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Energy loss of ions moving through high-density matter*

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An equation is obtained for the energy loss of ions to electrons in a high-density plasma of arbitrary degeneracy. The quantum-mechanical form of the dielectric function is used to produce a formula that is free of the usual Coulomb divergence, so that the introduction of *ad hoc* cutoff parameters is not required. The results show a decrease of 20% to 50% in the energy-loss rate when compared with other formulas that use the standard Coulomb logarithm.

I. INTRODUCTION

One method to obtain high thermonuclear energy gains for inertially confined fusion is to ignite only a small region at the center of the fuel and to allow the burn front to propagate into the relatively cold outer parts of the pellet.[1] The mechanism for burn propagation is that charged particles from fusion reactions deposit their kinetic energy back into the pellet so that cold fuel can be heated. Most formulas that describe this energy deposition contain a phenomenological cutoff parameter in the Coulomb logarithm.[2]

The necessity for a cutoff parameter reflects approximations made in the theoretical formulation that have caused logarithmic divergences in the equations. The cutoff parameter is usually estimated by physical arguments concerning the maximum and minimum impact parameters for a Coulomb collision. Since any error in this term appears inside of a logarithm, it is generally assumed that it will not significantly affect the slowing-down formula. Indeed, this is the case for a classical plasma where the argument of the logarithm is large, but it need not be true when the argument becomes small as can happen in the superdense plasmas anticipated for laser fusion. In this region, one requires a more exact formulation of the problem, one in which all parameters are well defined. Such a formalism is available for high-density matter, using the quantum mechanical form of the dielectric function.[3] It will be used here to obtain a general formula for the slowing down of ions by electrons in a semidegenerate plasma. The questionable logarithmic divergences do not occur in this calculation, and the final result does reduce to well-known expressions in the limits of strong and weak degeneracy.

The reason for the logarithmic divergence in a classical plasma is illustrated by writing the Coulomb logarithm in the form $\ln(b_{max}/b_{min})$, where the argument is the ratio of maximum to minimum impact parameters for a collision. These quantities are usually determined by two approaches, each of which causes a different kind of divergence. One approach is to describe the energy loss by means of two-body collisions using the Rutherford cross section. Then, b_{min} is determined by the maximum energy loss for a collision. However, b_{max} becomes infinite due to the long range of the Coulomb interaction, and it is necessary to introduce a cutoff parameter in order to prevent the logarithm from diverging.[4]

The second approach treats the plasma as a continuous medium described by a dielectric function, and considers the slowing down to result from a frictionlike force caused by plasma polarization. Now b_{max} becomes finite due to plasma shielding. Unfortunately, this treatment also gives b_{min} as zero, and hence, the logarithm will diverge at the other limit. This is caused by a breakdown of the classical dielectric function at small distances, and again requires the introduction of a cutoff parameter.[5]

These two approaches are seen to be complementary—one valid for close collisions, and the other for distant collisions. A standard way of combining them is to simply use b_{min} from the two-body approach, and b_{max} from the plasma dielectric treatment. The result is sufficiently accurate for a classical plasma in which the number of particles in a Debye sphere is large. But when the number of particles is small, as in high-density plasmas, a more systematic approach is needed.

For a quantum-mechanical plasma (i.e., one in which the interelectron distance is less than the Bohr radius), there is a more fundamental way to combine these two approaches. Instead of using the Rutherford cross section to calculate the energy loss, one can use the cross section obtained from the Born approximation with a Debye shielded potential. The resulting formula will not contain any divergent terms and will be correct at low velocities to within the accuracy of the Born approximation. However, at high velocities this approach is no longer valid, because the Debye potential around the moving ion can become distorted. Further, this approach cannot describe the energy loss to collective plasma oscillations. Here a more detailed description of the particle-plasma interaction is required, and this can be obtained from the random-phase-approximation (RPA) form of the quantum-mechanical dielectric function.

This dielectric function describes the linear response of electrons in a uniform high-density plasma, for both distant and close interactions with an external charge. Unlike its classical counterpart, the RPA dielectric function can treat short-distance phenomena fairly accurately. In a natural way, close encounters are described in terms of quantum-mechanical wave effects, and distant interactions by plasma shielding. In the past, the RPA dielectric function was used to find the energy loss for charged particles in a totally degenerate plasma,[3,6,7] i.e., at zero temperature. The extension to arbitrary temperature and degeneracy in a high-density plasma will be treated below.

