PHYSICAL REVIEW A VOLUME 16, NUMBER 4 OCTOBER 1977

Scattering of positrons by hydrogen in a modified Glauber method*

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(Received 12 April 1977)

The differential and integrated cross sections of elastic scattering of positrons by hydrogen atoms in the ground state are calculated with the modified Glauber amplitude recently proposed for atomic scatterings. While experimental data of this positron process are still not available for comparison, our results show a drastic difference from those obtained in the full-eikonal calculations by Foster and Williamson, as well as in other theoretical models. Thus the results of the analysis shown here can serve in the role of making the distinction between our Glauber model and others.

In some recent papers,[1,2] a new scattering amplitude (called the modified Glauber amplitude) has been proposed for the study of atomic scatterings at intermediate energies. This amplitude has been applied to analyze the elastic scattering of electrons by a hydrogen atom and the results were found to be in good agreement with experimental data acquired by absolute measurement. The choice of this amplitude for atomic scatterings was based on reasoning as follows.

Since the application of the Glauber amplitude[3] was initiated by Franco in his pioneering work a few years ago,[4] several calculations have been performed for various atomic and molecular processes with some degree of success[5] using the Glauber and Glauber-related methods. It is my belief that the Glauber amplitude may be a right candidate for these collision processes at intermediate energies, and its limited success in reproducing differential cross sections that agree well with data[5] may simply arise from the existence of some serious defect of some particular term contained in this amplitude. Obviously, the simplest way to improve the Glauber method is, therefore, to single out this defect and then adequately correct it. Since the term with a serious defect has been identified as the second-order term of its eikonal expansion (with its real part of a considerable magnitude disappearing from the amplitude and its imaginary part becoming singular in the forward direction), the Glauber amplitude should, therefore, be corrected as follows:

$$f_{\mathrm{GM}} = f_{\mathrm{G}} - f_{\mathrm{G}2} + f_{\mathrm{B}2}\,, \tag{1}$$

where f_{G} is the conventional Glauber amplitude, $f_{\mathrm{G}2}$ the second-order eikonal term, and $f_{\mathrm{B}2}$ its counterpart in the Born series. Note that with this choice of amplitude for atomic scatterings, one can furthermore avoid an unnecessary cutoff of eikonal terms such as $f_{\mathrm{G}4}$, $f_{\mathrm{G}5}$, ...from the scattering amplitude. These terms are usually found to be of considerable magnitude, and their neglect cannot, thereby, be very well justified.

Here, the modified Glauber amplitude will be applied to calculate cross sections of elastic positron scattering by a hydrogen atom in its ground state. Although experimental data for this positron process have not yet been available for comparison with our results, as will be seen subsequently, our calculations provide a set of theoretical values significantly distinct from those obtained in other models, especially in the full eikonal amplitude method.[6] It is, therefore, worthwhile to present the results of this analysis, since they certainly serve well in the role of distinguishing between our theoretical model and others. Furthermore, because of the absence of exchange effect in these positron processes, such a comparison among models would be somewhat more meaningful than a similar one made with electron processes.

In positron-hydrogen scattering, the interaction potential governing the process has an opposite sign to the one in the electron case. As a result, even-order terms of the Born series (in which the number of times of appearance of the potential in their expression is even) remain intact under the change from an electron beam to a positron one. The Glauber phase shift simply changes sign, and therefore all the even-order terms of the eikonal expansion including $f_{\mathrm{G}2}$ (which are all purely imaginary) will not be modified at all, while all the odd-order terms of this expansion (which are all real) will only change sign. Thus, compared to the electron case, the real part of the Glauber amplitude in the positron scattering changes sign, while its imaginary part remains the same. The three terms of the modified Glauber amplitude are calculated as follows. For the Glauber term, one can either use its closed form[7] or the original expression by Franco,[4]

$$f_{\mathrm{G}} = 2ik_\mu \int_0^{\pi/2} d\alpha \sin^3\alpha \cos\alpha (\sin^2\alpha - \tfrac{1}{2}q^2 \cos^2\alpha)(\sin^2\alpha + \tfrac{1}{4}q^2\cos^2\alpha)^{-4}$$

$$\times [1 - (|\cos 2\alpha|/\cos\alpha)^{-2i/k_\mu} |\cos 2\alpha| F(\tfrac{1}{2} - i/2k_\mu, 1 - i/2k_\mu, 1; \sin^2 2\alpha)] . \tag{2}$$

