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Magnetic field generation by resonance absorption of light

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We consider the self-generated dc magnetic fields due to resonant excitation of plasma waves by intense laser light. From Ampere's law we show that the time scale associated with the growth of the magnetic field is essentially the same as the time scale for the growth of the plasma wave. Further, we obtain a simple expression for the steady-state magnetic fields arising from a steady-state current derived in terms of the high-frequency excited fields. Computer simulations verify the theoretical conclusions on both the time variation and the steady-state value of the magnetic field. Difficulties of earlier calculations based on the so-called stress-tensor approach are discussed, and it is shown that the proper use of this method requires the use of a self-consistent kinetic theory for the plasma which takes into account the convection of momentum out of the resonance region.

I. INTRODUCTION

Recent calculations and experiments suggest that megagauss dc magnetic fields are generated by the absorption of intense laser light in laser-fusion plasmas.[1,2] The original work on this subject concentrated on thermoelectric sources for the magnetic field. Subsequently, it was suggested by Stamper and Tidman that radiation pressure gradients directly can provide a source of dc magnetic field.[3] A particular example of this mechanism arises when plane-polarized light is obliquely incident on a plasma with a density gradient. As is well known, resonance absorption or linear conversion of the light wave into plasma waves occurs.

Recently, the dc magnetic field generated as a concomitant of the resonance absorption process has been studied by simulation[4,5] and theoretically.[5,6] In that analysis the importance of dissipation on the plasma dynamics was stressed. The phenomenological manner in which dissipation was included, however, led to some perplexing results. For example, in Ref. 5 the time rate of change of the magnetic field B is calculated, but there is no mechanism included in the model used to saturate the growth of B even though the simulations clearly show a finite steady-state value for B in a system of fixed ions.

In this work we first begin by a brief review of the methodology of the calculations of $\dot{B}$ based on the stress tensor. The time rate of change of the magnetic field is said to result from the solenoidal electric field responding to a spatially varying dc force on the electrons. This force has two contributions: the time-averaged electromagnetic stress and the time-averaged particle stress. We show that the saturation of B can be treated with a corrected expression for the time-averaged particle stress tensor. This formulation achieves momentum balance by accounting for momentum convected out of the resonance region by electrostatic waves. We point out from the point of view of Ampere's law that the calculation of the $\dot{B}$ due to resonance absorption is particularly simple once the time scale for the growth of the high-frequency waves in the plasma is known. Finally, we show how the steady-state dc current can be calculated giving B directly.

II. STRESS-TENSOR FORMULATION

The usual procedure followed for the calculation of $\vec{B}_L$ starts by taking the time average of Faraday's law over a period $T = 2\pi/\omega_0$ of the high-frequency wave,

$$\dot{\vec{B}}_L = -c\vec{\nabla}\times\vec{E}_L \,. \tag{1}$$

We use the notation

$$\langle Q\rangle = \frac{1}{T}\int_{t-T/2}^{t+T/2} dt'\,Q(t') = Q_L, \quad \text{and } Q_H = Q - Q_L \,,$$

where Q is any plasma quantity. Thus, for example, $\vec{E}_L = \langle\vec{E}\rangle$ is a slowly varying electric field on the time scale of ω_0^{-1}, acting as a source for $\vec{B}_L$. To obtain $\vec{E}_L$, one considers the force density on electrons,

$$\vec{f}^e(\vec{r},t) = \vec{\nabla}\cdot\vec{T}^e(\vec{r},t) + \rho^e\vec{E}(\vec{r},t) + \vec{j}^e\times\vec{B}/c \,, \tag{2}$$

where ρ^e and $\vec{j}^e$ are the electron charge density and current, respectively, and $\vec{T}^e$ is the electron stress tensor

$$\vec{T}^e(r,t) = -m\sum_{\nu=1}^{N_e}\vec{v}^\nu(t)\vec{v}^\nu(t)\delta(\vec{r}-\vec{r}^\nu(t)) \,, \tag{3}$$

where $\vec{r}^\nu(t)$ and $\vec{v}^\nu(t)$ are the microscopic electron variables of position and velocity for the νth electron. In addition one invokes the identity

$$\vec{\nabla}\cdot T^{em} - \frac{1}{c}\frac{\partial}{\partial t}\frac{\vec{E}_H\times\vec{B}_H}{4\pi} = \rho_H\vec{E}_H + \frac{\vec{J}_H\times\vec{B}_H}{c} \,, \tag{4}$$

for the Lorentz force density in terms of the electromagnetic stress tensor,

$$\mathrm{T}^{\mathrm{em}} = \frac{1}{4\pi}\,[\vec{\mathrm{E}}_H\vec{\mathrm{E}}_H + \vec{\mathrm{B}}_H\vec{\mathrm{B}}_H - \vec{\mathrm{I}}\,(E_H^2 + B_H^2)/2]\,, \tag{5}$$

where $\rho = \rho^e + \rho^{\mathrm{ion}}$, $\vec{\mathrm{j}} = \vec{\mathrm{j}}^e + \vec{\mathrm{j}}^{\,\mathrm{ion}}$, to obtain

$$\vec{\mathrm{f}}_L^e = \vec{\nabla}\cdot(\langle\vec{\mathrm{T}}^e\rangle + \langle\mathrm{T}^{\mathrm{em}}\rangle) + \rho_L^e\vec{\mathrm{E}}_L + \frac{\vec{\mathrm{j}}_L^e\times\vec{\mathrm{B}}_L}{c} - \frac{1}{c}\frac{\partial}{\partial t}\frac{\langle\vec{\mathrm{E}}_H\times\vec{\mathrm{B}}_H\rangle}{4\pi}\,. \tag{6}$$

In (4) the identity in terms of the high-frequency fields is valid just as in the case of the total fields since the Maxwell equations are linear as is the operation of time-averaging.

