

## Theory of neutral atom scattering at long range from metal surfaces\*

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We examine possible interactions between neutral atoms and metal surfaces at separations greater than 500 Å, treating the Lifshitz dispersion force, dynamic forces, and fringe fields due to surface roughness. Using the derived interaction energies we compute the classical trajectories of atoms passing over metal cylinders and compare them with the results of a recent series of experiments. There is a significant discrepancy between theory and experiment, with theory predicting more scattering than is observed.

### I. INTRODUCTION

There has recently been an extensive experimental study of the interactions between neutral atoms or molecules and a conducting surface.<sup>1-4</sup> In this work the deflection of an atomic beam passing near a cylindrical surface is measured. One then attempts to match the observed decay of the beam intensity into the shadow with theoretically derived expressions. For the case when the incident beam consists of neutral atoms (no permanent dipole moment), the interaction potential energy at a distance  $R$  (much larger than the lattice constant) from the conducting surface is expected to follow the predictions of the Lifshitz<sup>5</sup> theory: decaying first as  $R^{-3}$ , then as  $R^{-4}$  when retardation effects become important. The explicit formula of Parsegian<sup>6</sup> yields this behavior in a form that is readily calculable given the frequency dependence of both the dielectric constant of the conductor and the polarizability of the atom—see Eq. (1). We stress that there are no arbitrary parameters in this theory.

The surprising conclusion of the experimental work<sup>4</sup> is that the above dispersion forces appear to overestimate the actual interaction significantly. Shih and Parsegian<sup>4</sup> are able to fit their data by an interaction of the Lifshitz form but assert that the deduced coupling constant is at least 60% smaller than the theoretical value, with the estimated experimental errors being less than (5–10)%. They go on to discuss several possible reasons for the discrepancy but cannot find a resolution.

In view of the important implications of their conclusions we have reexamined theoretically the possible interactions between a moving neutral atom and a conducting surface. We first essentially duplicated their calculation of the Lifshitz dispersion force and confirmed their result. Then we sought modifications of this force due to omitted effects. A strong restraint on this search is the fact that the experiments indicate a weaker interaction than expected. Since most of the possible

interactions arise from second-order perturbation theory, adding extra coupling mechanisms yields more attraction between the atom and the surface. However, a careful analysis of the atom's classical trajectory shows that more attraction need not imply more scattering. We find that the rate of spatial variation of the potential energy is an important independent parameter. Encouraged by this general result, we have studied models to describe the extra interaction energy due to surface roughness. In the most promising case we find a potential energy which varies as  $R^{-6}$ , at large  $R$ , with a strength that is controlled by parameters of the model.<sup>7</sup> This extra attraction does lead to a lower theoretical scattered intensity: but unfortunately the improvement is marginal with respect to the data, at least for what seem to be reasonable parameter choices. Still the model does show how surface topography might be an important variable in such experiments.

In Sec. II we discuss the Lifshitz dispersion force and explicitly show how it fails to explain the experiments. We also argue that modifications consistent with the dispersion force mechanism are negligible. Next in Sec. III we present a physical picture of our roughness models; the detailed derivations have been relegated to an Appendix. We give both a qualitative description of their effect on an atom's trajectory and a model calculation. Lastly we summarize our conclusions in Sec. IV.

### II. LIFSHITZ DISPERSION FORCE

We begin with the result of Parsegian<sup>6</sup> derived from the Lifshitz theory<sup>5</sup> of dispersion forces. The potential energy of a neutral atom at a distance  $R$  above a flat metal surface at zero temperature is

$$V_L(R) = -k_L/R^3, \quad (1)$$

with

$$k_L = \frac{\hbar}{4\pi} \int_0^\infty du \alpha(iu) \frac{\epsilon(iu) - 1}{\epsilon(iu) + 1} \times \left(1 + \frac{Ru}{c}\right)^2 \exp\left(\frac{-2Ru}{c}\right), \quad (2)$$

where  $\hbar$  is Planck's constant divided by  $2\pi$ ,  $c$  is the speed of light, and  $\alpha(iu)$  and  $\epsilon(iu)$  are, respectively, the polarizability of the atom and the dielectric function of the metal, with both evaluated at the imaginary frequency  $iu$ . The full effects of retardation are included in (2). They are suppressed when  $R \rightarrow 0$  (or formally  $c \rightarrow \infty$ ), yielding a constant for  $k_L$ . In the opposite limit of large  $R$ ,

$$k_L \underset{R \rightarrow \infty}{\sim} \frac{5}{16\pi} \frac{\hbar c \alpha(0)}{R}. \quad (3)$$

