Stimulated versus spontaneous emission as a cause of photon correlations

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Assertions that stimulated emission may cause photon correlations (or, properly, photocount correlations) are discussed. A particular association of terms in the density matrix rate equation for first order, linear, interactions is urged, following the literature of maser amplifiers. With this association of terms, a consistent picture results in which stimulated emission amplifies without altering the statistical character of an incident field. Spontaneous emission generates excess correlations, through its interference with the incident field. Objections to Webb's thermodynamic argument for an association between stimulated emission and the occurrence of photon correlations are also discussed.

INTRODUCTION

It is sometimes stated or implied that photoelectron correlations such as those of the Hanbury Brown–Twiss (HBT) effect may be caused by stimulated emission.[1-4] We note that the semiclassical analysis of the effect is based simply on the superposition of uncorrelated amplitudes, and that Glauber has shown in a fully quantum mechanical theory that superposition of contributions from independent sources is sufficient to derive the effect.[5]

Vorobev and Sokolovskii,[1] commenting on papers by Loudon[6] and by Chandra and Prakash,[7] state that "The processes of absorption and stimulated emission are essentially statistical processes. Therefore, even neglecting spontaneous emission, the initial distribution of photons must, in general, change when light propagates in a medium." This conclusion contradicts what Loudon terms the accepted view, that stimulated emission maintains the coherence properties of stimulating light, while spontaneous emission generates incoherent light.

The conclusion cited from Ref. 1 was made on the basis of the non-Poisson photon statistics claimed to result whenever coherent radiation interacts *through stimulated processes alone* with a collection of two-level atoms not all in the ground state. This interpretation follows from a particular correspondence of terms appearing in the density matrix equations with the physical processes of stimulated and spontaneous emission. Although in the absence of separate coupling constants for these two processes such identifications may be largely pedagogic, we argue that the correspondences made in Ref. 1 lead to physically unreasonable results and that an entirely different conclusion may be reached.

There is in the literature of maser amplifiers an alternative viewpoint on the effects of stimulated and spontaneous emission, one which is in much better agreement with the semiclassical description.[6-11] Adopting this viewpoint, we argue, in agreement with Carusotto,[11] that the HBT effect should, in the case of any *linear* interaction of an incident field with a collection of atoms, be interpreted as the result of interference occurring in the superposition of the spontaneously emitted chaotic field and a field statistically identical with the incident field. The effect of stimulated processes is simply to increase or decrease the intensity of the incident field. In the sense that stimulated processes do not change the normalized moments of the intensity probability distribution (or the equivalent normalized factorial moments of the counting distribution), they do not alter the statistics of electromagnetic fields.

Webb,[2] in a thermodynamic argument based on Einstein's derivation of Planck's blackbody formula,[12] claims that "... the process of stimulated emission is responsible for photons obeying Bose-Einstein statistics [and thus displaying the HBT correlations] in blackbody radiation." We will discuss our objections to the logic which led to this conclusion, and will point out the mechanism of approach to thermodynamic equilibrium which is implied in our alternative viewpoint.

THE SEMICLASSICAL VIEWPOINT: SUPERPOSITION OF INDEPENDENT SOURCES

When the problem of superposing field amplitudes from a large number of independent sources is considered classically, application of the central limit theorem leads immediately to the expected Gaussian probability distribution in the complex amplitude plane. It is the fluctuations associated with this distribution that lead to HBT intensity correlations.[13]

Semiclassically, the excess photoelectron correlations of the HBT effect will be present whenever the classical field intensity varies in time. The registration of a photoelectron is positively correlated with above-average fluctuations in intensity and therefore, since a field with finite bandwidth has a finite coherence time, with an above-average probability of registering a second count in neighboring time intervals. The excess correlations which result—correlations above and beyond those associated with the Poisson statistics of random events—are the manifestation of the HBT effect in experiments which deal directly with photoelectron coincidences.

The classical and semiclassical pictures are related quite generally through the equality of the moments of the classical intensity probability distribution function with the factorial moments of the semiclassical photon counting statistics.