To calculate $\langle\vec{\mathrm{T}}^e\rangle$ in (6) with $\vec{\mathrm{T}}^e = -m\int d^3v\,\vec{\mathrm{v}}\vec{\mathrm{v}}f^e(\vec{\mathrm{r}},\vec{\mathrm{v}},t)$, where f^e is the electron particle distribution, we introduce the transformation

$$f^e(\vec{\mathrm{r}},\vec{\mathrm{v}},t) = F(\vec{\mathrm{r}},\vec{\mathrm{v}} - \vec{\mathrm{u}}_H, t)\,, \tag{7}$$

where $\vec{\mathrm{u}}_H$ is the electron quiver velocity, $\dot{\vec{\mathrm{u}}}_H = -(e/m)\vec{\mathrm{E}}_H$. Then

$$\langle\vec{\mathrm{T}}^e\rangle = -mn_L^e\langle\vec{\mathrm{u}}_H\vec{\mathrm{u}}_H\rangle - m\int d^3v\,\vec{\mathrm{v}}\vec{\mathrm{v}}F_L - m\int d^3v\,(\vec{\mathrm{v}}\langle\vec{\mathrm{u}}_H F_H\rangle + \langle\vec{\mathrm{u}}_H F_H\rangle\vec{\mathrm{v}})\,, \tag{8}$$

where

$$n_L^e = \int d^3v\,F_L\,,$$

the dc component of the electron density. Some comments are in order concerning the transformation in (7). It is useful since the approximation of an oscillating Maxwellian for f^e corresponds to F_L being a Maxwellian and $F_H = 0$. Thus the transformation allows us to isolate the distortion of the electron distribution away from an oscillating Maxwellian due to wave-particle interaction. For example, in Maxwell's equations the current source for the high-frequency fields is given by

$$\vec{\mathrm{j}}_H^e = -en_L^e\vec{\mathrm{u}}_H + \Delta\vec{\mathrm{j}}_H\,, \tag{9}$$

where

$$\Delta\vec{\mathrm{j}}_H = -e\int d^3v\,\vec{\mathrm{v}}F_H\,. \tag{10}$$

The first term in (9) is just the isothermal fluid or oscillating Maxwellian contribution to the current, whereas $\Delta\vec{\mathrm{j}}_H$ denotes the deviation.

In (8) we use $F_L = F_L^0 + \delta F_L$, where F_L^0 is independent of the high-frequency fields. Thus δF_L as well as $\Delta\vec{\mathrm{j}}_H$ arise from non-Maxwellian effects. Then

$$\langle\vec{\mathrm{T}}^e\rangle = -mn_L^e\langle\vec{\mathrm{u}}_H\vec{\mathrm{u}}_H\rangle + \vec{\mathrm{P}}^e + \langle\delta\vec{\mathrm{T}}^e\rangle\,, \tag{11}$$

where the first term is just due to the quiver motion, the second term, $\vec{\mathrm{P}}^e = -m\int d^3v\,\vec{\mathrm{v}}\vec{\mathrm{v}}F_L^0$, is the background or field-independent electron pressure, and

$$\langle\delta\vec{\mathrm{T}}^e\rangle = -m\int d^3v\,\vec{\mathrm{v}}\vec{\mathrm{v}}\,\delta F_L + (m/e)(\langle\Delta\vec{\mathrm{j}}_H\vec{\mathrm{u}}_H\rangle + \langle\vec{\mathrm{u}}_H\Delta\vec{\mathrm{j}}_H\rangle) \tag{12}$$

represents the particle stress arising from finite-temperature wave-particle effects. Substituting (11) into (6), we obtain

$$\vec{\mathrm{f}}_L^e = \rho_L^e\vec{\mathrm{E}}_L + \frac{\vec{\mathrm{j}}_L^e\times\vec{\mathrm{B}}_L}{c} + \vec{\nabla}\cdot\{\vec{\mathrm{P}}^e + \langle\delta\vec{\mathrm{T}}^e\rangle + (1/4\pi)\times[\epsilon\langle\vec{\mathrm{E}}_H\vec{\mathrm{E}}_H\rangle + \langle\vec{\mathrm{B}}_H\vec{\mathrm{B}}_H\rangle - \tfrac{1}{2}\vec{\mathrm{I}}(\langle E_H^2\rangle + \langle B_H^2\rangle)]\} - \frac{1}{c}\frac{\partial}{\partial t}\frac{\langle\vec{\mathrm{E}}_H\times\vec{\mathrm{B}}_H\rangle}{4\pi}\,, \tag{13}$$

where $\epsilon = 1 - \omega_p^2/\omega_0^2$, and the high-frequency ion density and current response are neglected.

III. PREVIOUS RESULTS–LACK OF MOMENTUM BALANCE

Up to this point the discussion is completely general. Now if we may *assume* that the effect of $\langle\delta\vec{\mathrm{T}}^e\rangle$ may be accounted for by replacing ϵ by $\epsilon_R = 1 - \omega_p^2/(\omega_0^2 + \nu^2)$, where ν is a phenomenological effective-collision frequency, then we obtain the starting point of Ref. 5,

$$\vec{\mathrm{f}}_L^e = \rho_L^e\vec{\mathrm{E}}_L + \frac{\vec{\mathrm{j}}_L^e\times\vec{\mathrm{B}}_L}{c} + \vec{\nabla}\cdot\langle\vec{\mathrm{P}}^e + \vec{\mathrm{P}}^r\rangle - \frac{1}{c}\frac{\partial}{\partial t}\frac{\langle\vec{\mathrm{E}}_H\times\vec{\mathrm{B}}_H\rangle}{4\pi}\,, \tag{14}$$

where

$$\vec{\mathrm{P}}^r = (1/4\pi)[\epsilon_R\langle\vec{\mathrm{E}}_H\vec{\mathrm{E}}_H\rangle + \langle\vec{\mathrm{B}}_H\vec{\mathrm{B}}_H\rangle - \tfrac{1}{2}\vec{\mathrm{I}}(\langle E_H^2\rangle + \langle B_H^2\rangle)]\,.$$

This form for $\vec{\mathrm{f}}^e$ arises from the approximate form for $\langle\vec{\mathrm{T}}^e\rangle$ used in Ref. 5:

$$\langle\vec{\mathrm{T}}^e\rangle = \vec{\mathrm{P}}^e - \frac{mn_L^e}{1 + \nu^2/\omega_0^2}\langle\vec{\mathrm{u}}_H\vec{\mathrm{u}}_H\rangle\,, \tag{15}$$

as compared with (11).

In addition, the use of an effective collision frequency ν in the kinetic equation for f^e, as in Eq. (4) of Ref. 5, gives for the high-frequency current:

$$\vec{\mathrm{j}}_H = -\frac{en_L^e\vec{\mathrm{u}}_H}{1 + i\nu/\omega_0}\,, \tag{16}$$

as compared with (9).