The second-order eikonal term is given by

$$f_{\mathrm{G}2} = \frac{k_\mu i}{2!} \int db_2\, b_2 J_0(qb_2) \langle \phi_{1s} | \chi_{\mathrm{G}}^2 | \phi_{1s} \rangle , \tag{3}$$

where χ_{G} is the Glauber phase and ϕ_{1s} is the ground state of the hydrogen atom. The second-Born term is calculated approximately[1] by its average closure summation, as was usually considered for this term in the second-Born scattering theory,

$$\bar{f}_{\mathrm{B}2} = \frac{2}{\pi^2} \int d\vec{\mathrm{k}} \frac{1}{K_\mu^2 K_\nu^2} \frac{1}{k^2 - p_\mu^2 - i\epsilon}$$

$$\times \langle \phi_{1s} | e^{i\vec{\mathrm{q}}\cdot\vec{\mathrm{r}}} - e^{i\vec{\mathrm{K}}_\mu\cdot\vec{\mathrm{r}}} - e^{-i\vec{\mathrm{K}}_\nu\cdot\vec{\mathrm{r}}} + 1 | \phi_{1s} \rangle \tag{4}$$

where $\vec{\mathrm{q}}$ is the momentum transfer. Other notations used in these expressions have their usual meanings.[1] As was pointed out earlier, the change from an electron beam to a positron one only reverses the sign of the real part of f_{G}, while other terms of the modified Glauber amplitude remain the same. Consequently, the differential cross sections calculated with the modified Glauber amplitude for positron- and electron-atom scatterings differ from each other by a quantity equal to $4\mathrm{Re} f_{\mathrm{G}}\, \mathrm{Re} f_{\mathrm{B}2}$. Since the magnitudes of $\mathrm{Re} f_{\mathrm{G}}$ and $\mathrm{Re} f_{\mathrm{B}2}$ are both quite large at this range of intermediate energies, the differential cross sections of positron scattering are, therefore, expected to be much smaller than those of the electron scattering. This contrasts the first-Born and conventional Glauber approximations, which both predict an equality of these cross sections. The difference between the electron and positron cross sections in our model are so large that one may also expect to have the positron-atom cross sections lying lower than values obtained in most other theoretical models.

TABLE I. Differential cross sections of 1*s*-1*s* positron-hydrogen scattering at 50, 100, and 200 eV in the modified Glauber theory.

Angle (deg)	50 eV	100 eV	200 eV
0.5	2.765	2.038	1.556
1	2.676	1.995	1.551
3	2.332	1.763	1.384
5	2.011	1.505	1.150
10	1.336	9.564×10^{-1}	7.037×10^{-1}
15	8.593×10^{-1}	6.055×10^{-1}	4.285×10^{-1}
20	5.471×10^{-1}	3.884×10^{-1}	2.582×10^{-1}
25	3.490×10^{-1}	2.519×10^{-1}	1.563×10^{-1}
30	2.242×10^{-1}	1.656×10^{-1}	9.667×10^{-2}
40	9.571×10^{-2}	7.625×10^{-2}	4.088×10^{-2}
50	4.463×10^{-2}	3.943×10^{-2}	1.992×10^{-2}
60	2.456×10^{-2}	2.303×10^{-2}	1.098×10^{-2}
70	1.668×10^{-2}	1.492×10^{-2}	6.675×10^{-3}
80	1.342×10^{-2}	1.047×10^{-2}	4.394×10^{-3}
90	1.186×10^{-2}	7.811×10^{-3}	3.088×10^{-3}
100	1.092×10^{-2}	6.116×10^{-3}	2.292×10^{-3}
110	1.022×10^{-2}	4.982×10^{-3}	1.784×10^{-3}
120	9.644×10^{-3}	4.201×10^{-3}	1.448×10^{-3}
130	9.161×10^{-3}	3.653×10^{-3}	1.220×10^{-3}
140	8.765×10^{-3}	3.266×10^{-3}	1.065×10^{-3}
150	8.457×10^{-3}	2.999×10^{-3}	9.600×10^{-4}
160	8.237×10^{-3}	2.824×10^{-3}	8.924×10^{-4}
170	8.105×10^{-3}	2.724×10^{-3}	8.545×10^{-4}
180	8.061×10^{-3}	2.692×10^{-3}	8.423×10^{-4}