As an example of the effects of classical superposition and interference we consider the mixture of a coherent field

$$E_0 \cos(\omega t)$$

and a randomly phased noise source

$$E_1(t) \cos[\omega t + \varphi(t)] ,$$

where the noise source will be assumed to have a Gaussian amplitude distribution and hence an exponential intensity distribution

$$P(I_1) = \langle I_1 \rangle^{-1} \exp(-I_1/\langle I_1 \rangle) . \tag{1}$$

The combined field is

$$E_0 \cos(\omega t) + E_1(t) \cos[\omega t + \varphi(t)] .$$

The detected signal will be given by the square of the combined field

$$I = I_0 + I_1 + 2(I_0 I_1)^{1/2} \cos\varphi , \tag{2}$$

where $I_0 = \frac{1}{2}E_0^2$, $I_1 = \frac{1}{2}E_1^2$, and we have assumed that the detector averages over optical frequencies. The square of that signal will be

$$I^2 = (I_0 + I_1)^2 + 4I_0 I_1 \cos^2\varphi + (\text{terms in } \cos\varphi) . \tag{3}$$

Taking the time average and the implied average over intensities, we find (using $\langle I_1^2 \rangle = 2\langle I_1 \rangle^2$) that

$$\langle I \rangle = I_0 + \langle I_1 \rangle , \tag{4}$$

$$\langle I^2 \rangle = I_0^2 + 2I_0\langle I_1 \rangle + 2\langle I_1 \rangle^2 + 2I_0 \langle I_1 \rangle , \tag{5}$$

and

$$\langle I^2 \rangle - \langle I \rangle^2 = \langle I_1 \rangle^2 + 2I_0 \langle I_1 \rangle . \tag{6}$$

The last equation shows the correlations due both to the intrinsic fluctuations in the noise source and to interference between the two sources.

THE SEMICLASSICAL VIEWPOINT: ATOMIC INTERACTIONS AND MASER AMPLIFIERS

Semiclassically, the electromagnetic field interacts coherently with a stable population of atomic systems. (We neglect the complications of time-dependent atomic populations, and intend by the use of the term "coherently" to imply no change in the statistical character of the field, in the sense of invariant normalized moments.)

Depending on the atomic populations, the net effect of stimulated emission and absorption is to either increase or decrease the intensity of the field. These two processes enter the formalism in a completely symmetric way. (This latter fact will enter strongly into our later discussion.)

Spontaneous emission, which traditionally is introduced in an *ad hoc* fashion into semiclassical theories, has the effect of contributing an additional chaotic, or Gaussian, "noise" field. Since the spontaneously emitted field is to be combined with the amplified or attenuated input field through an addition of amplitudes rather than of intensities, there arises the possibility of interference effects. With a perfectly coherent input field, this interference may produce, as we have just shown, final-state intensity fluctuations far in excess of those which would be present in the spontaneously emitted field alone. Since there would be no such fluctuations in the absence of the spontaneously emitted field, we may say that the HBT effect is due not to stimulated emission, but rather to the superposition of the Gaussian spontaneous emission with the statistically unchanged incident field.

QUANTUM THEORY: SUPERPOSITION OF INDEPENDENT SOURCES

We will use the usual P representation, in which the density matrix for a single mode of the field is written in terms of the overcomplete set of "coherent states," which are eigenstates of the photon annihilation operator a.[5,14] That is,

$$a|\alpha\rangle = \alpha|\alpha\rangle \tag{7}$$

and

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha| d^2\alpha , \tag{8}$$

where α is a complex number.

The general superposition of two statistically independent fields of known P representations P_1, P_2 is described by a P representation which is a convolution of the two individual functions

$$P(\alpha) = \int P_1(\alpha')P_2(\alpha - \alpha') d^2\alpha' . \tag{9}$$

Glauber has shown that, in strict analogy with the classical random walk problem, the superposition of a large number of independent fields has a Gaussian P representation in the complex α plane.[5] For chaotic light,

$$P(\alpha)=(\pi\langle n\rangle)^{-1}\exp(-|\alpha|^2/\langle n\rangle)\ . \tag{10}$$

Such a state displays the photocount statistics expected of a semiclassical chaotic field.

