PHYSICAL REVIEW A VOLUME 16, NUMBER 4 OCTOBER 1977

Electron scattering from atoms in the presence of a laser field. III*

Marvin H. Mittleman
The City College of the City University of New York, New York, New York 10031
(Received 4 May 1977)

The development of the theory of the effect of a laser on electron-atom scattering is continued by the derivation of explicit relations between the observed electron-atom scattering cross sections in the presence of a laser and exact electron-atom scattering cross sections with no laser present. No approximation concerning the scattering interaction is made. The only approximations concerning the laser are that (1) the laser-atom interaction energy is small compared to atomic energies, (2) the Rabi frequency times the collision time is small, and (3) the laser intensity in appropriate units is small.

In two previous papers[1] the theory of the effect of a laser on electron-atom scattering was presented. Here, that development is completed by the derivation of a relation between the observed scattering in the presence of a laser and the usual cross sections measured and calculated in ordinary electron-atom scattering.

It is first necessary to review briefly the content of the first two papers. In the first (I), a formal theory of electron-atom scattering in the presence of a laser was developed. It was applied by using the approximation that the target-atom wave functions were unaffected by the laser except for the two target states which are resonantly coupled by the laser. These two were recoupled by the rotating-wave approximation with a classical description of the laser. These approximations are extremely good unless the lasers are very intense. The method was applied to a hydrogen target as an example, and various long-range interactions between the projectile and target were obtained. It was shown that some exotic effects, such as an oscillating dipole and permanent quadrupole, were induced in the atom by the laser. These potentials then produced some novel effects in the forward scattering.

These results could not be directly compared with experiment since the coupling of the system to the spontaneous radiation field was neglected. Typically, the atom will adiabatically pass into the laser and spend some time in it before the collision with the electron occurs. During that time hundreds of spontaneously radiated photons[2] (SRP's) will be emitted, so that the coupling to this field will be crucial in determining the atomic state that the electron encounters in the collision, that is, the "initial" state of the collisions described in I. This defect was remedied in the second paper (II), where the effect of the laser-atom-SRP coupling in both initial and final state was obtained. The dynamics of the SRP's was treated very crudely there, but more refined techniques, developed[3] for the resonance fluorescence problem, show that the results obtained are essentially exact. This gave a relation between the observed cross sections and those described in I. However, these still contained the laser in that they were electron-atom scattering cross sections in the presence of a laser. An application was made in which the electron-atom interaction was treated in the first Born approximation, and cross sections were then obtained which were directly comparable with experiment. However, because of the Born approximation they could only be expected to give good agreement at very high projectile energy. It is the purpose of this paper to obtain essentially the same relation between the laser-modified cross sections and the exact cross sections defined in the absence of the laser.

We proceed by defining laser-modified atomic states, $\phi_n(t)$. If the atomic states u_0 and u_1 are the two which are resonantly coupled by the laser, then

$$\phi_{0,1} = (e^{\pm\mu/2} u_0 e^{i\omega t/2} \pm e^{\mp\mu/2} u_1 e^{-i\omega t/2}) \times \frac{e^{\mp i\epsilon t}}{(2\cosh\mu)^{1/2}}\, e^{-i(W_0+W_1)t/2}, \qquad (1)$$

where ω is the laser frequency, W_n is the energy of u_n, and μ is defined by

$$\sinh\mu = \tfrac{1}{2}(\omega - W_{10})/|\Lambda|, \quad W_{10} = W_1 - W_0, \qquad (2)$$

with Λ being the coupling strength of the laser to the $u_0 \leftrightarrow u_1$ transition

$$\Lambda = \tfrac{1}{2} e \langle u_0 | \vec{r} | u_1 \rangle \cdot \vec{E} = |\Lambda| e^{i\varphi}. \qquad (3)$$

Also,

$$\epsilon = [|\Lambda|^2 + \tfrac{1}{4}(\omega - W_{10})^2]^{1/2}. \qquad (4)$$

For $n > 1$ the states are assumed to be undistorted and

$$\phi_n = u_n e^{-iW_n t}, \quad n > 1. \qquad (5)$$

The S matrix, in the presence of the laser, is given by

$$S(\vec{P}_f, n; \vec{P}_i n') = -i\left\langle \Omega_{P_f n}(0), V_0 \mathcal{A}_0 \left[1 + \left(i\frac{\partial}{\partial t} - H(t)\right)^{-1} V_0\right] \Omega_{P_i n'}(0)\right\rangle , \quad (6)$$

where $H(t)$ is the full Hamiltonian and

$$\Omega_{kn}(0) = \chi_k(0;t)\phi_n(1\ldots z;t) . \quad (7)$$

Here, $\chi_k(0;t)$ is the wave function of the zeroth electron with average momentum $\vec{k}$ in the field of the laser, and the notation $\phi_n(1\ldots z;t)$ means that the electrons 1 to z are in the target state n. V_0 is the Coulomb interaction of the zeroth electron with all other particles, and $\mathcal{A}_0$ is the antisymmetrization operator which takes care of the Pauli principle. Several approximations can now be made:

