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Nonlinear penetration of an inhomogeneous laser beam in an overdense plasma

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This paper presents an analysis of the penetration of an intense electromagnetic beam in an inhomogeneous plasma. It is seen that on account of the electron density gradient created by an intense transversely inhomogeneous beam a region is created in the overdense plasma where the beam can propagate. Typically a CO_2 laser beam of width $r_0 = 100$ μm, power $P = 15$ GW, and $\omega = 1.7\times 10^{14}$ sec^{-1} can penetrate to a region where $\omega_p^2/\omega^2 = 20$ in a 500-eV inhomogeneous plasma characterized by $\omega_p^2/\omega^2 = (\omega_{p0}^2/\omega^2)(1+Bz)$.

I. INTRODUCTION

In the presence of a Gaussian electromagnetic beam, a plasma experiences a ponderomotive force and is consequently redistributed. For beams of high power this mechanism should provide a transparent duct around the axis of the beam through which the beam can propagate in an otherwise overdense ($\omega_p \gg \omega$) plasma. The inhomogeneous duct thus formed causes the beam to converge (i.e., raises the axial intensity). Thus the phenomenon of self-focusing aids in the penetration of an overdense plasma by an intense beam. However, the earlier analyses[1-4] of the phenomenon of self-focusing are not applicable around and beyond critical electron density ($\omega \gtrsim \omega_p$) because of the underlying assumption $\Phi(EE^*) \ll \epsilon_0$; here $\epsilon_0\,(=1-\omega_p^2/\omega^2)$ is the linear part of the dielectric constant and Φ is the change in the dielectric constant due to nonlinearity. The limitation $\Phi \ll \epsilon_0$ is common to all the theoretical investigations of self-focusing in dielectrics also.[5-7]

In the present paper we investigate the penetration of a high-power Gaussian laser beam in an overdense inhomogeneous plasma in the paraxial ray approximation, taking self-focusing into account.

II. NONLINEAR DIELECTRIC CONSTANT

We consider the propagation of a Gaussian beam in a plasma along the z axis, the direction of density gradient. At $z=0$ the intensity distribution of the beam is given by

$$EE^*|_{z=0} = E_0^2 \exp(-r^2/r_0^2). \tag{1}$$

Because of nonuniformity in the intensity distribution along the wave front of the beam, the electrons experience a ponderomotive force and are redistributed in the transverse direction in a time scale $\tau \sim r_0/c_s$, where r_0 and c_s are the radius of the beam and ion sound speed, respectively. The modified electron density may be written[1]

$$n = n_0(z)\exp(-\alpha EE^*), \tag{2}$$

where $n_0(z)$ is the profile of electron density in the absence of the beam, and

$$\alpha = e^2/8m\omega^2 k_B T_0,$$

$-e$, m, and T_0 being the electronic charge, mass, and equilibrium plasma temperature, ω the frequency of the laser, and k_B Boltzmann's constant. Using Eq. (2) the nonlinear dielectric constant of the plasma may be written

$$\epsilon = 1 - \omega_{p1}^2/\omega^2,\qquad \omega_{p1}^2 = \omega_p^2\exp(-\alpha EE^*),\quad \omega_p^2 = 4\pi n_0 e^2/m. \tag{3}$$

III. SOLUTION TO WAVE EQUATION

The wave equation governing the propagation of the beam may be written

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{1}{r}\frac{\partial \vec{E}}{\partial r} + \frac{\partial^2}{\partial r^2}\vec{E} - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) + \frac{\omega^2}{c^2}\epsilon\vec{E} = 0. \tag{4}$$