Lachs has pointed out that interference in the addition of coherent and thermal light can increase the correlations in the Hanbury Brown-Twiss experiment.[15] The P representation which describes the superposition of a coherent field given by the two-dimensional δ function

$$P_C(\alpha)=\delta^{(2)}(\alpha-\alpha_0) \tag{11}$$

and a thermal field given by

$$P_T(\alpha)=(\pi\langle n_T\rangle)^{-1}\exp(-|\alpha|^2/\langle n_T\rangle) \tag{12}$$

is, according to Eq. (9), a displaced Gaussian function of α:

$$P(\alpha)=(\pi\langle n_T\rangle)^{-1}\exp(-|\alpha-\alpha_0|^2/\langle n_T\rangle)\ . \tag{13}$$

The first two factorial moments of the photocount statistics of this field are

$$\langle a^\dagger a\rangle=\langle n_T\rangle+|\alpha_0|^2 \tag{14}$$

and

$$\langle a^\dagger a^\dagger aa\rangle=2\langle n_T\rangle^2+4\langle n_T\rangle|\alpha_0|^2+|\alpha_0|^4\ . \tag{15}$$

The excess correlations are then

$$\langle a^\dagger a^\dagger aa\rangle-\langle a^\dagger a\rangle^2=\langle n_T\rangle^2+2\langle n_T\rangle|\alpha_0|^2\ . \tag{16}$$

These results are strictly analogous to our earlier classical analysis, and show clearly that interference effects are involved in the production of HBT correlations. [Compare Eq. (6).]

QUANTUM THEORY: ATOMIC INTERACTIONS

It remains to consider quantum treatments of the interaction of radiation with atomic systems. Rather than quoting formal results, we refer the reader to the literature of the subject. While it must be noted that in a quantized theory stimulated and spontaneous emission are formally inseparable, there is a general consensus that, to first order in the interaction, differing statistical consequences may be separately assigned to stimulated and spontaneous effects.

Carusotto,[11] analyzing the single-photon interaction of a field and a number of two-level atoms, expresses this consensus by concluding that "... in the interaction with the atomic system the initial field is amplified or attenuated, but this process generates incoherent light also. Clearly, since the amplified light is obtained by stimulated emission process and incoherent light is generated by spontaneous emission, our results confirm the accepted view that stimulated emission maintains the coherence properties of the stimulating light whereas spontaneous emission generates only incoherent light." This statement is in agreement with the results of maser amplifier or attenuator analyses by Shimoda *et al.*[8] and by Gordon, Walker, and Louisell.[9] It agrees also with treatments of parametric amplifiers by Gordon, Louisell, and Walker[16] and by von Foerster and Glauber,[17] among others. The statistical nature of the output field may always be described as a superposition of (i) the incident field amplified or attenuated without any change (in our prior sense) in statistical structure; and (ii) a thermal or chaotic field produced by the spontaneous emission process. (That part of the thermal light that passes through later portions of the amplifying medium is amplified as well.) Thus, all of the statistical properties of the output of a quantum amplifier can be understood in terms of the superposition effect we have already considered.[18]

In a fully quantized theory, as well as semiclassically, the effect of stimulated emission is to amplify coherently. (Formally, the effect is to produce a scale change in the α plane—not, as Shen[19] and Carusotto[11] state, a translation of the P distribution.) It is the necessarily accompanying spontaneous emission which, when superposed, changes the statistical character of any non-Gaussian field which undergoes interaction with atomic systems.

In the case of an amplifier with no input field (as in the experiment of Scarl and Smith[20]) the field at any point along the amplifier is Gaussian. The effect of any short incremental portion of the amplifier is simply to increase the mean of this thermal field by stimulated emission and also by the addition of thermal spontaneous noise.

QUANTUM THEORY: MASER AMPLIFIERS

Solutions of the maser amplifier atom-field interaction have been presented by many others. We will briefly review the solution for the radiation field density matrix in the number representation and in the P representation. We will attempt to show how the conclusions we have discussed flow naturally from the theory.