The transition between these two regimes occurs for the materials of interest—alkali atoms above gold surfaces—around  $R = 10^3 \text{ \AA}$  and is an important feature in explaining the data. For an evaluation of (1) we determine the dielectric constant of the metal (Au) from optical data, using either the Drude parameters of Johnson and Christy<sup>8</sup> or the more extensive results of the DESY group.<sup>9</sup> The atomic polarizability is calculated from

$$\alpha(iu) = \sum_i \frac{e^2}{m} f_{0i} (\omega_i^2 + u^2)^{-1}, \quad (4)$$

where  $m$  is an electron's mass and  $e$  is the magnitude of its charge. The oscillator strengths  $f_{0i}$  and the transition frequencies  $\omega_i$  are taken for Cs from Norcross.<sup>10</sup>

Given the potential energy, the computation of  $I/I_0$  vs  $s$  is a standard central-force problem of classical dynamics,<sup>1-4</sup> where  $I/I_0$  is the ratio of scattered to unscattered flux at the detector location (a distance  $l_2 \approx 55 \text{ cm}$  beyond the cylinder) and  $s$  is the depth of the detector in the shadow region. For a detailed description see Ref. 2. We only remark here that for the range of  $s$  over which data exist, 0.005 to 0.15 cm, corrections for the finite widths of the incident slit and the detector and for their possible misalignment as well as for the effective origin for  $s$  are generally negligible except at the lowest values of  $s$  where for example they amount to a few percent significance at  $s = 0.005 \text{ cm}$ . We neglect them here. Further, rather than average  $I/I_0$  over the energy distribution of the beam, we use an average energy  $\bar{E} = 1.4 k_B T$ , where  $k_B$  is Boltzmann's constant and  $T$  is the oven temperature. We have checked that the errors thereby incurred are at most a few percent.<sup>2</sup> Finally it is useful to plot not  $I/I_0$  but rather  $I/I_c$  vs  $s$ , where  $I_c/I_0$  is the scattering spectrum for a standard choice of potential energy. We compute  $I_c$  using the Lifshitz dispersion force, (1), with  $k_L$  set equal to  $14.6 D^2$  for all  $R$ . Here  $D$  is the Debye unit and

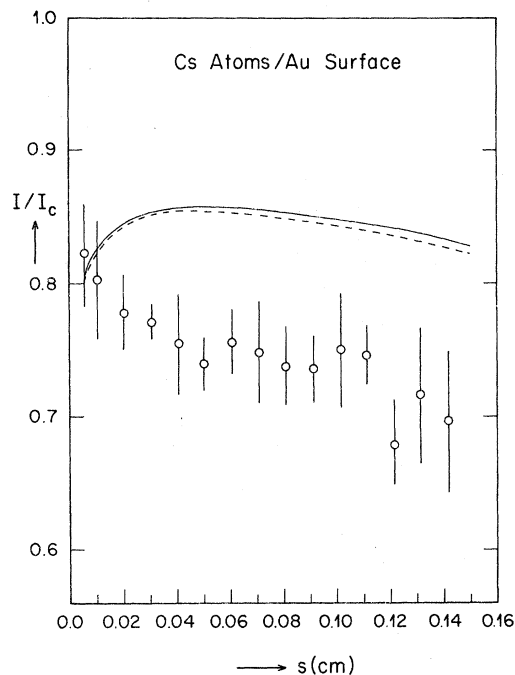


FIG. 1. Scattered intensity  $I/I_c$  vs distance into the shadow  $s$  for Cs atoms passing over a Au-plated cylinder. The experimental data and parameters are from Ref. 4. The error bars represent  $\pm$  one standard deviation. The theoretical curves are derived from the Lifshitz dispersion force, Eq. (1), using the optical data of Ref. 9, —; or Ref. 8, ----. The values  $I/I_c = 1$  represent the scattering due to a potential  $V = 14.6 D^2/R^3$ .