Using Maxwell's equation $\vec{\nabla}\cdot\epsilon\vec{E}=0$, taking

$$\frac{\partial}{\partial x} \sim \frac{\partial}{\partial y} \sim \frac{\partial}{\partial r} \sim \frac{1}{r_0},$$

and using Eq. (3), the fourth term in Eq. (4) may be neglected in comparison to the third term when

$$\frac{\partial}{\partial r}\left(\frac{E}{\epsilon}\frac{\partial}{\partial r}\epsilon\right) \ll \frac{E}{r_0^2} \quad\text{or}\quad \frac{\omega_{p1}^2}{\omega^2}\alpha EE^* \ll 1 - \frac{\omega_{p1}^2}{\omega^2}. \tag{4a}$$

For large values of αEE^* this implies that $\epsilon - 1 \ll 1$. Under this approximation Eq. (4) reduces to the scalar wave equation,[1]

$$\frac{\partial^2 E}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E\right) + \frac{\omega^2}{c^2}\epsilon E = 0. \tag{5}$$

To solve Eq. (5) we employ the WKB and paraxial-ray approximations. Then we may express

$$\epsilon=\epsilon_a^{(z)}+\gamma r^2,\quad \gamma r^2 \ll \epsilon_a\,, \tag{6}$$

and

$$E=A(r,z)\frac{\epsilon_a^{1/4}(0)}{\epsilon_a^{1/4}(z)}\exp\left[i\left(\omega t-\frac{\omega}{c}\int_0^z \epsilon_a^{1/2}dz\right)\right],$$

where

$$\epsilon_a=\epsilon(z,r=0),\quad \gamma=\frac{\partial\epsilon}{\partial(EE^*)}\left.\frac{\partial(EE^*)}{\partial r^2}\right|_{r=0}. \tag{7}$$

Substituting for E and ϵ in Eq. (5), we obtain

$$-2i\frac{\omega}{c}\epsilon_a^{1/2}\frac{\partial A}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial A}{\partial r}+\frac{\omega^2}{c^2}\gamma r^2 A=0\,. \tag{8}$$

On separating the real and imaginary parts of A as

$$A=A_0(r,z)\exp[-iS(r,z)]$$

and following Akhmanov, Sukhorukov, and Khokhlov[5] and Sodha, Ghatak, and Tripathi,[1] the solution of Eq. (8) satisfying the initial condition expressed by (1) may be written

$$S=\tfrac{1}{2}\beta(z)r^2+\phi_1(z)\,, \tag{9}$$

$$A_0^2=(E_0^2/f^2)\exp(-r^2/r_0^2f^2)\,, \tag{10}$$

$$\beta=\frac{\omega}{c}\epsilon_a^{1/2}\frac{1}{f}\frac{df}{dz},$$

where f is the beam width parameter governed by

$$\epsilon_a\frac{d^2f}{dz^2}+\frac{1}{2}\frac{d\epsilon_a}{dz}\frac{df}{dz}=\frac{c^2}{\omega^2r_0^4f^3}-\frac{\omega_p^2\alpha I}{\omega^2 f}e^{-\alpha I}, \tag{11}$$

where

$$I=\frac{E_0^2\epsilon_a^{1/2}(0)}{\epsilon_a^{1/2}f^2}\,.$$

The initial conditions on f are

$$f(z=0)=1\quad\text{and}\quad df/dz\,|_{z=0}=0$$

corresponding to an initially plane wave front. The first term on the right-hand side (RHS) of Eq. (11) corresponds to diffraction divergence and the second term corresponds to convergence due to nonlinearity.

A. Homogeneous plasma

When two terms on the RHS of Eq. (11) cancel each other at $z=0$, $d^2f/dz^2=0$; and since $df/dz=0$, $f=1$ at $z=0$ and for all values of z, i.e., the beam propagates without convergence or divergence. The condition for self-trapping is therefore

$$\alpha E_{0\text{cr}}^2\exp(-\alpha E_{0\text{cr}}^2)=c^2/r_0^2\omega_p^2\,. \tag{12}$$