In the adiabatic approximation for the atomic populations taken in its strictest sense—no population fluctuations—the time evolution of the radiation field density matrix is given by[21]

$$\dot{\rho}(t)=\tfrac{1}{2}\{A(a^\dagger\rho a-aa^\dagger\rho)+B(a\rho a^\dagger-a^\dagger a\rho)+\text{c.c.}\}\ , \tag{17}$$

where $A=2\Gamma^{-1}g^2N_a$, $B=2\Gamma^{-1}g^2N_b$, and g is the coupling constant between field and atom. N_a and N_b are the mean numbers of atoms in the states a and b (with $E_a>E_b$), and Γ is the width of the transition between the two states. In the number representation, where the density matrix is given by

$$\rho=\sum_{n,m}\rho_{nm}|n\rangle\langle m|\ , \tag{18}$$

this can be written in terms of the diagonal elements $\rho_n=\rho_{nn}$:

$$d\rho_n/dt=-A(n+1)\rho_n-Bn\rho_n+An\rho_{n-1}+B(n+1)\rho_{n+1}. \tag{19}$$

With an initially coherent field, for which

$$\rho_n(0)=\bar{n}^n(n!)^{-1}\exp(-\bar{n})\ , \tag{20}$$

the factorial moments after a short time Δt are, to first order in Δt,

$$\langle a^\dagger a\rangle_{\Delta t}=\bar{n}+(A-B)\Delta t\bar{n}+A\Delta t \tag{21}$$

and

$$\langle a^\dagger a^\dagger aa\rangle_{\Delta t}=\bar{n}^2+2\bar{n}^2(A-B)\Delta t+4A\Delta t\bar{n}\ . \tag{22}$$

The excess joint counting probability is given, also to first order in Δt, by

$$\xi(\Delta t)=\langle a^\dagger a^\dagger aa\rangle_{\Delta t}-\langle a^\dagger a\rangle^2{}_{\Delta t}=2A\Delta t\bar{n}\ . \tag{23}$$

The common interpretation of these equations is that the final-state field may be considered as the superposition of two fields.[10] First, there is the incident field, which has been amplified by the factor $1+(A-B)\Delta t$. Second, there is the Gaussian spontaneous emission field whose mean is $A\Delta t$. In this view the excess correlations represent interference effects between the incident light and the spontaneous noise field. [The excess due to the chaotic spontaneous emission alone would be of order $(A\Delta t)^4$.] This interpretation can be maintained for all moments and for all times. Use of the characteristic function of the density matrix leads to the same conclusions.[9,11]

In Ref. 1, certain terms in Eq. (19) were deleted in an attempt to omit the effects of spontaneous processes. The assumption was made that stimulated processes lead to terms of the form $m\rho_m$, and that the effect of spontaneous emission is to change certain such terms to the form $(m+1)\rho_m$. The equation that results from the deletions of Ref. 1 is

$$d\rho_n/dt=-An\rho_n-Bn\rho_n+A(n-1)\rho_{n-1}+B(n+1)\rho_{n+1}\ . \tag{24}$$

Noting that, except when N_a vanishes, an initially coherent field whose time evolution is described by this equation will develop excess correlations, the authors conclude that stimulated processes will in general change the statistical properties of the incident radiation.

Let us consider the implications which the assumed identification of terms has for the statistical effect of spontaneous emission alone. We assume that $N_b=0$, and delete the terms in Eq. (19) which Ref. 1 would have us associate with stimulated emission. Spontaneous emission is then the only interaction process assumed possible. Then we have

$$\begin{aligned}\rho_n(\Delta t)&=\rho_n(0)-A\Delta t\rho_n(0)+A\Delta t\rho_{n-1}(0)\ ,\\ \langle a^\dagger a\rangle_{\Delta t}&=\langle n\rangle_{\Delta t}=(1-A\Delta t)\langle n\rangle_0+A\Delta t\langle n+1\rangle_0\\ &=\langle n\rangle_0+A\Delta t\end{aligned} \tag{25}$$

and

$$\begin{aligned}\langle a^\dagger a^\dagger aa\rangle_{\Delta t}&=\langle n(n-1)\rangle_{\Delta t}\\ &=(1-A\Delta t)\langle n(n-1)\rangle_0+A\Delta t\langle(n+1)n\rangle_0\\ &=\langle n(n-1)\rangle_0+2A\Delta t\langle n\rangle_0\ .\end{aligned} \tag{26}$$

Thus, to first order in Δt,

$$\xi(\Delta t)=\langle n(n-1)\rangle_0-\langle n\rangle_0^2=\xi(0)\ . \tag{27}$$

This implication of Ref. 1, that to first order in Δt there is no change in the level of excess correlations, contradicts the conclusions of Eqs. (6) and (16). Superposition of randomly phased fields necessarily leads to excess correlations proportional to the product of the intensities of the two fields. The identification of terms made in Ref. 1 would imply that spontaneous emission never interferes with an incident beam.