$1 D^2$  equals  $10^{-36} \text{ erg cm}^3$ . This particular value we choose for  $k_L$  is the  $R = 0$  limit of Eq. (2) for Cs over Au using Refs. 9 and 10.

In Fig. 1 we compare the experimental data for Cs over Au with the predictions of the Lifshitz dispersion force both including and excluding the effects of retardation. The latter curve in our scheme is simply  $I/I_c = 1$  for all  $s$ . The data lie some (20–30)% below this curve and in order to make a fit with a potential of the form  $-k_3/R^3$  one needs  $k_3 = 7 D^2$ .<sup>4,11</sup> Although retardation effects on  $V_L$  can account for more than half the discrepancy, there remains a significant overestimate even in this theory. Note that the effect of different optical data is slight. We also remark that little significance should be attached to the downward plunge of the theoretical curves as  $s \rightarrow 0$ . The kinematic complications we have neglected rapidly become important below  $s = 0.005 \text{ cm}$ , so much so that the true predicted  $I/I_c$  probably is increasing with decreasing  $s$  for  $s < 0.0025 \text{ cm}$ . However, since all this structure occurs below the lowest experimental value of  $s$  and is tedious to calculate precisely, we ignore it.

Since it is obvious from Fig. 1 that theory and experiment do not agree, let us now consider possible modifications to the theory. The derivation of (1) is in essence a second order perturbation theory calculation requiring the virtual electronic excitation of both the neutral atom and the metal due to their mutual electromagnetic interaction. The specific form obtained depends further on the assumption that  $R$  is the dominant length in the problem. One assumes  $R$  is much larger than the dimension of the atom, the width of the metal's surface structure, and any screening length or lattice constant of the bulk metal. Although a detailed calculation of corrections to (1) is not easy,<sup>12,13</sup> it is reasonable that they must be of relative order  $\delta R/R$ , where  $\delta R$  is some average of the previously omitted small lengths and should be no greater than a few angstroms. Since the atomic trajectories of relevance to the experiment come no closer than about 500 Å, such corrections to  $V_L(R)$  are probably less than a few percent and hence inconsequential. This conclusion would be less tenable if there were a thick overlayer of surface contamination or a large degree of surface roughness. However the first possibility was eliminated by an extra experiment over a freshly deposited surface in high vacuum ( $<10^{-10}$  Torr) which gave the same results as previous work.<sup>4</sup> The second possibility is harder to analyze; but the fact that one is making a relative correction to (1), now of order  $(\delta R/R)^2$  due to its random nature, does not allow large changes in  $V_L$  even for roughness on the scale of  $\delta R \sim 100$  Å.

### III. ADDITIONAL INTERACTIONS

If one accepts our argument above that intrinsic corrections to the Lifshitz dispersion force are small, then what we need are new mechanisms of interaction. One possibility is a frictional force proportional to the atom's velocity. Such forces arise when one considers the first corrections to the Born-Oppenheimer approximation that we have so far invoked.<sup>14,15</sup> Unfortunately, a rough estimate of these, although complicated by the fact that the moving atom is not charged, indicate that they are several orders of magnitude too small.

A second possibility comes from the influence of large scale surface roughness on the kinematics. What we imagine here is that surface height variations along the length of the cylinder give rise to a random variation in the effective origin for  $s$ . This effect would necessitate an extra average of  $I/I_0$  over the distributions of origins. However, for  $s > 0.005$  cm an uncertainty in its true value of less than a micron is negligible. Moreover, since  $I/I_0$  has positive curvature for  $s$  in this range, the

average could only act to increase the predicted flux.

Another way that surface roughness can influence  $V(R)$  is by allowing a separate coupling mechanism to exist. The Lifshitz dispersion force requires the virtual excitation of both the neutral atom and the metal. For an infinite plane surface there are no static electric fields that exist beyond the surface electronic dipole layer. This result remains true for a closed surface of arbitrary shape as long as the dipole layer is uniform. The presence of surface roughness will however allow the existence of significant fringe or patch fields well outside the metal surface, so we now turn to models that allow the calculations of such fields.