(i) We may neglect the interaction of the laser with the projectile. This is an expansion in the parameter

$$\frac{e}{2m}\frac{pE}{\omega^2} \simeq \frac{\mathcal{R}}{2\hbar\omega}\left(\frac{I}{2\times 10^{16}\ \mathrm{W/cm^2}}\right)^{1/2},$$

where $\mathcal{R}$ is the Rydberg constant and I is the laser intensity. This is very small for all currently contemplated experiments.

(ii) We may neglect the laser-atom interaction in all but the states u_0 and u_1 which leads to (5). Further, we may neglect the laser-atom interaction in these states, u_0 and u_1, when they occur as intermediate states. This is simply the statement that an intermediate state must exist for a time which is less than the total scattering time, and during the scattering time the lasers contemplated are not intense enough to pump between the two resonant states. This was discussed in II in the first section.

The first of these approximations allows the replacement of the states χ_k in (7) by plane waves, and the two together allow the replacement of $H(t)$ in (6) by the Hamiltonian H_0 for scattering without the laser. The resulting S matrix then contains mention of the laser only through the laser-distorted states ϕ_0 and ϕ_1 when they occur as initial or final states. Since the states all have exponential time dependence and this is the only remaining time dependence left in Eq. (6), then $i\,\partial/\partial t$ can be replaced by an energy and the S matrix can be related to conventional T matrices.

For example, if we are interested in scattering between excited states n and $n'>1$, then the laser has no affect at all and the observed cross section will be identical with the no-laser cross section.

A more interesting example occurs when we scatter from states coupled by the laser to a state $n>1$ unaffected by the laser. In that case,

$$\begin{aligned} S(\vec{P}_f, n; \vec{P}_i, n')_{n'=0,1} = {} & -2\pi i\delta\left(\frac{p_f^2}{2m} - \frac{p_i^2}{2m} + W_{n0} + \frac{\chi}{2} \mp \epsilon\right)\langle \vec{P}_f, n| T\left(\frac{p_i^2}{2m} + W_0 - \frac{\chi}{2} \pm \epsilon\right) |\vec{P}_i, 0\rangle \frac{e^{\mp\mu/2}}{(2\cosh\mu)^{1/2}} \\ & - 2\pi i\delta\left(\frac{p_f^2}{2m} - \frac{p_i^2}{2m} + W_{n1} - \frac{\chi}{2} \pm \epsilon\right)\langle \vec{P}_f, n| T\left(\frac{p_i^2}{2m} + W_1 + \frac{\chi}{2} \pm \epsilon\right) |\vec{P}_i, 1\rangle (\pm)\frac{e^{\pm\mu/2 - i\varphi}}{(2\cosh\mu)^{1/2}} , \end{aligned} \quad (8)$$

where

$$\chi = \omega - W_{10} , \quad (9)$$

and where the notation $|\vec{P}, n\rangle$ means the time-independent states

$$e^{i\vec{P}\cdot\vec{r}_0} u_n(1\ldots z) . \quad (10)$$

The two energy δ functions in (8) give different final energies for the projectile; so the two T matrices add incoherently. The square of the individual T matrices would be proportional to conventional cross sections were it not for the fact that the T matrices are slightly off shell by an energy of the order of χ and ϵ. These have been assumed to be small compared to W_{10} in the derivation, so it is consistent to neglect them. In that case only on-shell T matrices occur.

In addition, it has been shown in II [Eq. (2.14)] that the only observable is the averaged cross section

$$\frac{d\bar{\sigma}}{d\Omega}(n; \vec{P}_f, \vec{P}_i) = P_+ \frac{d\sigma}{d\Omega}(\vec{P}_f, n; \vec{P}_i, 0) + P_- \frac{d\sigma}{d\Omega}(\vec{P}_f, n; \vec{P}_i, 1) , \quad (11)$$

where $P_\pm$ are the probabilities of an electron finding the states ϕ_0 or ϕ_1, as an "initial" state for the collision. These are given by

$$P_\pm = e^{\pm 2\mu}/2\cosh 2\mu . \quad (12)$$

Combining (8), (11), and (12) we obtain the two possible cross sections which leave the atom in the excited state $n>1$. For final electron energy given by