At this point it is convenient to derive an equivalent expression for (14) which makes its dependence on ν more transparent and allows one to assess the validity of (15) and (16). Evaluating $\vec{\nabla}\cdot\vec{\mathrm{P}}^r$, we have

$$\vec{\nabla}\cdot\vec{P}^r=(1/4\pi)[\langle\vec{\nabla}\cdot(\epsilon_R\vec{E}_H)\vec{E}_H\rangle+(\epsilon_R-1)\langle\vec{E}_H\cdot(\vec{\nabla}\vec{E}_H)\rangle - \langle\vec{E}_H\times(\vec{\nabla}\times\vec{E}_H)\rangle-\langle\vec{B}_H\times(\vec{\nabla}\times\vec{B}_H)\rangle]. \quad (17)$$

From Maxwell's equations, the last two terms on the right-hand side of (17) can be written $(1/c)(\partial/\partial t)\times\langle\vec{E}_H\times\vec{B}_H\rangle$. Substituting the resulting expression for (17) into (14) we find

$$\vec{f}^e_L=\rho^e_L\vec{E}_L+\frac{\vec{j}^e_L\times\vec{B}_L}{c}+\frac{1}{4\pi}\left(\langle\vec{\nabla}\cdot(\epsilon_R\vec{E}_H)\vec{E}_H\rangle+(\epsilon_R-1)\langle\vec{E}_H\cdot\vec{\nabla}\vec{E}_H\rangle+4\pi\frac{\langle\vec{j}_H\times\vec{B}_H\rangle}{c}\right). \quad (18)$$

If we use (16) for $\vec{j}_H$, the last term in (18) can be written

$$4\pi\langle\vec{j}_H\times\vec{B}_H\rangle/c=(\epsilon_R-1)(\vec{\nabla}\langle E^2_H\rangle/2-\langle\vec{E}_H\cdot\vec{\nabla}\vec{E}_H\rangle-\nu\langle\vec{E}_H\times\vec{B}_H\rangle/c).$$

Finally if we use the continuity equation and (16), we have

$$\vec{\nabla}\cdot(\epsilon_R\vec{E}_H)=-\vec{\nabla}\cdot\left(\frac{\omega_p^2}{\omega_0^2+\nu^2}\,i\frac{\nu}{\omega_0}\,\vec{E}_H\right).$$

Substituting these expressions into (18), we have

$$\vec{f}^e_L=\rho^e_L\vec{E}_L+\frac{\vec{j}^e_L\times\vec{B}_L}{c}+\nabla\cdot\vec{P}^e+(1-\epsilon_R)\left\{-\vec{\nabla}\frac{\langle E^2_H\rangle/2}{4\pi}+\nu\frac{\langle\vec{E}_H\times\vec{B}_H\rangle}{4\pi c}+\frac{\nu}{\omega_0}\,\mathrm{Im}\frac{(\vec{\nabla}\cdot\vec{E}_H\vec{E}^*_H)}{2\pi}\right\}, \quad (19)$$

where we write

$$\vec{E}_H(\vec{r},t)=\vec{E}_H(x)\exp[i(\omega t-k_y y)]+\text{c.c.}\,.$$

In (19) the first term in the braces is just the ponderomotive force and gives no contribution to the solenoidal electric field. The second term has a simple interpretation at least for a pure electromagnetic wave when we note that $\langle\vec{E}_H\times\vec{B}_H\rangle/4\pi$ is just the field momentum density for the light wave, and thus the second term corresponds to the rate of momentum transfer due to absorption of the light wave. In the event that the wave has an electrostatic component, the third term also must be included. Continuing to follow the procedure of Refs. 3 and 5, we take $f^e_{Ly}\approx 0$ and solve for E_{Hy} in the limit of fixed ions to find near-critical density n_c:

$$E_{Ly}=\frac{1}{en_c}\left(\frac{\nu}{\omega_0}\right)\frac{\omega_p^2}{\omega_0^2}\left(\frac{\partial}{\partial x}\,\mathrm{Im}\frac{\langle E_{Hx}E^*_{Hy}\rangle}{2\pi}+k_0\sin\theta\,\frac{\vec{E}_H\cdot\vec{E}^*_H}{2\pi}\right). \quad (20)$$

With $\dot{B}_L=-c(\partial E_{Ly}/\partial x)$ we have

$$\dot{B}_L\sim-\frac{c}{en_c}\,\frac{\nu}{\omega_0}\,\frac{\partial^2}{\partial x^2}\,\mathrm{Im}\frac{\langle E_{Hx}E^*_{Hy}\rangle}{2\pi}, \quad (21)$$

having dropped the second term in (20) so long as $k_0\sin\theta\ll\partial/\partial x$. Since

$$\mathrm{Im}\,E_{Hx}E^*_{Hy}\sim\mathrm{Re}\,E_{Hx}E^*_{Hy}=\tfrac{1}{2}\langle E_{Hx}E_{Hy}\rangle,$$

we can write

$$\dot{B}_L\sim 7\times10^{-14}\,\frac{1}{k_0^2}\,\frac{\partial^2}{\partial x^2}\left(\alpha\,\frac{\langle E_{Hx}E_{Hy}\rangle_{\text{sat}}}{\langle E_0^2\rangle}\right)\frac{\nu}{\omega_0}\,I,$$

measured in MG/psec, where I is the intensity in W/cm^2 and $\alpha\sim\frac{1}{2}$ is some fraction of $\langle E_{Hx}E_{Hy}\rangle_{\text{sat}}$, the saturated value of $\langle E_{Hx}E_{Hy}\rangle$, to account for the fact that steady state has not been reached.

For the simulation of Ref. 5, we use $I=2.2\times10^{16}$ W/cm^2 and the data recorded in Fig. 2(a) of Ref. 5 to obtain $\dot{B}_L\sim 7\times10^2$ Mg/psec. This result is 100 times greater than observed. However this estimate can be in error by a factor of 2 because of the difficulty of obtaining the value of $\partial^2\langle E_{Hx}E_{Hy}\rangle_{\text{sat}}/\partial x^2$ from Fig. 2(a). As we see from the analysis leading to Eq. (21), it is a straightforward result of (14) and (16), which are a direct consequence of the use of a phenomenological dissipation ν in the kinetic equation for f^e. We chose to derive Eq. (21) from (14) to show explicitly the dependence of $\dot{B}_L$ on ν. Alternatively, one may deal directly with (14), as was done in Ref. 5 to obtain their Eq. (9):

$$\dot{B}_L=-\frac{\partial}{\partial x}\,\frac{c}{en_0}\,\frac{\partial}{\partial x}\left(\epsilon_R\,\mathrm{Re}\,\frac{\langle E_{Hx}E_{Hy}\rangle}{2\pi}\right)\approx\frac{\partial^2}{\partial x^2}\,\frac{c}{en_c}\,\frac{\nu}{\omega_0}\,\frac{\langle E_{Hx}E^*_{Hy}\rangle}{2\pi}, \quad (22)$$

having used

$$\epsilon_R\frac{\langle E_{Hx}E_{Hy}\rangle}{4\pi}=\left(1-\frac{x}{L}\right)\frac{\langle E_{Hx}E^*_{Hy}\rangle}{2\pi}\approx\frac{\nu}{\omega_0}\,\mathrm{Re}\,\frac{\langle E_{Hx}E^*_{Hy}\rangle}{2\pi}.$$

Thus Eq. (9) of Ref. 5 gives essentially the same numerical result as Eq. (21). We expect that the good agreement with the observed value obtained in Ref. 5 by analytic evaluation of the fields was coincidental. In fact, the analytic form for the fields is strictly valid for $\sin^2\theta\gg\nu/\omega_0$,[7] which is not satisfied for the present example with $\theta=11°$ and $\nu/\omega_0=0.08$. A more fundamental problem with (21) is that it implies that B_L does not saturate. Alternatively, with $\dot{B}_L=0$ in steady state, we must have $\partial E_{Ly}/\partial x=0$ or $E_{Ly}=0$, whereas from (14),

$$E_{Ly}=\frac{1}{en_c}\,\frac{\partial}{\partial x}\,\epsilon_R\,\frac{\langle E_{Hx}E_{Hy}\rangle}{4\pi}\neq 0$$

so long as the laser is on. Thus we must conclude that the theory used in Ref. 5 does not treat saturation effects adequately. As will be shown later, Eq. (19) is deficient since it does not include the time-averaged drag force on the current J_{Ly}.