There are at least two ways in which surface roughness can make the electronic dipole layer nonuniform. Over the face of a single microcrystalline grain, one can imagine steps between various plateaus which in effect discontinuously raise or lower the dipolar charge density. On a larger transverse scale the different exposed microcrystalline faces cause an additional nonuniformity of the dipole layer due to variations in the work function with crystal face, giving rise to the classical patch fields.<sup>16</sup> Although both effects are simultaneously present, for simplicity we treat them separately (see the Appendix for details). In each case we can derive a static but spatially fluctuating electric field outside the metal. This force acts on the neutral atom to produce in second order perturbation theory an additional attractive interaction:

$$\delta V(R) = -\frac{1}{2}\alpha |\vec{\mathcal{E}}(R)|^2, \quad (5)$$

where  $\alpha = \alpha(0)$  is the atom's static polarizability and  $\vec{\mathcal{E}}(R)$  is the roughness produced electric field at the atom's location. To make our models tractable, we replace the squared field in (5) by the mean-square field at distance  $R$  above the mean surface  $\langle\langle |\vec{\mathcal{E}}(R)|^2 \rangle\rangle$ , where the average is taken over the surface roughness, which is represented by a Gaussian correlation function. We thereby obtain an expression for  $\delta V(R)$  whose consequences can still be analyzed as a central force problem. The complete formulas are in the Appendix, here we merely give their large  $R$  variation.

For the model of steps on a single-crystal face we have for  $R \gg \Lambda_s$ ,

$$\delta V_s(R) \sim -\frac{4\pi}{4}\alpha(\pi e D \Sigma \Lambda_s)^2 / R^6, \quad (6)$$

where  $4\pi e D$  is the change in electrostatic potential due to the dipole layer of the smooth surface. The parameter  $\Lambda_s$  characterizes the transverse dimensions of the plateau while  $\Sigma$  describes the height of the steps. In the opposite limit  $R \ll \Lambda_s$ ,  $\delta V_s$  saturates at a finite value.

The strong spatial dependence of  $\delta V_s$  in the asymptotic region is an important feature. To clarify this point we make a brief digression to discuss the classical mechanics of an atom's trajectory. In the limit of large deflections we have found an approximate analytic solution of the equations of motion. It predicts

$$I/I_0 \propto \exp[-(\frac{1}{2}b)^{1/2}sr_0/l_2], \quad (7)$$

where  $r_0$  is the cylinder radius and  $s/l_2$  is the angle of deflection. The parameter  $b$  is defined by

$$bE = -\left. \frac{d^2V}{dR^2} \right|_{R=R_0}, \quad (8)$$

where  $E$  is an atom's initial (and final) kinetic energy,  $V(R)$  is the total interaction potential energy with the cylinder, and  $R_0$  locates the maximum in the effective potential energy  $V_{\text{eff}}$ , which controls the radial motion:

$$V_{\text{eff}} = L^2/2M(r_0 + R)^2 + V(R), \quad (9)$$

where  $M$  is an atom's mass and  $L = (2ME)^{1/2}(r_0 + a)$ , with  $r_0 + a$  the impact parameter, is the conserved angular momentum. One can easily show to an excellent approximation<sup>2</sup> that for all reasonable  $V(R)$ ,  $V_{\text{eff}}$  has a single maximum at some  $R = R_0$ , independent of  $a$ , or equivalently of  $s$ ; that the maximum value  $V_{\text{eff}}(R_0)$  grows linearly with  $a$ ; and that the shape of  $V_{\text{eff}}$  near  $R_0$  depends only on  $b$ . Consequently, there is a value of  $a = a_0$  at which the maximum value of  $V_{\text{eff}}$  equals  $E$ . For  $a < a_0$  the atoms will crash into the surface, while for  $a > a_0$  the radial motion will recoil against  $V_{\text{eff}}$  at a value of  $R > R_0$ . Typical values of  $a_0$ ,  $R_0$ , and  $r_0$  are 500 Å, 400 Å, and 1.25 cm, respectively. The key point is that for  $a > a_0$  how quickly an atom reverses its radial velocity determines the extent it is scattered: a slow reversal implies a large deflection and vice versa. The speed of the reversal depends in turn, for  $a$  sufficiently close to  $a_0$  (i.e., for large enough  $s$ ) only on  $b$ . Equation (7), which is the analytic result of this argument, is formally valid only in the limit of large deflections but in practice its predicted dependences are apparent for deflection angles as small as  $s/l_2 \approx 0.002$  rad, which are well within the range of the data.