Recently, Brysk[8,9] obtained a formula that interpolates between the limits of strong and weak degeneracy. However, he had to introduce the Coulomb logarithm in a rather *ad hoc* manner. The plan here is to obtain the energy-loss formula, for arbitrary temperature, by starting with the RPA dielectric function. It will not be necessary to introduce the Coulomb logarithm or any cutoff parameters in the calculation. Only the loss to electrons will be considered, as the loss to ions should be adequately described by the classical formula.[4,5] Section II will give a brief description of the dielectric formulation of the problem; Sec. III will analyze the resulting equations; and Sec. IV will contain a discussion of the range of validity for the equations, as well as a comparison with the work of Brysk.

II. FORMALISM

A charged particle passing through a plasma will induce an electric field $\vec{E}_{\rm ind}$ by polarizing the medium. This field will then act back on the particle and cause it to lose kinetic energy W according to the formula

$$\frac{dW}{dx}=\frac{Ze}{v_0}\vec{v}_0\cdot\vec{E}_{\rm ind}[\vec{r}(t),t], \tag{1}$$

where Z, $\vec{v}_0$, and $\vec{r}$ are the charge, velocity, and position of the incident particle. The induced electric field can be related to the dielectric function $\epsilon(\vec{k},\omega)$ of the medium through its Fourier transform,[5] so that Eq. (1) can be rewritten

$$\frac{dW}{dx}=\frac{Z^2e^2}{2\pi^2 v_0}\int d\vec{k}\,\frac{\vec{k}\cdot\vec{v}_0}{k^2}\,\mathrm{Im}\frac{1}{\epsilon(\vec{k},\vec{k}\cdot\vec{v}_0)}\ . \tag{2}$$

Here the approximation was made that the incident particle moves in a straight line. This should be quite accurate for MeV ions.

In addition to the loss described by Eq. (2), the particle can also lose (or gain) energy by scattering from the electric-field fluctuations within the plasma.[5] In general, this is a relatively small effect for ions, but it can become important when the ion energy drops below the electron thermal energy. Since this paper is only concerned with the energy loss from MeV ions in a keV plasma, this term will not be considered here.

The dielectric function to be used is obtained from quantum-mechanical considerations. In the RPA, it is given by[3,10]

$$\epsilon(\vec{k},\omega)=1+\sum_s\frac{4\pi Z_s^2e^2}{\hbar k^2}\int d\vec{v}\,\frac{f_s(\vec{v})-f_s(\vec{v}-\hbar k/m_s)}{\omega-\vec{k}\cdot\vec{v}+\hbar k^2/2m_s+i\delta}\ , \tag{3}$$

where the sum is over all charged species, and f is the single-particle distribution fuction for the unperturbed plasma. The term $i\delta$ is an infinitesimal increment which indicates how to treat the pole. It is seen that this expression does reduce to the classical form, namely,

$$\epsilon(\vec{k},\omega)=1+\sum_s\frac{4\pi Z_s^2e^2}{m_s k^2}\int d\vec{v}\,\frac{\vec{k}\cdot\partial f_s/\partial\vec{v}}{\omega-\vec{k}\cdot\vec{v}+i\delta}\ , \tag{4}$$

in the limit $\hbar\to 0$. The important point about the quantum-mechanical expression, Eq. (3), is that it provides a good description for the linear response of the plasma electrons for *all* wave numbers k[3,7] whenever the average interparticle distance is less than the Bohr radius, i.e.,

$$(\tfrac{4}{3}\pi n)^{-1/3}<\hbar^2/me^2\ . \tag{5}$$

This is not the case for the classical expression in which large-k phenomena are not treated adequately, as is seen from the well-known Coulomb divergence for small impact parameters. Condition (5) means that large-k interactions are dominated by quantum wave effects, which are already contained in the quantum-mechanical dielectric function. For a *DT* plasma, condition (5) is satisfied for densities greater than 20 times solid (10^{24} atoms/cm^3). This contains the burn region of interest for laser-induced fusion.