IV. STEADY-STATE FIELDS AND MOMENTUM BALANCE

Equation (13), on the other hand, in steady state implies that

$$E_{Ly}=\frac{1}{e}\frac{\partial}{\partial x}\langle\delta T^{e}_{xy}\rangle+\frac{\partial}{\partial x}\left(\epsilon\frac{\langle E_{Hx}E_{Hy}\rangle}{4\pi}\right). \tag{23}$$

We now show that in a *collisionless* kinetic theory, (23) leads exactly to $E_{Ly}=0$ in steady state so long as the high-frequency fields $\vec{E}_H$ satisfy Maxwell's equations. To obtain [see (12)] $\partial\langle\delta T^{e}_{xy}\rangle/\partial x$ we need $(\delta/\delta x)\int d^3v\, v_x v_y\, \delta F_L$. Using the transformation (7) it is straightforward to obtain the kinetic equation for F_L (see the Appendix). The v_y velocity moment of this kinetic equation in steady state gives

$$\frac{\partial}{\partial x}\int d^3v\, v_x v_y\, \delta F_L=\int d^3v\, v_y\left\langle\vec{\nabla}(\vec{v}\cdot\vec{u}_H)\cdot\frac{\partial}{\partial\vec{v}}F_H\right\rangle-\int d^3v\, v_y\langle\vec{u}_H\cdot\vec{\nabla}F_H\rangle, \tag{24a}$$

or

$$-m\frac{\partial}{\partial x}\int d^3v\, v_x v_y\, \delta F_L=-\frac{m}{e}\left(\frac{\partial}{\partial x}\langle\Delta j_{Hy}u_{Hx}\rangle+\left\langle\Delta\vec{j}_H\cdot\frac{\partial}{\partial y}\vec{u}_H\right\rangle\right), \tag{24b}$$

if we use the definition for $\Delta\vec{j}_H$ in (10). Therefore, from (12),

$$\frac{\partial}{\partial x}\langle\delta T^{e}_{xy}\rangle=\frac{m}{e}\left(\frac{\partial}{\partial x}\langle\Delta j_{Hx}u_{Hy}\rangle-\left\langle\Delta\vec{j}_H\cdot\frac{\partial}{\partial y}\vec{u}_H\right\rangle\right). \tag{25}$$

Now using the steady-state Maxwell equations for the fields where the only time dependence is sinusoidal,

$$\Delta j_{Hx}=-\frac{1}{4\pi}\left(i\omega_0(\sin^2\theta-\epsilon)E_{Hx}-c\sin\theta\frac{\partial}{\partial x}E_{Hy}\right), \tag{26a}$$

$$\Delta j_{Hy}=-\frac{1}{4\pi}\left[i\omega_0\left(-k_0^{-2}\frac{\partial^2}{\partial x^2}-\epsilon\right)E_{Hy}-c\sin\theta\frac{\partial}{\partial x}E_{Hx}\right], \tag{26b}$$

we can evaluate (25) to obtain

$$\frac{\partial}{\partial x}\langle\delta T^{e}_{xy}\rangle=-\frac{\partial}{\partial x}\left(\epsilon\frac{\langle E_{Hx}E_{Hy}\rangle}{4\pi}\right), \tag{27}$$

which exactly cancels the second term in (23). Therefore, from (23), $E_{Ly}=0$ and $\dot{B}_L=0$ in steady state in a collisionless plasma so long as the fields $\vec{E}_H$ satisfy the steady-state Maxwell equations (26). Thus we may conclude that no additional physical effects need be invoked to achieve steady state for B_L as was suggested in Ref. 5. Rather, the source of B_L identically vanishes when the correct form for the particle stress is used, and the high-frequency fields $\vec{E}_H$ have reached steady state, satisfying Maxwell's equations. The steady state for B_L is a direct consequence of steady state having been achieved by the $\vec{E}_H$.

The following plausible physical picture can be proposed to explain these rigorous results. First, we note that T^{e}_{xy} is the y momentum (density) convected across a unit surface with normal in the x direction. It is reasonable in view of our results in Secs. I and II to identify the second term in (23) with the momentum *added* to the plasma by the incident electromagnetic field. (As we saw in Sec. II, this term with ϵ replaced by ϵ_R gives a nonzero, time-independent contribution to $\dot{B}_L$.) The first term in (23), which cancels the second in steady state, we identify with the momentum convected *out* of the resonant region by the electrostatic waves. A similar momentum balance can be achieved without convection if momentum is lost due to collisional dissipation as we will show at the end of the next section.

V. CALCULATION OF B_L USING AMPERE'S LAW

Considering the complexity of obtaining E_{Ly} to find $\dot{B}_L$, it appears more straightforward to start with Ampere's law (neglecting the displacement current which is easily seen to be small):

$$B_L(x)=-\frac{4\pi}{c}\int^{x}dx'\, J_{Ly}(x'), \tag{28}$$

giving the magnetic field directly in terms of the generated low-frequency current. Now $J_{Ly}=J_{Ly}\{\vec{E}_H\}$; i.e., J_{Ly} is a functional of the high-frequency fields. Taking the time derivative of (28) and using the functional-derivative chain rule, we write

$$\dot{B}_L(x)=-\frac{4\pi}{c}\int dx'\int dx''\,dt''\left[\sum_m\frac{\delta J_{Ly}(x',t)}{\delta E_{Hm}(x'',t'')}\times\frac{\partial}{\partial t''}E_{Hm}(x'',t'')+\text{c.c.}\right]. \tag{29}$$

Thus $\dot{\vec{E}}_H=0$ implies $\dot{B}_L=0$ to within a time response $\delta J_{Ly}/\delta E_{Hm}=K_m(x,t;x',t')$. This time response depends on the transit time τ_P of particles through the high-frequency field. Generally $\tau_P\ll\tau_H$, where τ_H is the time scale for the E_H to attain steady state. Thus we expect B_L to essentially follow the E_H fields adiabatically, since the E_H develop on a slow time scale compared to τ_P.