The most useful aspect of (7) is that it clearly shows  $I/I_0$  to depend on more than the mere strength of  $V$ . For a pure inverse power-law potential energy, say  $V = -k_n/R^n$ , a larger  $k_n$  gives a larger  $I/I_0$  since  $b \propto (1/k_n)^{1/(n+1)}$ .<sup>11</sup> However, if we consider a mixture of power laws, say

$$V(R) = -k_3/R^3 - k_m/R^m, \quad m > 3, \quad (10)$$

then it is possible as we increase  $k_m$  from zero, that  $b$  may increase and thereby lower  $I/I_0$ , even though the strength of  $V$  is increasing. In fact for

a  $V$  of the form (10) we can show that  $I/I_0$  at large  $s$  will initially decrease with increasing  $k_m$  if  $m > 4$ . The encouraging point is that if in addition to the Lifshitz potential energy one can justify an interaction between the atoms and the metal which varies rapidly with  $R$ , then there is hope that  $I/I_0$  will be lowered even though the extra interaction is also attractive.

Now reconsider (6) which matches our criteria. We expect (and find) that the addition of  $\delta V_s$  to  $V_L$  will lower the predicted  $I/I_0$ . In this regard the alternative patch model of surface roughness is not successful. Its strongest spatial dependence is also at large  $R$  when  $R \gg \Lambda_p$  and

$$\delta V_p \sim -\frac{3}{4}\alpha(\pi e\delta D\Lambda_p)^2/R^4, \quad (11)$$

where  $4\pi e^2\delta D$  measures half of the average variation of work function with crystal face and  $\Lambda_p$  characterizes the transverse grain size. Hence its inclusion acts to increase  $I/I_0$ . Similarly for both  $\delta V_s$  and  $\delta V_p$  with  $R_0$  away from the asymptotic region, one expects only increases in  $I/I_0$ .

We have carried out many model calculations, all of which confirm the above qualitative remarks. For instance if we augment  $V_L$  with  $\delta V_p$  using  $4\pi e^2\delta D = 0.06$  eV, then  $I/I_0$  increases in roughly uniform fashion for all  $s > 0.01$  cm up to a maximum enhancement of approximately 2% as  $\Lambda_p$  is increased to  $10^3$  Å. For further increases in  $\Lambda_p$ ,  $I/I_0$  is reduced back to its value for a smooth surface. Of course the maximum enhancement depends on the value of  $\delta D$ . Our choice here is based on the calculations of Lang and Kohn,<sup>17</sup> but we stress that for no choice does  $I/I_0$  decrease below its smooth surface values.

The situation is not quite as bad when we instead add  $\delta V_s$  to  $V_L$ . Now as long as we keep  $\Lambda_s \approx 50$  Å to ensure the applicability of (6), the predicted  $I/I_0$  will initially decrease as the product  $D\Sigma\Lambda_s$  increases; see Fig. 2. The maximum reduction occurs for  $D\Sigma\Lambda_s \approx 150$  Å. Further increases of  $D\Sigma\Lambda_s$  eventually make  $I/I_0$  greater than its smooth surface values. In addition, if  $\Lambda_s$  is increased beyond 50 Å at constant  $D\Sigma$ , then the predicted  $I/I_0$  returns to its smooth surface values.

Unfortunately, physical estimates suggests that the product  $D\Sigma\Lambda_s < 2$  Å. To illustrate, we use  $D = 0.0128/\text{Å}$  (i.e.,  $4\pi e^2D = 2.32$  eV) to correspond to a jellium estimate of the dipole-layer contribution to the work function of Au (Ref. 17); we take  $\Sigma = 2.36$  Å, the spacing of the (111) planes of Au (Ref. 18); and we set  $\Lambda_s = 50$  Å.<sup>19</sup> Combining these yields  $D\Sigma\Lambda_s = 1.5$  Å. Even a factor of 10 enhancement (somehow spread over these estimates) still gives a product  $D\Sigma\Lambda_s$  far too small to resolve the discrepancy between theory and experiment. Calculations for the data on K or Rb over Au are similarly