Equation (12) is the same as Eq. (4.18) of Ref. 1 (where it is derived in the limit of $\Phi\ll\epsilon_0$). The power of the beam is related to E_0^2 through the expression

$$P=\frac{c}{8\pi}\int_0^\infty 2\pi r\,dr\,E_0^2e^{-r^2/r_0^2}\epsilon^{1/2}$$

$$=\frac{c\,r_0^2}{8\,\alpha}\left[\ln\left(\frac{2e^{\alpha E_0^2}-\omega_{p0}^2/\omega^2}{2-\omega_{p0}^2/\omega^2}\right)-2\frac{\omega_{p0}^2}{\omega}(1-e^{-\alpha E_0^2})\right].$$

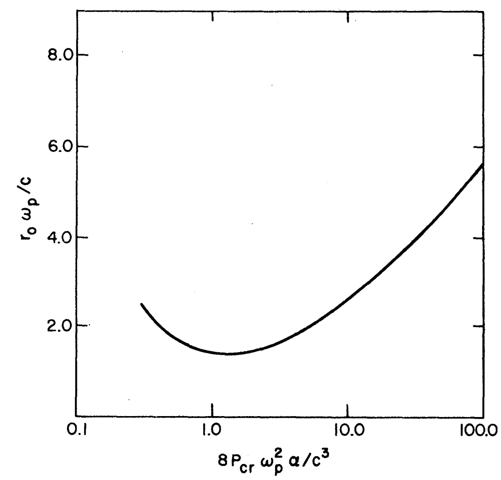

FIG. 1. Variation of critical power for self-focusing with $r_0\omega_p/c$ for $\omega_p^2/\omega^2=0.5$.

The variation of critical power P_{cr} with $r_0\omega/c$ is displayed in Fig. 1.

Equation (12) has two roots $E_{0\text{cr}1}$ and $E_{0\text{cr}2}$ ($E_{0\text{cr}1}<E_{0\text{cr}2}$). $E_{0\text{cr}1}$ increases with decreasing $r_0\omega_p/c$, whereas $E_{0\text{cr}2}$ decreases with decreasing $r_0\omega_p/c$. At $r_0\omega_p/c=e^{1/2}$ the two roots are coincident. For $r_0\omega_p/c<e^{1/2}$, Eq. (14) does not have any real root, and hence self-trapping can not occur. The beam can be self-focused only when $E_{0\text{cr}1}<E_0<E_{0\text{cr}2}$ and the range ($E_{0\text{cr}2}-E_{0\text{cr}1}$) increases rapidly with increasing $r_0\omega_p/c$.

In the range $P_{\text{cr}1}<P<P_{\text{cr}2}$ the second term on the RHS of Eq. (11) is larger than the first term, and hence the beam gets focused. In this case Eq. (11) can be integrated once to obtain

$$\left(\frac{df}{dz}\right)^2=\frac{2}{\epsilon_a}\left[\frac{c^2}{2\omega^2r_0^4}\left(1-\frac{1}{f^2}\right)-\frac{1}{r_0^2}\int_1^f\frac{\omega_p^2}{\omega_1^2}\alpha I e^{-\alpha I}df\right]. \tag{13}$$

We have solved this equation numerically and the variation of beam width parameter with distance of propagation is displayed in Fig. 2 (for $P_{\text{cr}1}<P<P_{\text{cr}2}$); f is seen to be an oscillatory function of z. This may be explained as follows. For a given value of $r_0\omega_p/c$, when $E_0>E_{0\text{cr}1}$ the second term on the RHS of Eq. (11) is greater than the first term at $z=0$ and d^2f/dz^2 is negative. Consequently, f decreases with z. At some value of z, where