From this general argument we conclude that

$$\frac{1}{B_L}\frac{\partial}{\partial t}B_L \approx \frac{1}{E_H}\frac{\partial}{\partial t}E_H , \qquad (30)$$

where the E_H in (30) is the field component which grows at the slower rate. As a confirmation of (30) consider Fig. 1, where the time history of B_L is shown. The maximum value at a given time is plotted. In this simulation a fixed linear profile with $k_0L = 12.5$ was taken, the incident intensity was $eE_0/m\omega_0 c = v_0/c = 0.025$, and the plasma temperature was $(2T_e/mc^2)^{1/2} = 0.05$. From the figure we find $\dot{B}_L/B_L \approx \dot{B}_L/\frac{1}{2}(B_L)_{\rm sat} = \nu_{B_L} \approx 3 \times 10^{-2}\omega_0$. This value for ν_{B_L} should be compared with $\nu_{E_H} \approx 1\times 10^{-2}\omega_0$, which is of the same order of magnitude as $\nu \approx (v_e/\omega_0 L)^{2/3}\omega_0$, the relevant time scale for the growth of E_H when plasma-wave convection out of the critical region is responsible for the absorption rate.

As we have shown, to use (13) or the so-called stress-tensor approach to obtain E_{Ly} from which $\dot{B}_L$ is obtained without properly accounting for $\langle \delta T^e_{xy}\rangle$ leads to quantitative error and does not properly account for saturation. On the other hand, a correct evaluation of $\langle \delta T^e_{xy}\rangle$ in a collisionless theory would require the second velocity moment of the low-frequency distribution as shown by Eq. (12) along with the time history of the high-frequency fields. Therefore this approach is difficult to use correctly and unnecessarily complex particularly in the light of the simple result of (30). Indeed, if F_L is known accurately enough to obtain the second velocity moment $\int d^3v\, v_x v_y F_L$, it would seem more straightforward to obtain the first moment $\int d^3v\, v_y F_L$ from which B_L may be obtained directly.

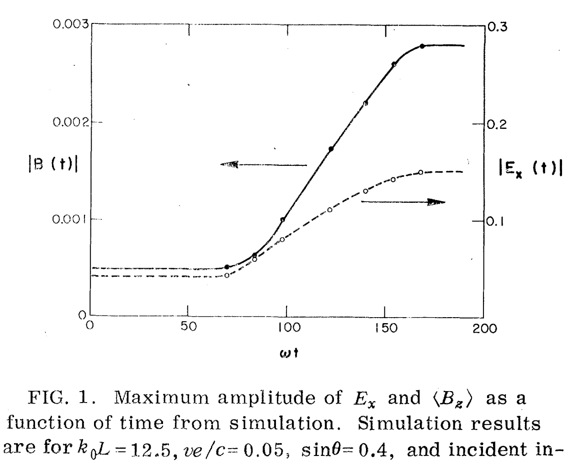

FIG. 1. Maximum amplitude of E_x and $\langle B_z\rangle$ as a function of time from simulation. Simulation results are for $k_0L = 12.5$, $ve/c = 0.05$, $\sin\theta = 0.4$, and incident intensity $E_0 = 0.025$. All fields are expressed in units of $mc\omega_0/e \approx 100$ MG.

We now turn to a direct calculation of B_L by obtaining J_{Ly} in Eq. (28). As an approximation for J_{Ly}, one might use the dc beat current $-e\langle n_H u_{Hy}\rangle$ as suggested by fluid theory, where n_H is the high-frequency density. In general, the current is given by

$$\langle J_y\rangle = J_{Ly} = -e\left\langle \int d^3v\, v_y f^e(\vec{r},\vec{v},t)\right\rangle . \qquad (31)$$

Using the transformation (7), we can write

$$J_{Ly} = -e\langle n_H u_{Hy}\rangle + \delta J_{Ly} , \qquad (32)$$

where $\delta J_{Ly} = -e\int d^3v\, v_y F_L(\vec{r},\vec{v},t)$. Thus, in addition to the fluidlike contribution to the current, we find that there is a low-frequency current δJ_{Ly} resulting from a low-frequency modification of the distribution function. To calculate F_L a consistent kinetic theory is required. Such a kinetic theory was formulated in Ref. 8, and is generalized to three-dimensions in the Appendix of this paper. This theory is valid so long as $|\vec{v}\cdot\vec{u}_H/v_{l_1}(x)v_{\rm eff}| \ll 1$, where $v_{l_1}^{-1}(x) = \omega_0^{-1}\partial \ln u_{Hx}/\partial x$ and $v_{\rm eff}^{-1} = \partial \ln F_L/\partial v_x$. That relatively low velocities do indeed make the dominant contribution to δJ_{Ly} can only be seen *a posteriori*. However, some qualitative argument is possible that is suggestive of this conclusion. In terms of $n_L^e(v_y,\vec{r})$, the density of particles for a given v_y, $\delta J_{Ly} = -e\int dv_y\, n_L^e(v_y,\vec{r})$. Now $n_L^e(v_y,\vec{r})$ has its main contribution from unheated particles so long as most of the heating occurs in the tail of the distribution. In addition, we note that relatively little y-heating is generally observed in typical simulations. Thus we expect that the dominant contribution to δJ_{Ly} is due to the distortion of the low-velocity portion of the distribituon function by the dc beats of the high-frequency fields.

From the Appendix we have in steady state:

$$\vec{v}\cdot\vec{\nabla}F_L + \vec{a}\cdot\frac{\partial}{\partial\vec{v}}F_L = \langle I_H(t)F_H(t)\rangle , \qquad (32a)$$

where

$$I_H(t) \equiv \vec{\nabla}[\vec{v}\cdot\vec{u}_H(t)]\cdot\frac{\partial}{\partial\vec{v}} - \vec{u}_H(t)\cdot\vec{\nabla} ,$$

$$\vec{a} \equiv -\frac{e}{m}\left(\vec{E}_L + \frac{\vec{v}\times\vec{B}_L}{c}\right) - \vec{\nabla}\frac{1}{2}\langle u_H^2\rangle .$$

If we ignore the coupling to F_H in (32a), then the low-frequency acceleration of $\vec{a}$ accounts for the perturbation of F_L arising from the ponderomotive force $-\frac{1}{2}m\vec{\nabla}\langle u_H^2\rangle$, $\vec{E}_L$, and $\vec{B}_L$. F_H satisfies a similar equation as (32a):

$$\frac{\partial}{\partial t}F_H(t) + \vec{v}\cdot\vec{\nabla}F_H(t) = I_H(t)F_L , \qquad (32b)$$

where we have neglected the corresponding term due to the acceleration $\vec{a}$, since in steady state $F_H(t)=F_H e^{-i\omega_0 t}+\text{c.c.}$, and we are considering sufficiently weak gradients that $\omega_0 F_H \gg \vec{a}\cdot\partial F_H/\partial\vec{v}$. We can solve (32b) for F_H in terms of F_L:

$$F_H=\int_{-\infty}^{x} dx' \frac{\exp[i(\omega_0/v_x)(x-x')]}{|v_x|} I_H F_L(x',\vec{v}) \quad (33)$$

for

$$\sin\theta\, v_y/c \ll 1, \quad v_x>0,$$

with a similar expression for $v_x<0$, and substitute into (32a) to obtain a closed equation for F_L. [Note that when the explicit time dependence is not shown for a high-frequency quantity, for example I_H, we mean the amplitude in the representation $I_H(t)=I_H e^{-\omega_0 t}+\text{c.c.}$].