$$\frac{p_f^2}{2m} = \frac{p_i^2}{2m} - W_{n0} ,$$

$$\frac{d\bar{\sigma}}{d\Omega}(n; \vec{P}_f, \vec{P}_i) = (1 - \beta'^2)\frac{d\sigma}{d\Omega}(\vec{P}_f, n; \vec{P}_i, 0) ; \quad (13)$$

for final electron energy given by

$$\frac{p_f^2}{2m}=\frac{p_i^2}{2m}-W_{n1},$$

$$\frac{d\bar{\sigma}}{d\Omega}(n;\vec{P}_f,\vec{P}_i)=\beta'^2\frac{d\sigma}{d\Omega}(\vec{P}_f,n;\vec{P}_i,1)\,, \tag{14}$$

and

$$\beta'^2=(2\cosh 2\mu)^{-1}. \tag{15}$$

The bar over the cross section denotes the observed cross section in the presence of the laser, while the cross sections on the right-hand side of these equations are the exact cross sections measured in the absence of the laser.

Finally, the final atomic state can be either ϕ_0 or ϕ_1, but in that case the final state is also strongly coupled to the laser and the SRP's; therefore, as was pointed out in II, the only observable "elastic" cross section, after the on-shell approximation is made, is

$$\frac{d\bar{\sigma}_\nu}{d\Omega}(\mathrm{el};\vec{P}_f,\vec{P}_i)=\frac{d\bar{\sigma}_\nu}{d\Omega}(0;\vec{P}_f,\vec{P}_i)+\frac{d\bar{\sigma}_\nu}{d\Omega}(1;\vec{P}_f,\vec{P}_i)\,. \tag{16}$$

The integer ν is defined by

$$\frac{p_f^2}{2m}=\frac{p_i^2}{2m}+\nu\omega \tag{17}$$

so that $\nu=0$ is truly elastic, $\nu=1$ is superelastic, and $\nu=-1$ is inelastic. These three cross sections are given by

$$\frac{d\bar{\sigma}_0}{d\Omega}(\mathrm{el};\vec{P}_f,\vec{P}_i)=(1-\beta'^2)\frac{d\sigma}{d\Omega}(\vec{P}_f,0;\vec{P}_i,0)+\beta'^2\frac{d\sigma}{d\Omega}(\vec{P}_f,1;\vec{P}_i,1)\,, \tag{18}$$

$$\frac{d\bar{\sigma}_1}{d\Omega}(\mathrm{el};\vec{P}_f,\vec{P}_i)=\beta'^2\frac{d\sigma}{d\Omega}(\vec{P}_f,0;\vec{P}_i,1)\,, \tag{19}$$

$$\frac{d\bar{\sigma}_{-1}}{d\Omega}(\mathrm{el};\vec{P}_f,\vec{P}_i)=(1-\beta'^2)\frac{d\sigma}{d\Omega}(\vec{P}_f,1;\vec{P}_i,0)\,. \tag{20}$$

These results are a generalization of those obtained in a first Born approximation in the scattering potential obtained in II.

Note that our results allow for at most the transfer of one quantum of energy between the projectile and the laser. This result is inherent in our approximation: which (i) does not allow for direct coupling between the projectile and the laser, and (ii) does not allow for the atom to be pumped *during* the collision. These results allow for a connection between laser-modified cross sections and the usual electron-atom scattering cross sections and permit the measurement of cross sections which are otherwise not measurable.

Finally, it should be pointed out that if either of the cross sections on the right-hand side of (16) were measurable, then interference terms between T matrices such as

$$\langle\vec{P}_i,0\,|\,T\,|\,\vec{P}_i,0\rangle\langle\vec{P}_f,1\,|\,T\,|\,\vec{P}_f,1\rangle$$

would occur. These cancel in the sum. These interference terms contain the exotic effects pointed out in I which are not observable here. The measurement of either of the cross sections on the right-hand side of (16) individually requires an identification of the final state of the collision as either ϕ_0 or ϕ_1. This could be accomplished if the atom could be extracted from the laser immediately after the collision. Here, "immediately" means after only a few SRP's have been emitted. This would appear to be a formidable technological problem.

*Research supported by the U. S. Office of Naval Research under Contract No. 00014-76-C-0014.

[1]J. I. Gersten and M. H. Mittleman, Phys. Rev. A 13, 123 (1976); M. H. Mittleman, Phys. Rev. A 14, 1338 (1976), and Erratum, Phys. Rev. A 15, 1355 (1977). These are referred to as I and II, respectively.

[2]N. D. Bhaskar, B. Jaduszliwer, and B. Bederon, Phys. Rev. Lett. 38, 14 (1977).

[3]B. R. Mollow, Phys. Rev. A 12, 1919 (1975).