III. ANALYSIS

The energy loss of an ion to plasma electrons will now be calculated for artibrary electron degeneracy. The loss to plasma ions will not be considered here, as the classical expression[4] is probably sufficiently accurate. It is only for the electron contribution that the Coulomb logarithm becomes small and is questionable. Equations will be obtained for the case where the ion velocity v_0 is less than the average electron velocity $\langle v\rangle$, as this is the main region of interest for a thermonuclear burn.

The dielectric function of Eq. (3) is evaluated using the Fermi-Dirac single-particle distribution function

$$f(\vec{v}) = 2(m/h)^3[\exp(mv^2/2T - \eta) + 1]^{-1},$$

where m is the electron mass, T is the temperature in energy units, and h is Planck's constant. The degeneracy parameter η is chosen to satisfy the normalization condition

$$\int d\vec{v}\, f(\vec{v}) = n_e .$$

After some algebraic manipulation, the electron contribution to the dielectric function becomes

$$\epsilon(\vec{k}, \vec{k}\cdot\vec{v}_0) = 1 + \frac{2e^2m^2T}{\pi\hbar^4k^3}\int_{-\infty}^{\infty}\ln\left(1+\frac{-mv^2}{e^{2T}}+\eta\right)\left(\frac{1}{v_0\mu - v + \hbar k/2m + i\delta} - \frac{1}{v_0\mu - v - \hbar k/2m + i\delta}\right)dv, \quad (6)$$

where $\mu = \vec{k}\cdot\vec{v}_0/kv_0$. This will now be expanded, keeping the parameter $(\hbar k/2m)/\langle v\rangle$ to only first order. Such an approximation was found to be fairly accurate, as the main contribution to the slowing-down formula came when $\hbar k/2m$ was less than the average electron velocity. For small v_0 ($v_0 < \langle v\rangle$), the imaginary part of the dielectric function was found to be

$$\text{Im}\epsilon(\vec{k}, \vec{k}\cdot\vec{v}_0) = \vec{k}\cdot\vec{v}_0\frac{2m^2e^2}{(\hbar k)^3}\left[\exp\left(\frac{\hbar^2k^2}{8mT} - \eta\right) + 1\right]^{-1}. \quad (7)$$

The result for the real part is

$$\text{Re}\epsilon(\vec{k}, \vec{k}\cdot\vec{v}_0) = 1 + (k_D^2/k^2)F'_{1/2}(\eta)/F_{1/2}(\eta), \quad (8)$$

where k_D is the Debye wave number

$$k_D^2 = 4\pi ne^2/T .$$

The function $F_{1/2}$ is the standard Fermi integral

$$F_{1/2}(\eta) = \int_0^\infty \frac{x^{1/2}dx}{e^{x-\eta}+1},$$

and is related to the electron number density by

$$n = 4\pi/h^3(2mT)^{3/2}F_{1/2}(\eta) .$$

Equation (8) is the classical expression for the real part of the dielectric function with the Debye screening length modified by the factor $(F_{1/2}/F'_{1/2})^{1/2}$. This term is the result of electron degeneracy and was previously derived by Salpeter.[12]

The expression for the dielectric function, Eqs. (7) and (8), can now be used with Eq. (2) to obtain a formula for the energy loss to electrons. The result is

$$\frac{dW}{dx} = -\sqrt{W}\, n_e\frac{Z^2e^4}{T^{3/2}}\left(\frac{m_e}{M}\right)^{1/2}\sqrt{\pi} \times\frac{8}{3}\left(\frac{\sqrt{\pi}}{2F_{1/2}(\eta)}\,\frac{1}{e^{-\eta}+1}\right)\ln\Lambda_{\text{RPA}} \quad (9)$$

Here a factor $\ln\Lambda_{\text{RPA}}$ has been separated out to facilitate comparison with other slowing-down formulas. This term is a generalization of the classical Coulomb logarithm, and is defined by an integral over k,

$$\ln\Lambda_{\text{RPA}} = (1+e^{-\eta})\int_0^\infty dk\,\frac{k^3}{(k^2+k_0^2)^2}\left[\exp\left(\frac{\hbar^2k^2}{8mT}\right) - \eta\right]^{-1}, \quad (10)$$

where

$$k_0^2 = k_D^2F'_{1/2}(\eta)/F_{1/2}(\eta) .$$

As will be shown below, $\ln\Lambda_{\text{RPA}}$ reduces to standard expressions in the limits of strong and weak degeneracy. However, unlike the classical Coulomb logarithm, $\ln\Lambda_{\text{RPA}}$ is well defined and does not result from a divergent integral.