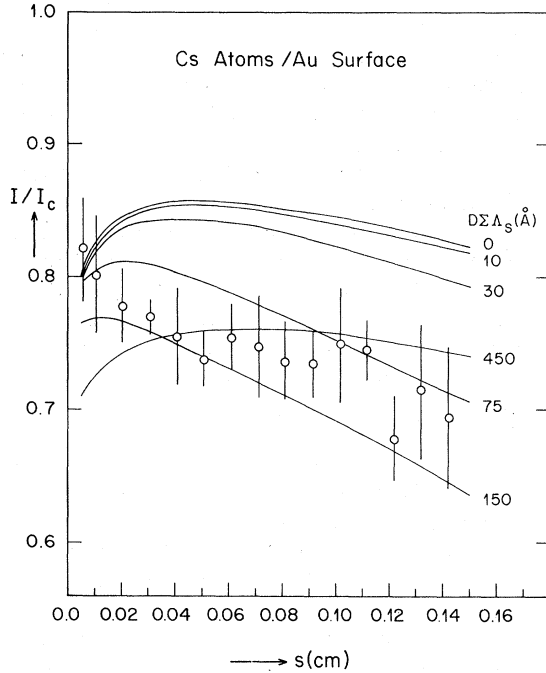


FIG. 2. Scattered intensity  $I/I_c$  vs distance into the shadow  $s$  for Cs atoms passing over a Au-plated cylinder. The experimental data and parameters and the normalization are as in Fig. 1. The theoretical curves are calculated using for the interaction potential energy the sum of the Lifshitz expression (1) and the step roughness model (A6). The latter is evaluated here only in the asymptotic limit (6) and is parametrized by the product  $D\Sigma\Lambda_s$ .

disappointing. Our mechanism acts in the right direction but with apparently too weak a magnitude.

#### IV. DISCUSSION

The outstanding point is the significant discrepancy between theory and experiment. We have argued that neither intrinsic corrections to the Lifshitz dispersion force nor new mechanisms of interaction through surface roughness can resolve the disagreement. However, even though surface roughness effects seem too small, they at least show a possible way to reduce  $I/I_0$  without rejecting the Lifshitz theory.

We urge that similar experiments be done, but with a particular regard to characterizing the surface. This will aid in the construction of suitable models and choice of parameters. An experiment over a liquid metal droplet would probably resolve the importance of surface roughness effects.<sup>4</sup> It would be especially useful in ruling out large scale roughness for which our averaging procedure is inappropriate. Note that a more extensive range of  $s$  will be of little use: the small  $s$  region is

complicated by kinematic corrections and the large  $s$  region is determined by (7).

If further work fails to remove the discrepancy, one may have to question the basic Lifshitz calculations, a point of view we have not explored here.

#### ACKNOWLEDGMENT

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#### APPENDIX

To derive the potential energy of an atom above a rough surface we need by Eq. (5) the static electric field produced by the surface. We calculate this field by modeling the spatial variation of the metal's charge density. For a smooth surface, we consider the net charge density  $\rho$  to depend only on the coordinate  $x$  normal to the surface and on the crystal face exposed. The direct effect of lattice periodicity on the fields produced far above the surface (with respect to the lattice constant) is negligible.<sup>12</sup> Assuming the metal is neutral, the first relevant moment of the charge density is the dipole moment/unit area:

$$D(\vec{X}) = \frac{1}{e} \int_{-\infty}^{\infty} dx x \rho(x, \vec{X}), \quad (\text{A1})$$

where  $\vec{X}$  is a two-dimensional vector in the surface plane. The  $\vec{X}$  variation of  $D(\vec{X})$  is due to the work function variation among crystal faces. We remark that a charged cylinder would produce such a slowly varying average field—proportional to  $r_0/(r_0 + R)$ —that it would be of little consequence for the experimental range of deflections and in any case would only raise  $I/I_0$ .<sup>1</sup>

Now consider a rough surface. Over a single patch  $D(\vec{X})$  will be constant so we write

$$D(\vec{X}) = D + \delta D(\vec{X}), \quad (\text{A2})$$

where the average of  $\delta D(\vec{X})$  over  $\vec{X}$  vanishes. To account for steps in the surface height within a patch, we use the smooth surface density profile shifted in  $x$  by  $\Sigma(\vec{X})$ . The origin of  $x$  locates the mean surface and we require the average of  $\Sigma(\vec{X})$  over  $\vec{X}$  to vanish. Note that our model omits any "healing" of the charge density in response to the distortions of the rough surface and hence probably overestimates the fields produced.