We have numerically integrated the differential cross sections of positron-hydrogen elastic scattering in its ground state at 50, 100, and 200 eV

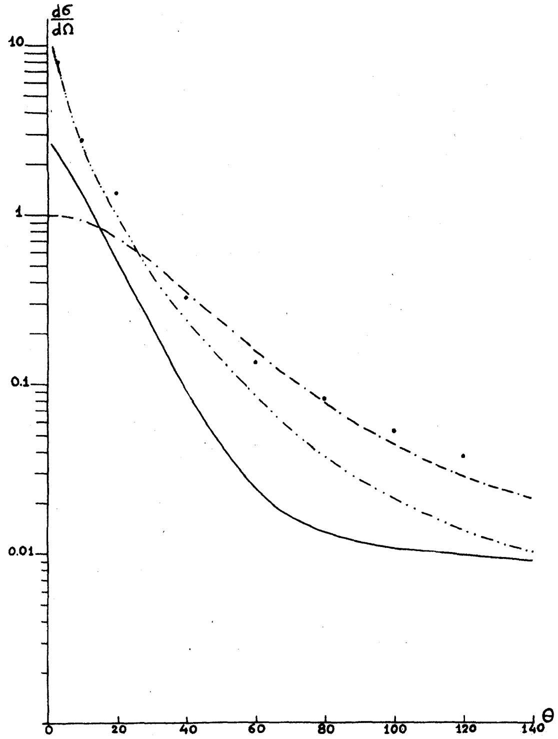

FIG. 1. Differential cross sections of 1*s* positron-hydrogen elastic scattering at 50 eV in $a_0^2\,\mathrm{sr}^{-1}$ units. ——, modified Glauber; — — • — — •, first Born; — • • — • • —, conventional Glauber; dot points are values of the full eikonal method.

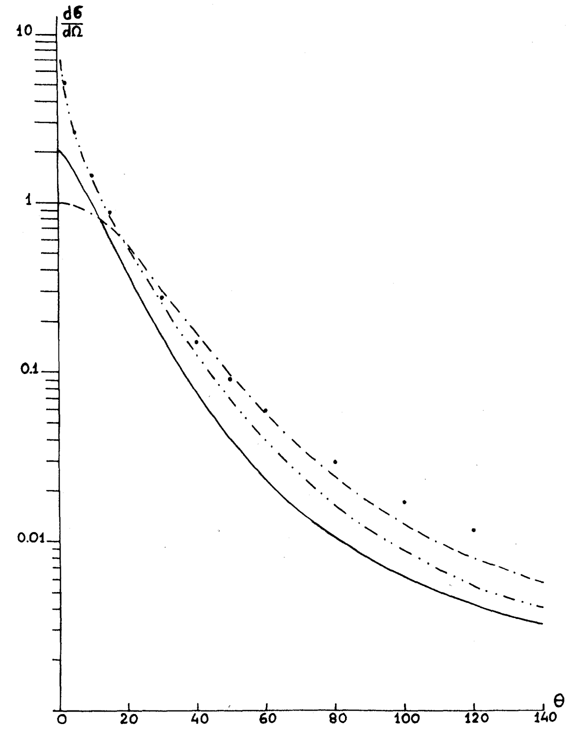

FIG. 2. Same as in Fig. 1, at 100 eV.