In our view, the density matrix terms identified in Ref. 1 as describing the effects of stimulated emission are also influenced by interference between spontaneous emission and the original field. The conclusion that there is a cause-and-effect relationship between stimulated emission and the HBT effect is thus not logically justified.

We may seek alternatively to identify terms in the description of the evolution of the field density matrix in terms of the P representation rather than the number representation. Equation (9) leads to[21,7]

$$\frac{\partial P(\alpha,t)}{\partial t} = -\frac{(A-B)}{2}\left(\frac{\partial}{\partial\alpha}[\alpha P(\alpha,t)] + \frac{\partial}{\partial\alpha^*}[\alpha^* P(\alpha,t)]\right) + A\frac{\partial^2}{\partial\alpha\partial\alpha^*}P(\alpha,t) \ . \quad (28)$$

Here there are no terms proportional to n or to $(n+1)$. In order to gain insight into the meaning of terms in this equation, we calculate the mean intensity at time Δt. Integrating by parts, and assuming that $P(\alpha)\to 0$ as $|\alpha|\to\infty$, we find

$$\langle a^\dagger a\rangle_{\Delta t} = \int |\alpha|^2 P(\alpha,\Delta t)\,d^2\alpha = \bar{n}_0 + (A-B)\Delta t\bar{n}_0 + A\Delta t \ , \quad (29)$$

where

$$\bar{n}_0 = \int |\alpha|^2 P(\alpha,0)\,d^2\alpha \ . \quad (30)$$

The second term in Eq. (29) represents the stimulated contribution to the intensity, since it is proportional to the incident field intensity. The last term represents the spontaneous contribution. On the basis of this association, stimulated effects are assumed, in both Eqs. (28) and (29), to be given by terms proportional to $(A-B)$. Spontaneous effects are described by terms proportional to A alone.[22]

We will now consider the effects of the terms in Eq. (28) separately. If we drop the last term on the right-hand side, and retain only the "drift" term, the equation has a Green's function solution[23]

$$P(\alpha,t\,|\,\beta,0) = \delta^{(2)}(\alpha - \beta e^{\gamma t/2}) \ , \quad (31)$$

where $\gamma = (A-B)$.

We see that an initial superposition of coherent states

$$\rho(0) = \int P(\beta,0)\,|\beta\rangle\langle\beta|\,d^2\beta \quad (32)$$

has the time evolution

$$\rho(t) = \int P(\alpha,t)\,|\alpha\rangle\langle\alpha|\,d^2\alpha \ , \quad (33)$$

where

$$P(\alpha,t) = \int P(\alpha,t\,|\,\beta,0)P(\beta,0)\,d^2\beta = P(\alpha e^{-\gamma t/2},0) \ . \quad (34)$$

Thus, the stimulated effects term alone causes only a scale change in the α plane. It follows that

$$\frac{\partial}{\partial t}\int |\alpha|^{2n}P(\alpha,t)\,d^2\alpha = n\gamma\int |\alpha|^{2n}P(\alpha,t)\,d^2\alpha \ . \quad (35)$$

From this, it is easily shown that the normalized moments do not change with time:

$$\frac{\partial}{\partial t}\frac{\int|\alpha|^{2n}P(\alpha,t)\,d^2\alpha}{(\int|\alpha|^2P(\alpha,t)\,d^2\alpha)^n} = 0 \ . \quad (36)$$

Only the intensity changes.

The preceding is the standard result for an absorber[19] (for which $A=0$), but we see that this behavior results from stimulated emission as well as stimulated absorption. We thus have retained the semiclassical symmetry between these two processes.

If only the term proportional to A is kept in Eq. (28), it is readily shown that the solution is

$$P(\alpha,t) = \int P(\alpha',0)[\pi n(t)]^{-1} \times \exp[-|\alpha-\alpha'|^2/n(t)]\,d^2\alpha' \ , \quad (37)$$

where $n(t) = At$. In this case the output field is seen to be simply the direct superposition of the initial field with an independent Gaussian field which can be explained simply as spontaneously generated noise.