$$\frac{E_0}{f}\frac{\epsilon_a^{1/4}(0)}{\epsilon_a^{1/4}}=E_{0\text{cr}2}\;,$$

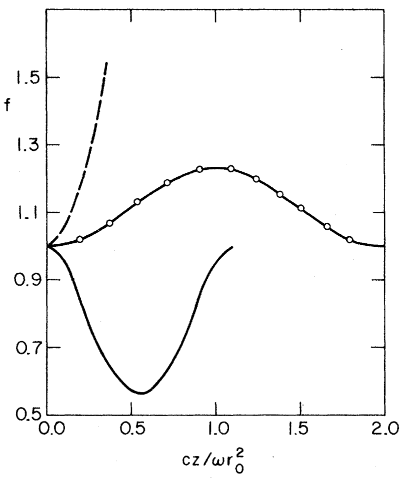

FIG. 2. Variation of beam width parameter with distance of propagation on short time scale for $\omega_p^2/\omega^2=0.9$, $r_0\omega_p/c=30$, and three values of αE_0^2. Solid line, $\alpha E_0^2=6$, $(E_{0cr1}<E_0<E_{0cr2})$; dash-circle line, $\alpha E_0^2=12(E_0>E_{0cr2})$; dashed line, $\alpha E_0^2=0.002(E_0<E_{0cr1})$.

the two terms on the RHS cancel each other; for larger values of z, the RHS becomes positive. However, the beam still continues to converge due to the curvature it has already gained though $\partial f/\partial z$ becomes less and less negative. At $z=z_f$, $df/dz=0$ and $f=f_{min}$. Beyond this point, f increases with z, attains a maximum $f=1$ at $z=2z_f$, and then repeats its behavior. Thus the beam propagates in a self-made oscillatory waveguide. For a given value of E_0 the effect of increasing $r_0\omega_p/c$ is to increase the value of E_{0cr2} and hence to increase the value of $(E_0/f_{min})[\epsilon_a^{1/4}(0)/\epsilon_a^{1/4}]$ (which is always greater than E_{0cr2}); i.e., f_{min} decreases with increasing $r_0\omega_p/c$.

For $P<P_{cr1}$ and $P>P_{cr2}$ the diffraction effects predominate over nonlinear effects and the beam suffers divergence; in the former case f is a monotonically increasing function of z whereas in the latter case it is an oscillatory function of z.

B. Inhomogeneous plasma

Equation (11) is valid for all profiles of unperturbed electron density. However, for the sake of explicitness we have solved it numerically for a linear profile, viz.,

$$\omega_p^2/\omega^2=(\omega_{p0}^2/\omega^2)(1+Bz) .$$

Figure 3 shows the variation of axial wave intensity [viz., $E_0^2\epsilon_a^{1/2}(0)/\epsilon_a^{1/2}f^2$] as a function of distance of penetration into the plasma. As a competition of self-focusing and diffraction effects the intensity varies in an oscillatory manner. As the beam penetrates in the plasma, the axial dielectric constant decreases, and one expects a turning point

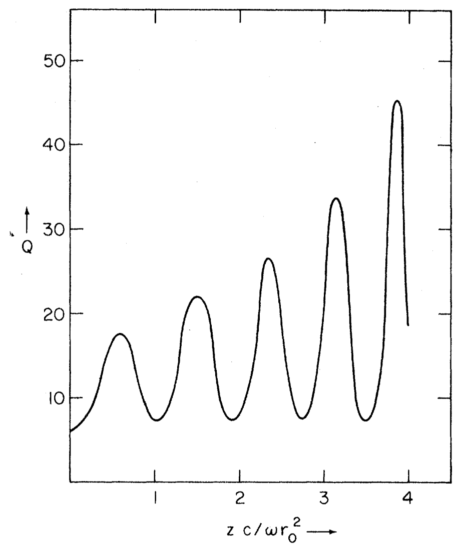

FIG. 3. Variation of beam width parameter f and axial intensity of the beam $[Q=\alpha E_0^2\epsilon_a^{1/2}(0)/\epsilon_a^{1/2}f^2]$ as a function of distance of propagation for $r_0\omega/c=30$, $\omega_{p0}^2/\omega^2=0.5$, $\alpha E_0^2=4.0$, and $B(\omega/c)r_0^2=5.0$.

where $\epsilon_a=0$. However, the present treatment is not applicable around this point. Hence, in order to have an idea of the depth of penetration as a function of the power of the beam, we have introduced a typical distance $z=z_t$ where $\epsilon_a=0.7$. The variation of z_t with the power of the beam is displayed in Fig. 4. It is obvious from the figure that z_t varies rapidly with power; i.e., a beam of higher power penetrates much deeper in the plasma.