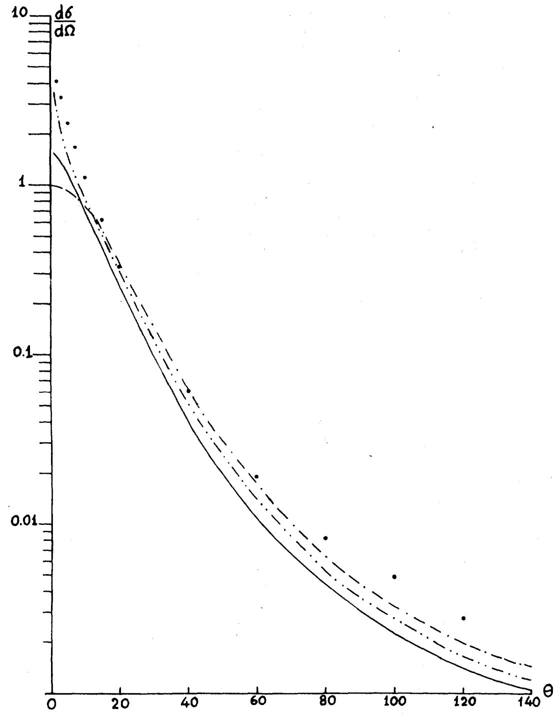

FIG. 3. Same as in Fig. 1, at 200 eV.

and at different angles from 0.5 to 180° using the modified Glauber amplitude proposed above. The results are shown in Table I. In Figs. 1, 2, and 3, these results are plotted versus scattering angles along with those of other theoretical models (all recalculated by us, except the values of the full eikonal amplitude method which we quoted from the work by Foster and Williamson[6]). We find that the differential cross sections calculated with our modified Glauber method, exactly as predicted earlier, are much smaller than those given by other theoretical models considered here, except at very small angles, where they can only exceed the values of the first-Born approximation. At small angles, our results differ more appreciably from the values of both conventional Glauber and full eikonal amplitudes. At larger scattering angles, the difference between the results of our model and the conventional Glauber amplitude becomes, however, narrower, while the gap between our values and those of the full eikonal amplitude becomes wider. In general, these curves obtained with different theoretical models are clearly distinct from each other and can easily be discriminated by experimental data. Unfortunately, to the best of our knowledge, at present no experimental data for this process are available for comparison. As the energy of the incident positrons increases, the difference between these curves narrows down but is still easily noticeable. Using these differential cross section values, we have integrated over scattering angles to obtain the integrated cross sections for 1s elastic positron-hydrogen scattering in various models. They are shown in Table II. The integrated cross sections calculated with our modified Glauber amplitude are naturally found to be smaller than those obtained in most other models. Especially, they differ sharply from the value given by the full eikonal amplitude,[6] a model which predicts an integrated cross section even greater than the one calculated with the conventional Glauber amplitude. Finally, the optical theorem has been used to calculate the total cross sections predicted by our method and others. They are shown in Table III for comparison.

TABLE II. Integrated cross sections of elastic positron-hydrogen scattering in its ground state.

Energy (eV)	Cross sections in a_0^2 units Born[a]	Glauber[a]	Full Eikonal[a]	Modified Glauber
50	1.60	2.01	2.98	0.85
100	0.91	0.91	1.38	0.62
200	0.47	0.47	0.75	0.39

[a] Values quoted from Foster and Williamson (Ref. 6).

In summary, we have analyzed the elastic scattering of positrons by hydrogen atoms in their ground state with the modified Glauber amplitude. Contrary to the first-Born and conventional Glauber approximations, this amplitude yields results of electron and positron scatterings drastically different from each other at all angles. In general, our values are found to be smaller than those calculated in other theoretical models considered here. In particular, while the full eikonal method[6] predicts cross sections of the positron greater than those of the electron, our modified Glauber amplitude predicts the opposite. Experimental data for this process, if available, can easily discriminate among these models. We have also calculated the integrated cross sections for elastic positron-hydrogen scattering and the results, as expected, are found to be smaller than values of many other models, which in turn are also quite different among themselves. Thus the analysis of the positron-hydrogen process shown here may at least serve in the role of distinguishing between our Glauber model and others.

TABLE III. Total cross sections of positron-hydrogen scattering deduced from optical theorem.

Energy (eV)	Total cross sections in a.u.			
	Born	Glauber	Full eikonal	Modified Glauber
50	6.555	∞	∞	9.555
100	4.635	∞	∞	6.562
200	3.278	∞	∞	4.041

I wish to thank the National Research Council of Canada for its financial support of this work.

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*Work supported in part by the National Research Council of Canada (Operating Grant A-3962).

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[5] See a rather clear and complete review article by E. Gerjuoy and B. K. Thomas, Rep. Prog. Phys. 37, 1345 (1974) and references therein.

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