If we keep both terms in Eq. (28), the solution may be written[23]

$$\begin{aligned} P(\alpha,t\,|\,\beta,0) &= [\pi n(t)]^{-1}\exp[-|\alpha-\bar{\alpha}(t)|^2/n_T(t)] \\ &= [\pi n(t)]^{-1}\int \exp[-|\alpha-\alpha'|^2/n_T(t)] \times \delta^{(2)}[\alpha'-\bar{\alpha}(t)]\,d^2\alpha' \ , \end{aligned} \quad (38a)$$

where

$$\bar{\alpha}(t) = \beta\exp(\gamma t/2) \ ,$$
$$n_T(t) = A(A-B)^{-1}[\exp(\gamma t) - 1] \ . \quad (38b)$$

Thus, in general,

$$P(\alpha,t) = [\pi n_T(t)]^{-1}\int \exp[-|\alpha-\alpha'|^2/n_T(t)] \times P(\alpha' e^{-\gamma t/2},0)\,d^2\alpha' \ . \quad (39)$$

This result differs slightly from those based on a consideration of the two terms separately. The Gaussian noise field has now been amplified by stimulated effects as well as increased by the addition of spontaneous emission. The general picture which we have presented still stands, however, in that the incremental effect of stimulated processes is only to change the mean intensity of the incident field, while spontaneous emission adds Gaussian noise.[24]

In our view, the $(n+1)\rho_n$ that appears in the number representation analysis is a seductive enticement to an erroneous assignment of cause and effect. Having found in the P representation a different basis for assigning terms to stimulated and spontaneous effects, we transform our results back into the number representation, using Eq. (28) and the relation

$$\rho_n = \int |\alpha|^{2n}(n!)^{-1} \exp(-|\alpha|^2)P(\alpha)d^2\alpha \ . \qquad (40)$$

The result is

$$d\rho_n/dt = (A-B)[n\rho_n - (n+1)\rho_{n+1}] + A[(n+1)\rho_{n+1} - (2n+1)\rho_n + n\rho_{n-1}] \ . \qquad (41)$$

This equation is equivalent to Eq. (19).

Equation (41) could have been generated from Eq. (19) simply by insisting that all terms proportional to B must appear in the combination $(A-B)$. As our P representation discussion showed, or as may easily be confirmed by direct calculation of the first two factorial moments at time Δt, these terms describe stimulated emission and absorption in a sense that exactly parallels the classical and semiclassical viewpoint. The remaining terms, which are proportional to A alone, represent spontaneous emission effects.

If, as we urge, the identification of stimulated emission with terms in the formalism is made in a manner consistent with the maser amplifier literature we have discussed, the conclusions of Ref. 1 no longer follow. The terms retained in going from Eq. (19) to Eq. (24) do not describe stimulated emission alone. Using our identification of terms, stimulated emission is found to be no more involved in the production of photon correlations than is absorption in their removal. In particular, stimulated emission then can not be said to produce correlations when the incident field is initially fully coherent.

It should be pointed out that, in general, any pretense of distinction between the processes of stimulated and spontaneous emission can be maintained only in first-order calculations. (For example, Kimble and Mandel have presented more general results.[25]) In the spirit that the first-order distinction is at least pedagogically useful, however, we offer the interpretations made here as being consistent not only with ideas of power gain in the interaction of field and atom, but also with ideas of coherence and interference.

THE THERMODYNAMIC ARGUMENT

Webb has made an argument for a connection between photon correlations and stimulated emission on grounds entirely independent of those discussed above.[2] Following Einstein's derivation[12] of Planck's spectral distribution formula for blackbody radiation, he begins with the equilibrium statement

$$N_n(A_{nj} + B_{nj}\rho_\nu) = N_j B_{jn}\rho_\nu \ . \qquad (42)$$

Here ρ_ν is the density of radiation in the enclosure and N_n and N_j are the numbers of oscillators in states n and j (with energies $E_n > E_j$). A_{nj} is the spontaneous transition rate and the two stimulated emission coefficients are B_{nj} and B_{jn}.

