in the $\overline{V} = 0$ approximation the computed values cannot account for the data, thus confirming the inadequacy of the plane-wave approximation. In Fig. 6(a) data taken in the $\overline{V} = 20$ eV eikonal approximation are compared with computed f_v and f_t *ee* factors. The data are in good agreement with calculated values, but it is not possible definitely to discriminate between the two factors.

The goodness of the t matrix in describing the $(e, 2e)$ cross section is better tested from the data taken at $q_0 = 0.7$ a.u. The data are reported in Fig. 7. Also in this case, Fig. 7(b) refers to data taken under plane-wave kinematic conditions. Once again strong disagreement is found between measured and computed values both for f_v and f_t *ee* factors. The data reported in Fig. 7(a), relative to $\overline{V} = 20$ eV eikonal models, are in good agree-

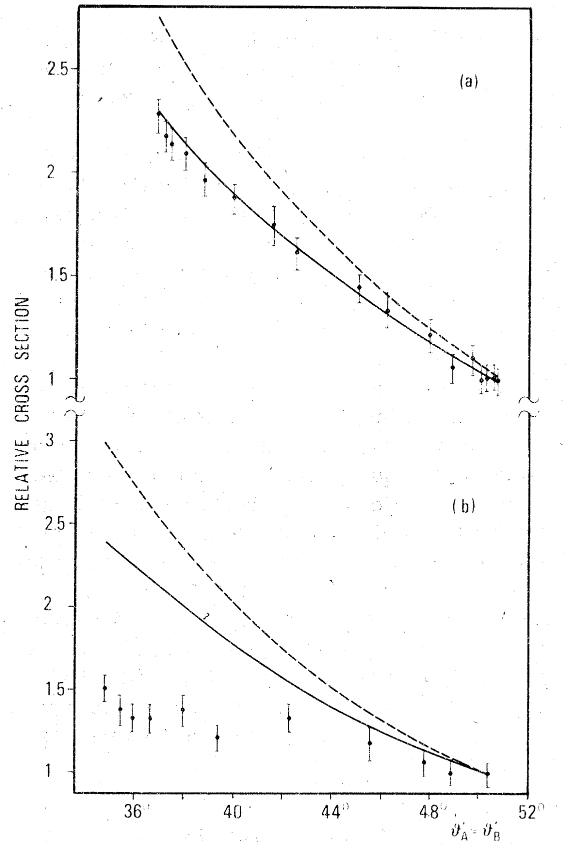

FIG. 7. Relative cross section for the ejection of 1s electrons from He in the noncoplanar ϕ variation so as to keep $q_0 = 0.70$ a.u. Incident energy $E_0 = 400$ eV. In abscissa the scattering angle θ' between the incident and final electrons has been reported. (a) Data taken in eikonal approximation $\overline{V} = 20$ eV are compared with computed f_v (dashed line) and f_t (solid line) *ee* factors. (b) Data taken in plane-wave kinematic conditions ($\overline{V} = 0$) are compared with f_v (dashed line) and f_t (solid line) *ee* factors.

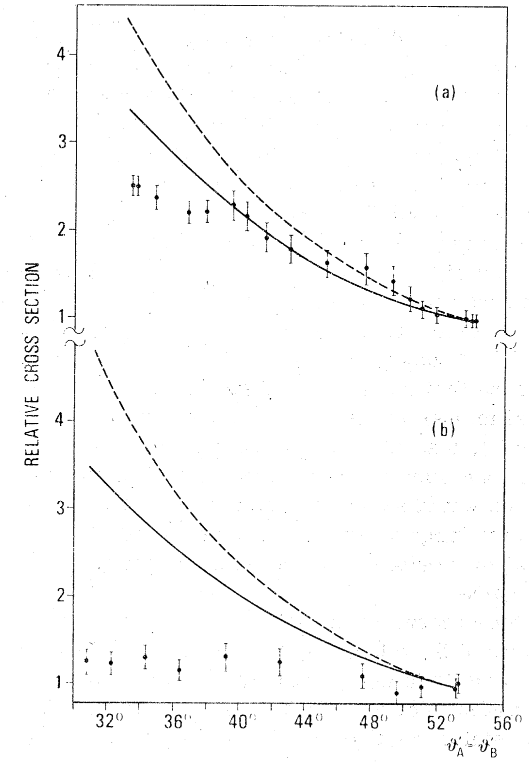

FIG. 8. Relative cross section for the ejection of 1s electrons from He in the noncoplanar ϕ variation so as to keep $q_0 = 1.0$ a.u. Incident energy $E_0 = 400$ eV. On the abscissa the scattering angle θ' has been reported. (a) Data taken in eikonal approximation $\overline{V} = 20$ eV are compared with f_v (dashed line) and f_t (solid line) *ee* factors. (b) Data taken in plane-wave kinematic conditions ($\overline{V} = 0$) are compared with computed f_v (dashed line) and f_t (solid line) *ee* factors.