IV. DISCUSSION

Figure 1 shows how $\ln\Lambda_{\text{RPA}}$ varies as a function of temperature for different electron densities. As seen, the main effect of electron degeneracy is to make $\ln\Lambda_{\text{RPA}}$ independent of temperature below the Fermi temperature (indicated by a vertical line on each curve). In the limits of strong and weak

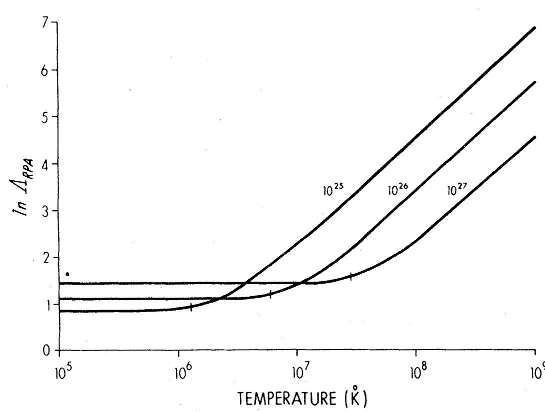

FIG. 1. RPA Coulomb logarithm as a function of temperature for different electron densities. The vertical line on each curve indicates the Fermi temperature.

degeneracy, analytic expressions can be obtained for $\ln\Lambda_{RPA}$.

For a nondegenerate plasma $(\eta < 1)$ $\ln\Lambda_{RPA}$ becomes

$$\ln\Lambda_{RPA} = \tfrac{1}{2}(1+\gamma)e^{\gamma}\int_{\gamma}^{\infty}\frac{e^{x}}{x}\,dx - \frac{1}{2},$$

where γ is related to Λ_S, the standard argument of the Coulomb logarithm, by

$$\gamma = \tfrac{3}{2}/\Lambda_S^2$$

and

$$\Lambda_S^2 = 12mT/\hbar^2 k_D^2 \tag{11}$$

In the limit of weak degeneracy, γ is much less than 1, so that further simplification is possible, namely,

$$\ln\Lambda_{RPA} \approx \tfrac{1}{2}[\ln(1+\Lambda_S^2) - 2]. \tag{12}$$

The logarithmic part is the usual high-temperature expression.[4] The nonlogarithmic term is, in part, the result of electron shielding, and causes about a 20% correction at these densities.

To investigate the limit of strong degeneracy $(\eta \gg 1)$, it is useful to rewrite Eq. (10) in the form

$$\ln\Lambda_{RPA} = (1+e^{-\eta})^{1/2}\int_0^{\infty}\frac{d\phi}{dx}\,\frac{dx}{e^{x-\eta}+1},$$

where

$$\phi = -x/(x+\gamma) + \ln(1+x/\gamma)$$

and

$$x = \hbar^2 k^2/8mT.$$

Now the Sommerfeld lemma[11] can be conveniently applied to obtain the result

$$\ln\Lambda_{RPA} \approx \tfrac{1}{2}[\ln(1+\Lambda_D^2) - 1], \tag{13}$$

which is in agreement with the expression obtained by Ritchie.[7] The variable Λ_D, and is defined by

$$\Lambda_D^2 = (8mT/\hbar^2 k_D^2)F_{1/2}(\eta)/F'_{1/2}(\eta).$$

For high degeneracy, Λ_D can be further simplified to

$$\Lambda_D^2 = \Lambda_S^2(T_F/T)^2,$$

where

$$T_F = \tfrac{2}{3}\eta T$$

is the Fermi temperature. Since Λ_S is proportional to T, it is seen that Λ_D is just the high-temperature expression evaluated at T_F. When Eq. (13) is compared with Ref. 9, it is seen that Brysk *et al.* used only the logarithmic part. This has a value around 2 at these densities, so that the second term will make about a 50% correction.