Equation (42), together with a density-of-states factor and the Boltzmann population ratio leads to the Planck formula

$$\rho_\nu d\nu = (8\pi h\nu^3/c^3)[\exp(h\nu/kT) - 1]^{-1}d\nu \ . \qquad (43)$$

Application of the Einstein-Fowler fluctuation equation,

$$\langle(\Delta E_\nu)^2\rangle = kT^2(\partial E_\nu/\partial T) \ , \qquad (44)$$

and use of the quantum relationship $E = nh\nu$ then gives, for a single cell of phase space, the usual expression for photon statistics:

$$\langle(\Delta n)^2\rangle = \bar{n}^2 + \bar{n} \ . \qquad (45)$$

Returning to Eq. (42), Webb then claims to remove the effects of the stimulated emission process by dropping the B_{nj} term from Eq. (42). There results the Wien spectral law in place of the Planck law. The final result on photon statistics becomes

$$\langle(\Delta n)^2\rangle = \bar{n} \ . \qquad (46)$$

Webb then notes that "... the fluctuations of radiation derived in this case, with the stimulated emission term omitted, give only the one term $\bar{n}$ which is characteristic of particles alone." (The $\bar{n}^2$ term corresponds to the fluctuations of energy density characteristic of waves alone.)

On the basis of a comparison of Eqs. (45) and (46), Webb concludes that "... the process of stimulated emission is responsible for photons obeying Bose-Einstein statistics in blackbody radiation." Since we have argued that the HBT effect which is evidenced in the Bose-Einstein photon statistics of blackbody radiation is due to random spontaneous emission rather than to stimulated emission, these statements require reconciliation.

In our view, removing the B_{nj} term in Eq. (42) does not, as Webb's logic requires, produce a description of light which differs from the usual theory only in that the possibility of stimulated emission has been eliminated. No theory of light which has the necessary wavelike behavior in the classical limit can be described by such an equation. Time reversal connects the processes of stimulated emission and absorption in any classical theory which uses such terms as $\vec{v}\cdot\vec{E}$ to describe the exchange of energy between field and matter. Hermitian conjugation similarly connects these two processes in the usual quantum theory. In neither realm can the possibility of stimulated emission be removed and that of stimulated absorption retained without fundamentally altering the underlying physical assumptions. The comparison which has been made is thus between a thermodynamic equation based on the usual theory of light, in which a dual wave and particle nature is exhibited, and one based on a theory which may display no wave nature at all. It is not surprising that the statistical results in the latter case are those expected on the basis of a classical particle theory. The excess correlations characteristic of the HBT effect in thermal light or, in the language of Ref. 2, of Bose-Einstein statistics, result from the wave nature of light, not simply from the stimulated emission process.

It should be noted that the basic result of Webb's original thermodynamic argument follows solely from the temperature dependence of the portion of Eq. (43) which is included within the square brackets. The statistical result of Eq. (45) applies mode-by-mode and in no way requires the final spectral distribution of Eq. (43), which includes the density-of-states factor. The determining assumption, brought in implicitly with the use of the Einstein-Fowler equation, is that of thermodynamic equilibrium within one mode, not the more general equilibrium between many modes which leads to the spectral result.

In equilibrium, each mode of the field within a blackbody cavity displays the intensity fluctuations leading to Eq. (45). This behavior is to be expected on the basis of our earlier discussion. Since spontaneous emission always transfers energy from the atoms to the field, the net effect of stimulated emission and absorption in equilibrium situations will be to extract energy from the field. This energy is removed coherently (in the sense of decreased mean and unchanged normalized moments). The energy contributed by spontaneous emission is added chaotically. It follows that any specified initial state of a given mode will tend ultimately toward Gaussian statistics, and that the equilibrium state will satisfy Eq. (45).

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[1]F. A. Vorobev and R. I. Sokolovskii, Opt. Spectrosk. 32, 842 (1972) [Opt. Spectrosc. 32, 446 (1972)]. Mandel (see Ref. 20) discusses an argument similar to that made by Vorobev and Sokolovskii.

[2]J. H. Webb, Am. J. Phys. 40, 850 (1972).

[3]M. Bertolotti, in *Photon Correlation and Light Beating Spectroscopy*, edited by H. Z. Cummins and E. R. Pike (Plenum, New York, 1974), p. 41.

[4]M. Sargent, III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, Mass., 1974), p. 22, 279.

[5]R. J. Glauber, Phys. Rev. 131, 2766 (1963). In this connection see also U. Fano, Am. J. Phys. 29, 539 (1961).