It is convenient to integrate by parts on x' in (33) to obtain

$$F_H=(1/-i\omega_0)I_H F_L+\delta F_H\ , \quad (34)$$

where

$$\delta F_H=\int_{-\infty}^{x} dx' \exp\left[i\frac{\omega_0}{v_x}(x-x')\right]\frac{1}{i\omega_0}\frac{\partial}{\partial x'}(I_H F_L)\,. \quad (35)$$

This procedure can be continued to form a series expansion in $(v_x/\omega_0)\partial/\partial x$, valid so long as $v_x \ll v_{l_1}(x)$. As an aid to understanding the terms appearing in (34), we calculated the high-frequency current $\Delta\vec{j}_H$ which enters the Maxwell equations for the high-frequency fields. It was useful to perform a second integration by parts with the result

$$F_H=\frac{1}{-i\omega_0} I_H F_L - v_x\frac{1}{(i\omega_0)^2}\frac{\partial}{\partial x}(I_H F_L) + \int_{-\infty}^{x} dx'\, v_x \exp\left[\frac{i\omega_0(x-x')}{v_x}\right]\frac{\partial^2}{\partial x'^2}(I_H F_L)\,. \quad (36)$$

In this calculation the current linear in the fields was obtained with F_L approximated by its zero-order limit, a local Maxwellian $F_m(v^2)$. Then the first term in (36) gave no contribution to $\Delta\vec{j}_H$, the second term accounted for the effect of wave convection, and the last term, nonlocal in the field, included the effect of Landau damping. The Maxwell equations for the fields were solved, leading to good agreement with simulations for moderately intense laser light.[9]

If we apply the same limit to $\langle I_H(t)F_H(t)\rangle$ in (32a), we find

$$v_x\frac{\partial}{\partial x}F_L+\frac{e}{m}\partial_x\phi\frac{\partial}{\partial v_x}F_L=\langle I_H(t)\delta F_H(t)\rangle\,, \quad (37)$$

where

$$\phi=\phi_L-\frac{m}{e}\left[\frac{1}{2}\langle u_H^2\rangle+\frac{1}{\omega_0}\left(-\frac{v_y}{\omega_0}\left\langle\frac{\partial}{\partial t}u_{Hx}\frac{\partial}{\partial x}u_{Hy}\right\rangle + k_y\vec{v}\cdot\langle\vec{u}_H u_{Hy}\rangle\right)\right]\,.$$

For $v_x \ll v_{l_1}(x)$ the source term in (37) is small and

$$F_L=F_m(v^2-2(e/m)\phi)\,, \quad v_x<v_{l_1}(x)\,. \quad (38)$$

Thus the effect of the fields can be understood as a shift in the velocity away from an isotropic Maxwellian in velocity. For those velocities satisfying $v_x<v_{l_1}(x)$, the distribution F_L of (38) contributes to δJ_{Ly}

$$-e\int d^3v\, v_y F_m\left(v^2-\frac{2e\phi}{m}\right) = -en_L^e\left(\frac{1}{\omega_0^2}\left\langle\frac{\partial}{\partial t}u_{Hx}\frac{\partial}{\partial x}u_{Hy}\right\rangle-\frac{k_y}{\omega_0}\langle u_y^2\rangle\right)\,. \quad (39)$$

This result for δJ_{Ly} follows by taking the first term in (36) for F_H. If we calculate $n_H=\int d^3v\, F_H$ in the same approximation, then

$$-e\langle n_H u_{Hy}\rangle=-en_L^e\left[\frac{1}{\omega_0^2}\left\langle\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}u_{Hx}\right)u_{Hy}\right\rangle + \frac{k_y}{\omega_0}\langle u_{Hy}^2\rangle\right]\,. \quad (40)$$

Therefore we find

$$J_{Ly}=\frac{-en_L^e}{\omega_0^2}\frac{\partial}{\partial x}\left\langle\frac{\partial}{\partial t}u_{Hx}u_{Hy}\right\rangle = -en_L^e c k_0^{-1}\frac{\partial}{\partial x}2\,\text{Im}\left(\frac{u_{Hx}}{c}\frac{u_{Hy}^*}{c}\right)\,. \quad (41)$$

Finally, with (28),

$$\frac{\omega_{B_L}}{\omega_0}=2\frac{\omega_p^2}{\omega_0}\text{Im}\left(\frac{u_{Hx}}{c}\frac{u_{Hy}^*}{c}\right)\,, \quad (42)$$

where ω_{B_L} is the gyrofrequency eB_L/mc. It should be stressed that this simple expression for the steady-state magnetic field is valid only in a region localized around the critical surface since it requires $v_x \ll v_{l_1}(x)$, or equivalently, restriction to a region where δF_H in (34) can be ignored. This correction is important when the phase velocity $v_l(x)$ has decreased to the point that wave damping is significant. Thus (42) does not include all the effects of Landau damping. Wave damping is included in the fields, however. Further, it is limited to moderate laser power where the linear approximation for $\Delta\vec{j}_H$ is possible.

In Fig. 2(a) we have plotted Eq. (42) for ω_{B_L}/ω_0 using the steady-state fields E_x and E_y as obtained from the electromagnetic PIC (particle-in-cell) code wave[10] for the simulation parameters $v_0/c=0.025$, $k_0L=12.5$, $v_e/c=0.05$, and a fixed density profile. For the sake of comparison we also show the same quantity as obtained directly from the simulation. As can be seen the agreement is quite good, the spatial profiles and peak magnitudes being quite similar, even in reproducing secondary structure. In Fig. 3(a) we show the magnetic field as one would determine from the partial current $-e\langle n_H u_{Hy}\rangle$. The agreement comparing with Fig. 2(b) is rather poor. Thus it is essential that the contribution to B_L from δJ_{Ly} be included to obtain the correct expression for the total y current. Finally, we wish to point out an interesting property of Eq. (42) for the magnetic field. For a given sign of k_y for the incident electromagnetic wave, E_{Hx} and E_{Hy} have a definite polarization relative to one another at the critical surface. For the opposite sign of k_y, E_{Hx} changes sign and E_{Hy} retains the same sign as before. Thus B_L has the parity of k_y. This polarization dependence of B_L has recently been observed experimentally.[11] In the attempt at an analytic explanation of this very beautiful experiment, Eq. (1) of this paper identifies the source current for the magnetic field as $-e\langle n_H \vec{u}_H\rangle$. As we have illustrated in Fig. 3(a), such a current can give only a very poor representation for the magnetic field observed in both computer simulation and in their experiments. The contribution from $\delta\vec{J}$ is essential to obtain Eq. (42), and this expression agrees quantitatively with the experiment.[11]