ment with the computed f_t *ee* factor. In contrast the f_v *ee* factor cannot account for the data and has to be ruled out.

The last set of data taken in θ and ϕ variation concerns the value $q_0 = 1.0$ a.u. This value has been chosen in order to investigate in different kinematic conditions the failure observed in coplanar geometry at low-momentum transfer. Data are shown in Fig. 8 and, as usual, Fig. 8(b) concerns plane-wave kinematics. Once more plane-wave models fail completely in describing the scattering. In Fig. 8(a), the reported data are taken in the $\overline{V} = 20$ eV eikonal approximation. The data are in good agreement with the curve computed for the f_t factor only in the region of θ' greater than 38°. For lower angles the same type of failure observed in coplanar geometry is found.

VI. CONCLUSIONS

The first definite result of this investigation is the verification of the eikonal form of the distorted-wave impulse (t-matrix) approximation over a wide range of q at an energy where the plane-wave approximation fails completely. The eikonal form of the distorted-wave Born (v-matrix) approximation is ruled out at 400 eV, although it is roughly as good as the impulse approximation at 800 eV. Previous evidence of the inadequacy of the v matrix has been given in measurements of the overall behavior of the absolute cross section at $q = 0$ as a function of incident energy.[7] The t matrix was adequate to fit these data.

There are two important approximations whose validity has been investigated. The first is the impulse approximation

$$T = \langle \chi_A^{(-)} \chi_B^{(-)} (f|t|g) \chi_0^{(+)} \rangle . \tag{13}$$

This is the most complete approximation to the quasi-three-body problems that can be reasonably expected in terms of two-body operators.

The second is the factorized form of Eq. (13), which is necessary for computation. Only in the eikonal approximation is the factorization of Eq. (13) exact, because of the simplicity of transformation from partial to relative coordinates. This is the reason for treating the present investigation in terms of this model. It is essential to know in what range the eikonal approximation gives a reasonable estimate of Eq. (13).

If the eikonal approximation is reasonably valid, it means that there is an effectively constant internal momentum in each distorted wave. For larger q, a smaller radial region of the bound-state wave function is relevant to the amplitude and one would expect the approximation to break down for large q (small r) where the effective internal momentum is higher than its average surface value.

In the present work on He the borderline for the validity of the eikonal approximation is about $q=1$ a.u. This is in agreement with previous work[2] on other atoms for valence states. The deeper valence states require larger values of $\bar{V}$, but this does not change the general conclusions that the approximation is valid for q less than roughly 1 a.u.

The breakdown of the factorization approximation, since it depends essentially on the eikonal approximations, must depend on q, which is the only variable in the distorted-wave factor.

In several different experiments, Figs. 3 (coplanar symmetric), 5 (constant q-factor coplanar), and 8 (constant q-factor noncoplanar), we have a situation where the factorized impulse approximation works well for a large range of q, but breaks down as a function of $\theta_A+\theta_B$ at almost identical points, namely, less than 70°.

This constitutes strong evidence that it is the impulse approximation itself, not the factorization (or eikonal) approximation, that is breaking down. It is possible to speculate that this is to be expected for electrons emerging with smaller relative momentum, where long-range three-body effects due to screening of the two-body potentials may be more likely.

The present investigation provides incentive for further work on the reaction mechanism in two areas. First a reliable unfactorized calculation would settle the question of whether the impulse approximation, Eq. (13), is being sufficiently accurately evaluated. Second, relative differences have been observed between data and different approximations (e.g., f_t and f_v) for different kinematic conditions, but it is not clear in what kinematic range the calculations are failing. For example, in distinguishing between f_t and f_v at 400 eV, absolute cross sections with an accuracy in the vicinity of 20% would establish the range of validity (if any) of f_v.

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