Equations (12) and (13) can be conveniently combined by means of the interpolation formula

$$\ln\Lambda_{RPA} \approx \tfrac{1}{2}[\ln(1+\Lambda^2) - 1], \tag{14}$$

where

$$\Lambda = \Lambda_S[0.37 + (T_F/T)^2]^{1/2}.$$

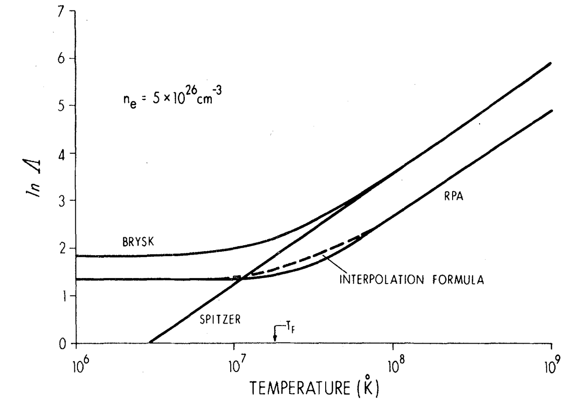

FIG. 2. Comparison of the RPA Coulomb logarithm [Eq. (10)] with the interpolation formula [Eq. (14)] and with the expressions of Spitzer (Refs. 2 and 4) and Brysk *et al.* (Ref. 9).

A comparison between this approximation and the numerically integrated RPA expression, Eq. (10), is shown in Fig. 2. The two are seen to differ in only a small region around the Fermi temperature, and that difference is less than 10%. Also shown are the results using Spitzer's expression[2,4] ($\ln\Lambda_S$) and the formula of Brysk *et al.*[9] As explained above, both of these differ from RPA at high temperature by about 20%, and at low temperatures, Brysk's differs by about 50%.

A number of approximations were made in deriving the energy loss formula (9) from the exact (and more cumbersome) RPA expression. To find the region of validity, calculations were performed to evaluate the range of 3.5-MeV α particles using both the full RPA expression and Eq. (9). For densities above 10^{25} electrons/cm^3, the two differed by less than 5% at all temperatures. Below this density, the approximations in Eq. (9) caused quite large errors for temperatures less than 10^7 °K. Above 10^7 °K, there was good agreement at all densities. Note that the range of validity depends on the velocity of the incident ion, and will be different for each of the charged reaction products. However, the region of validity is sufficiently large to make Eq. (9) adequate for many thermonuclear burn calculations.

For these calculations the range was defined as the distance over which the α particles lost 90% of their energy. The ion contribution was included by assuming equal electron and ion temperatures,

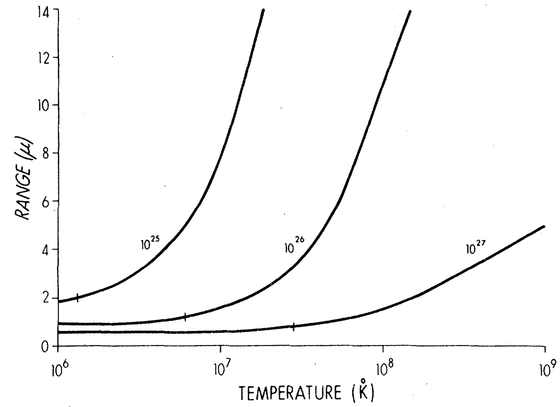

FIG. 3. Range of 3.5-MeV α particles in a hydrogen plasma as a function of temperature for different electron densities. The vertical line on each curve indicates the Fermi temperature.

and by using the classical slowing-down formula[4] for the loss to ions. Figure 3 shows how the range varies as a function of temperature, for three values of the electron density. The low-temperature part of this graph supplements the results presented in Ref. 1 and shows how the range becomes temperature independent below the Fermi temperature.

To summarize, the RPA was used to derive an expression, Eq. (9), for the energy loss of an ion to plasma electrons at high density and arbitrary degeneracy. This approach contained a sufficient amount of physics so that the usual Coulomb logarithmic divergence did not occur, and it was not necessary to introduce any *ad hoc* cutoff parameters. Calculations were made for the range of 3.5-MeV α particles, and to evaluate the generalized expression for $\ln\Lambda$ [Eq. (10) or (14)]. The results showed how both become independent of temperature when the plasma falls below the Fermi temperature. When compared with the results of other authors, there was found to be a 20% correction in the high-temperature limit,[4] and in the limit of strong degeneracy, there was about a 50% correction to the expresion used by Brysk *et al.*[9]

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