To summarize, we have considered the approach of using the stress tensor to obtain $\dot{B}_L$. It appears as if all attempts to use this method without including $\langle\delta\vec{T}\rangle$ have resulted in conclusions which are incorrect. It has been shown that B_L can be obtained directly by using the steady-state current J_{Ly} in Ampere's law. We show that this current has an important contribution δJ_{Ly} in addition to the usual beat current $-e\langle n_H u_{Hy}\rangle$, which when combined with the beat current yields a result for the magnetic field which closely reproduces the observed spatial profile of the magnetic field. To obtain δJ_{Ly} for a collisionless plasma, a consistent kinetic theory is needed for calculating the electron distribution function in the presence of high-frequency fields. We have constructed such a theory and interpret the calculation of the correct value for δJ_{Ly} using this theory as evidence for its validity.

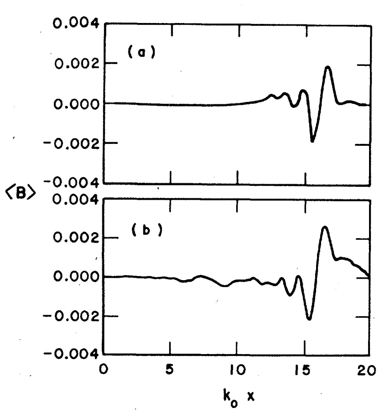

FIG. 2. (a) Magnetic fields $\langle B_z\rangle$ as predicted by theory using the simulation fields. (b) Magnetic field $\langle B_z\rangle$ entirely from simulation,

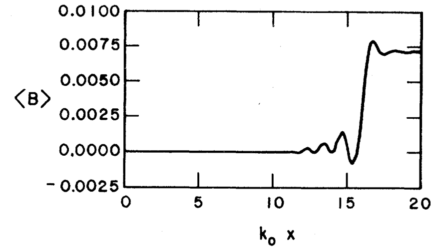

FIG. 3. Magnetic field $\langle B_z\rangle$ with partial current $-e\langle n_H u_{Hy}\rangle$ as source.

Finally, we wish to point out that the expression for the magnetic field of Eq. (42) may be derived in a very simple fashion[12] by noting that the time-averaged drag force $\nu m\langle J_y\rangle e^{-1}$ should appear on the right-hand side of the expression for f_{Ly} in Eq. (19). Thus Eq. (19) should read

$$f_{Ly} = -en_L^e E_{Ly} + \frac{\nu}{\omega_0}\frac{\omega_p^2}{\omega_0^2}\left(\frac{\partial}{\partial x}\,\mathrm{Im}\,\frac{\langle E_{Hx}E_{Hy}^*\rangle}{2\pi} + k_0 \sin\theta\,\frac{\vec{E}_H\cdot\vec{E}_H^*}{2\pi}\right) + \nu m J_{Ly} e^{-1}\,. \qquad (43)$$

This *phenomenological* drag force *cannot* be incorporated in the divergence of the stress tensor, whereas in the collisionless theory we found an additional stress-tensor contribution $\langle\delta\vec{T}\rangle$ which led to convective saturation.

We have in place of Eq. (21) for quasisteady state, $f_{Ly}\approx 0$,

$$\dot{B}_L = -c\frac{\partial}{\partial x}E_{Ly} \sim -\frac{c}{en_L^e}\frac{\partial}{\partial x}\left[\frac{\nu}{\omega_0}\frac{\partial}{\partial x}\,\mathrm{Im}\,\frac{\langle E_{Hx}E_{Hy}^*\rangle}{2\pi} + \nu m J_{Ly}e^{-1}\right]. \qquad (44)$$

Thus in steady state we recover Eq. (41) for the current J_{Ly} with $\dot{B}_L = 0$ and $E_{Ly} = 0$. From this simple derivation we may conclude that the use of a phenomenological ν in itself does not lead to incorrect results. However, in this case the total force is *not* given by Eq. (14) as assumed in Ref. 5. The time-averaged drag force must be explictly added to the right-hand side of (14). The stress-tensor arguments leading up to Eq. (14) are thus deficient. We conclude then that a steady-state current and therefore a steady-state B_L can arise from balancing the momentum imparted to the plasma by the high-frequency fields with either the momentum convected out of the resonance region in a collisionless theory or with momentum dissipated by local collisional drag forces.

In the collisional theory, the steady-state current is determined by balancing the drag force against the driving force and yields the localized magnetic field calculated above which is independent of the collision frequency ν. In problems *without* a driving source, the drag force produces the familiar time-dependent diffusion of B,

$$\dot{\vec{B}} = c(\vec{\nabla}\times\vec{E}) = c\vec{\nabla}\times(\vec{J}/\sigma) = (c^2/4\pi\sigma)\vec{\nabla}\times\vec{\nabla}\times\vec{B}\,,$$

due to the finite conductivity $4\pi\sigma = \omega_p^2/\nu$. Our calculated steady-state magnetic field [Eq. (42)], which includes the effect of the collisional drag force, is localized in a region which is only a small fraction of an electron Larmor radius even for thermal electrons. This localized field does *not* diffuse further into the overdense region and, contrary to the assertions of Ref. 5, cannot have a significant effect on the transport of electrons into the overdense plasma.

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APPENDIX

Suppose we apply the transformation of Eq. (7) to the Vlasov equation for f^e:

$$\frac{\partial}{\partial t} f^e(\vec{r},\vec{v},t) + \vec{v}\cdot\vec{\nabla} f^e(\vec{r},\vec{v},t) - \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}_L}{c}\right)\frac{\partial}{\partial\vec{v}} f^e(\vec{r},\vec{v},t) = 0\,. \quad (\mathrm{A1})$$