[6]R. Loudon, Phys. Rev. A 2, 267 (1970). See also Ref. 11.

[7]N. Chandra and H. Prakash, Phys. Rev. Lett. 22, 1068 (1969). Further work by the same authors appears in Ind. J. Pure Appl. Phys. 9, 409 (1971).

[8]K. Shimoda, H. Takahasi, and C. H. Townes, J. Phys. Soc. Jpn. 12, 686 (1957).

[9]J. P. Gordon, L. R. Walker, and W. H. Louisell, Phys. Rev. 130, 806 (1963).

[10]R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1973), pp. 232–241.

[11]S. Carusotto, Phys. Rev. A 11, 1629 (1975).

[12]A. Einstein, Phys. Z. 18, 121 (1917) [English translation: *Sources of Quantum Mechanics*, edited by B. L. van der Waerden (Dover, New York, 1967), p. 63].

[13]See, for example, Loudon (Ref. 10), p. 96.

[14]We will not be concerned with fields that have no P representation, but note that all fields which have a classical analog can be so described.

[15]Gerard Lachs, Phys. Rev. 138, B1012 (1965). For discussions of an experiment see Hans Morawitz, Phys. Rev. 139, A1072 (1965); D. B. Scarl, Phys. Rev. 175, 1661 (1968); and G. Present and D. B. Scarl, Appl. Opt. 11, 120 (1972).

[16]J. P. Gordon, W. H. Louisell, and L. R. Walker, Phys. Rev. 129, 481 (1963).

[17]T. von Foerster and R. J. Glauber, Phys. Rev. A 3, 1484 (1971).

[18]An interpretation of experimental results presented by E. B. Aleksandrov and V. M. Kulyasov, Zh. Eksp. Teor. Fiz. 55, 766 (1968) [Sov. Phys. JETP 28, 396 (1969)] contradicts our viewpoint here. Further experiments and modifications in interpretation are reported by Aleksandrov and Kulyasov in Zh. Eksp. Teor. Fiz. 56, 784 (1969) [Sov. Phys.-JETP 29, 426 (1969)] and Zh. Eksp.Teor. Fiz. 61, 2259 (1972) [Sov. Phys.-JETP 34, 1210 (1972)].

[19]Y. R. Shen, Phys. Rev. 155, 921 (1967).

[20]D. B. Scarl and S. R. Smith, Phys. Rev. A 10, 709 (1974). L. Mandel, in a recent Comment [Phys. Rev.

A 14, 2351 (1976)], objects to the language used by Scarl and Smith. Scarl and Smith have presented a short reply to Mandel's Comment [Phys. Rev. A 14, 2355 (1976)]. We take the experimental results simply as proof that stimulated emission does not involve the additional particlelike correlations associated event-by-event with such processes as Compton scattering. (It was this conceptual framework that was employed in the original report.) The present discussion deals with the interpretation of terms involved in calculations whose final results enjoy general acceptance.

[21]Sargent, Scully, and Lamb (Ref. 4), Chap. 16.

[22]This assignment has also been made by Chandra and Prakash (Ref. 7).

[23]W. H. Louisell, in *Quantum Optics, Proceedings of the International School of Physics, "Enrico Fermi", Course XLII*, edited by R. J. Glauber (Academic, New York, 1969), p. 713. (There are erroneously placed signs of inclusion in the relevant equation.) Also see R. J. Glauber, in the same volume, p. 41, and W. H. Louisell, in *Quantum Optics, Proceedings of Scottish University Summer School, 10th; Edinburgh, 1969*, edited by S. M. Kay and A. Maitland (Academic, New York, 1970), p. 198.

[24]When amplified without change in normalized moments, a field which originally had nonzero excess correlations will, of course, be found to have a greater number of photoelectron pairs. This effect is described simply by a scale change in the field variables of the classical and semiclassical pictures. As we have shown, it is describable also by a scale change of the P representation in a quantum description. This effect is qualitatively different from the action of spontaneous emission, which introduces an additional field which in general changes the normalized moments of the intensity distribution function of the total field. Here, as in other places in the literature, confusion is likely to result when the term "photon correlations" is used in any sense other than "photoelectron correlations." In this paper we use the terms without any implied distinction.

[25]H. J. Kimble and L. Mandel, Phys. Rev. A 13, 2123 (1976).