The electrostatic field is given by  $\vec{\mathcal{E}} = -\vec{\nabla}\phi$  with

$$\begin{aligned} \phi(\vec{r}) = & \int \frac{d^2Q}{2\pi Q} \int d^2X' e^{i\vec{Q}\cdot(\vec{X}-\vec{X}')} \\ & \times \int dx' e^{-Q|x-x'|} \rho(\vec{r}'), \end{aligned} \quad (\text{A3})$$

where  $\vec{Q}$  is a two-dimensional vector in the surface plane,  $Q = |\vec{Q}|$ , and the three-dimensional vector  $\vec{r} = (x, \vec{X})$ . Retardation effects do not enter (A3) since this potential is due to a static charge distribution.

We want to evaluate (A3) at  $x \approx R$ , far beyond the dipolar charge density. Hence we replace  $Q|x - x'| = Qx - Qx'$  and expand  $e^{Qx'} \approx 1 + Qx' + \dots$ . Then using our approximation for  $\rho$ , we find up to first order in  $\delta D$  and  $\Sigma$ ,

$$\begin{aligned} \vec{\mathcal{E}}(\vec{r}) &\approx \int \frac{d^2 Q}{2\pi Q} (Q, -i\vec{Q}) \exp(i\vec{Q} \cdot \vec{X} - Qx) \\ &\quad \times \int d^2 X' e^{-i\vec{Q} \cdot \vec{X}'} [eQ(D + \delta D(\vec{X}')) + \dots] \\ &\quad \times [1 + Q\Sigma(\vec{X}') + \dots] \\ &\approx -e \int \frac{d^2 Q}{2\pi} (-Q, i\vec{Q}) \exp(i\vec{Q} \cdot \vec{X} - Qx) \\ &\quad \times [\delta D(\vec{Q}) + QD\Sigma(\vec{Q})], \end{aligned} \quad (\text{A4})$$

where  $\delta D(\vec{Q})$  and  $\Sigma(\vec{Q})$  are two-dimensional Fourier transforms. The second equality in (A4) shows how in our approximation the effects of patches and steps separate. We further assume that there is no correlation between the random functions  $\delta D(\vec{X})$

and  $\Sigma(\vec{X})$  so that when we form the mean-square field we need only specify averages of  $\delta D(\vec{Q})\delta D(\vec{Q}')$  and  $\Sigma(\vec{Q})\Sigma(\vec{Q}')$ . To parametrize these we make the ansatz

$$\begin{aligned} \langle\langle \Sigma(\vec{X})\Sigma(\vec{X}') \rangle\rangle &= \Sigma^2 \exp(-|\vec{X} - \vec{X}'|^2/\Lambda_S^2), \\ \langle\langle \delta D(\vec{X})\delta D(\vec{X}') \rangle\rangle &= (\delta D)^2 \exp(-|\vec{X} - \vec{X}'|^2/\Lambda_p^2), \end{aligned} \quad (\text{A5})$$

where the averages on the left-hand sides of (A5) are over the center-of-mass variable  $\frac{1}{2}(\vec{X} + \vec{X}')$ . Our use of an average expression here implies an assumption that the roughness is not too large.

From the Fourier transforms of (A5) we can readily compute  $\langle\langle |\vec{\mathcal{E}}(R)|^2 \rangle\rangle$ , which separates into two parts. Using (5) we obtain finally  $\delta V = \delta V_S + \delta V_p$ , where

$$\begin{aligned} \delta V_S(R) &= -2\pi^2 \alpha (eD\Sigma\Lambda_S)^2 \\ &\quad \times \int_0^\infty dQ Q^5 \exp(-2QR - \frac{1}{4}\Lambda_S^2 Q^2) \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} \delta V_p(R) &= -2\pi^2 \alpha (e\delta D\Lambda_p)^2 \\ &\quad \times \int_0^\infty dQ Q^3 \exp(-2QR - \frac{1}{4}\Lambda_p^2 Q^2). \end{aligned} \quad (\text{A7})$$

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<sup>19</sup>The experimental characterization (Refs. 3 and 4) is only that the several-thousand-angstrom-thick Au films were vacuum deposited on drawn-glass or stainless-steel cylinders, the latter polished to 250 Å.