Then it is a simple matter to show that the function $F(\vec{r},\vec{v},t)$ satisfies the equation

$$\frac{\partial}{\partial t} F(\vec{r},\vec{v},t) + \vec{v}\cdot\vec{\nabla}F(\vec{r},\vec{v},t) + \left[-\frac{e}{m}\left(\vec{E}_L + \frac{\vec{v}\times\vec{B}_L}{c}\right) - \vec{\nabla}\,\frac{1}{2}\,[\vec{u}_H(t)\cdot\vec{u}_H(t)]\right]\cdot\frac{\partial}{\partial\vec{v}} F(\vec{r},\vec{v},t) = \vec{\nabla}[\vec{v}\cdot\vec{u}_H(t)]\cdot\frac{\partial}{\partial\vec{v}} F(\vec{r},\vec{v},t) - \vec{u}_H(t)\cdot\vec{\nabla}F(\vec{r},\vec{v},t) \quad (\mathrm{A2})$$

or

$$\left(\frac{\partial}{\partial t} + L(t)\right) F(t) = I_H(t)F(t)\,,$$

where the operators L and I_H are defined in (32a). We write

$$F(t) = F_L + F_H(t)\,, \quad (\mathrm{A3})$$

where $F_L = \langle F(t)\rangle$, and $F_H(t) = F(t) - \langle F(t)\rangle$. Now since the plasma is modulated by coherent high-frequency fields, oscillating at the frequency ω_0, we may expect that $F_H(t)$ may be written

$$F_H(t) = F_H^{(1)}(t)\ \ F_H^{(2)}(t) + \cdots\,, \quad (\mathrm{A4})$$

where, for example, $F_H^{(1)}(t) = F_H^{(1)}e^{-i\omega_0 t} + \text{c.c.}$, and $F_H^{(2)}(t) = F_H^{(2)}\,e^{-i2\omega_0 t} + \text{c.c.}$, etc. Thus the harmonic modulation leads to a harmonic expansion for $F_H(t)$. However, the transformation of Eq. (7) leading to F is useful only if $F_H^{(1)} \gg F_H^{(2)} \gg F_H^{(3)} \gg \cdots$, so that $F_H(t)$ may be approximated by $F_H^{(1)}(t)$, say; otherwise, Eq. (A2) would lead to a set of coupled equations for F_L and the various harmonic components $F_H^{(n)}$. Physically we expect the transformation to be useful for relatively low velocities, i.e., when a particle of a given velocity v_x transits the localized, high-frequency field structure in a long time compared with the period of oscillation. In this case the velocity of the particle has a rapidly oscillating component which the transformation removes. Contrariwise, if the particle is moving through the field structure so fast that it experiences a more or less time-independent field, then we expect the original distribution function f^e to be nonoscillatory, and the transformation to the oscillating velocity frame would introduce an artificial time dependence, making it impossible to truncate (A3). Of course many other transformations are possible. We have restricted ourselves to Eq. (7) since it gives results which are more or less easy to interpret.

To obtain the coupled set of equations for F_L and F_H, we substitute (A3) and (A2) and take the time average of the resulting equation to find

$$\left(\frac{\partial}{\partial t} + \langle L(t)\rangle\right) F_L = I_H F_H^{(1)*} - L_H F_H^{(2)*} + \text{c.c.} \quad (\mathrm{A5})$$

Subtracting Eq. (A5) from Eq. (A2), we find for the first harmonic amplitude $F_H^{(1)}$:

$$(-i\omega_0 + \langle L(t)\rangle)F_H^{(1)} = I_H F_L - L_H F_H^{(1)*} + I_H^* F_H^{(2)}\,. \quad (\mathrm{A6})$$

Similarly, we have for $F_H^{(2)}$:

$$(-i\omega_0 + \langle L(t)\rangle)F_H^{(2)} = I_H F_H^{(1)} - L_H F_L + I_H^* F_H^{(3)}\,, \quad (\mathrm{A7})$$

and so on for $F_H^{(3)}$, $F_H^{(4)}$, etc.

Our objective is to determine under what conditions $F_H^{(1)} \gg F_H^{(2)} \gg F_H^{(3)} \cdots$ allowing for truncation. Therefore we assume $F_H^{(3)} \ll F_H^{(1)}$ in Eq. (A7), neglecting the last term on the right-hand side compared to the first, and solve for $F_H^{(2)}$ using the formal inverse operator $(-i\omega_0 + \langle L(t)\rangle)^{-1} \sim 1/(-i\omega)$. Substituting the resulting expression into Eq. (A6), we have for $F_H^{(1)}$:

$$-i\omega_0+\langle L(t)\rangle - I_H^*(-i\omega_0+\langle L(t)\rangle)^{-1}I_H = I_H F_L - I_H^*(-i\omega_0+L(t)\rangle)^{-1}L_H F_L - L_H F_H^{(1)*} . \quad \text{(A8)}$$

Inspecting Eq. (A8), we see that so long as

$$\frac{L_H}{\omega_0} = \vec{\nabla}\,\frac{(1/2)\vec{u}_H^*\cdot\vec{u}_H}{\omega_0}\cdot\frac{\partial}{\partial\vec{v}} \sim \frac{\partial}{\partial v_x}\,\frac{(1/2)\vec{u}_H^*\cdot\vec{u}_H}{\omega_0}\,\frac{\partial}{\partial v_x} \ll 1 , \quad \text{(A9)}$$

we have for $F_H^{(1)}$,

$$\{-i\omega_0+\langle L(t)\rangle - I_H^*(-i\omega_0+\langle L(t)\rangle)^{-1}I_H\}\,F_H^{(1)} = I_H F_L \quad \text{(A10)}$$

and

$$\left(\frac{\partial}{\partial t}+\langle L(t)\rangle\right) F_L = \langle I_H^{(1)}(t) F_H^{(1)}(t)\rangle . \quad \text{(A11)}$$

These results, however, are subject to the condition $F_H^{(3)} \ll F_H^{(1)}$. Now from the corresponding equation for $F_H^{(3)}$, etc., we see that $F_H^{(3)} \ll F_H^{(1)}$ so long as

$$\frac{I_H}{\omega_0} = \vec{\nabla}\,\frac{\vec{v}\cdot\vec{u}_H}{\omega_0}\cdot\frac{\partial}{\partial\vec{v}} - \frac{\vec{u}_H\cdot\vec{\nabla}}{\omega_0} \sim \frac{\partial}{\partial x}\,\frac{\vec{v}\cdot\vec{u}_H}{\omega_0}\,\frac{\partial}{\partial v_x} - \frac{u_{Hx}\,\partial/\partial x}{\omega_0} \ll 1 . \quad \text{(A12)}$$

Since $u_H/v_{I_1}(x) \ll 1$ quite generally for all cases of interest and u_H/v_{eff} is of order one or less, the only significant limitation on (A10) and (A11) is the requirement that

$$\left|\frac{\vec{v}\cdot\vec{u}_H}{v_{I_1}(x)v_{\text{eff}}}\right| \ll 1 . \quad \text{(A13)}$$

Thus the transformation to oscillating velocity coordinates leading to (A10) and (A11) is limited to sufficiently low velocities as is consistent with the qualitative arguments presented earlier. Finally, it is important to note that although (A10) and (A11) are approximately valid, it is a simple matter to show that they satisfy particle and momentum conservation.

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