Thus it is seen that intense beams can penetrate overdense plasmas. In a typical case, a 15-GW

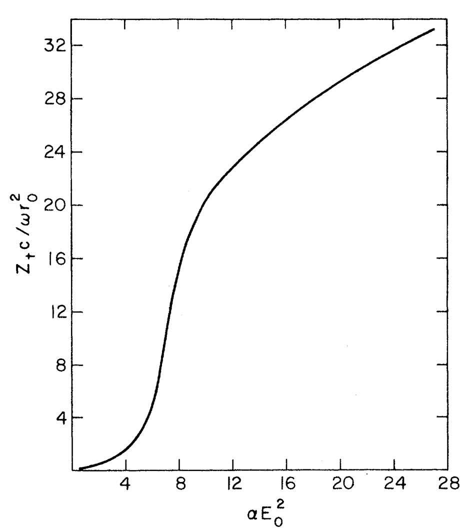

FIG. 4. Variation of depth of penetration z_t with the power of the beam for $r_0\omega/c=30$, $\omega_{p0}^2/\omega^2=0.5$, $B(\omega/c)r_0^2=5.0$.

CO_2 laser of width 100 μm can penetrate a plasma up to a region where $\omega_p^2/\omega^2 \simeq 20$ in the absence of the beam. It will be interesting to include this aspect of propagation in the computer codes for simulation of laser fusion.

In the present analysis we have neglected the filamentation instability of the beam. For a perturbation of size a the threshold for filamentation is given by [cf. Eq. (6.14), Ref. 1]

$$\frac{\epsilon_2 EE^*}{\epsilon_0} = \frac{\pi^2}{4k^2a^2}, \quad k = \frac{\omega}{c}\epsilon^{1/2}, \quad \epsilon_2 = \frac{\omega_{p1}^2}{\omega^2}\alpha .$$

The relevant scale length for filamentation in the present case would be $a \lesssim r_0/2$; hence the threshold is given by

$$\frac{\epsilon_2 EE^*}{\epsilon_0} = \frac{\pi^2}{k^2 r_0^2};$$

this has two roots, E_1 and E_2, and filamentation occurs when $E_1 < E < E_2$. On the other hand, the threshold for self-focusing corresponds to

$$\frac{\epsilon_2 EE^*}{\epsilon_0} = \frac{1}{k^2 r_0^2}$$

having roots E_{cr1} and E_{cr2} where $E_{cr1} < E_1$ and $E_{cr2} > E_2$. Thus there exists a range of values of beam power in which self-focusing is important and the filamentation does not take place. Moreover, in the present case as we are more concerned with the depletion of the plasma from the axial region and not so much with the self-focusing, we employ powers as high as possible which essentially correspond to fields greater than E_2 at which filamentation does not occur.

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[1]M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. 13, 169 (1976).

[2]V. V. Demchenko and A. M. Hussein, Physica 65, 396 (1973).

[3]K. B. Dysthe, Phys. Lett. 27A, 59 (1968).

[4]A. G. Litvak, Zh. Eksp. Teor. Fiz. 57, 629 (1969) [Sov. Phys. JETP 30, 344 (1970)].

[5]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys. Usp. 10, 609 (1968)].

[6]B. R. Suydam, IEEE J. Quantum Electron. QE-10, 837 (1974).

[7]O. Svelto, Prog. Opt. 12, 1 (1974), see also references cited therein.