Millimeter spectrum and molecular constants of silicon monoxide*

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A new high-temperature millimeter-wave microwave spectrometer has been constructed and used to measure a wide range of vibrational and rotational states of silicon monoxide. This work results in accurate rest frequencies for all of the intersteller SiO maser transitions that have been observed as well as accurate measurements or predictions of all transitions that are likely to be of astrophysical interest. In addition, the Dunham spectral and potential constants are calculated for the three major isotopic species. For ²⁸Si¹⁶O: $Y_{01} = 21787.453(11)$ MHz, $Y_{11} = -151.026(11)$ MHz, $Y_{21} = 70.5(24)$ kHz, $Y_{02} = -29.38(13)$ kHz, $a_0 = 5.390(22) \times 10^5$ cm⁻¹, $a_1 = -2.9899(41)$, $a_2 = 5.7(7)$, $a_3 = -9.0(5)$. The calculated equilibrium parameters for ²⁸Si¹⁶O are $B_e = 21787.5(10)$ MHz, $\omega_e = 1252.(3)$ cm⁻¹, $\omega_e x_e = 5.96(71)$ cm⁻¹, and $r_e = 1.50973(4)$ Å.

INTRODUCTION

In 1968, Törring¹ reported the laboratory measurement of the $J=0 \rightarrow 1$ rotational transitions of ²⁸Si¹⁶O in the 43-GHz region for four vibrational states, v=0,1,2,3. Meanwhile, a maser signal at 86 243 MHz, observed by radioastronomers² in the nebular source Orion A, has been assigned to the $J=2\rightarrow 1$, v=1 transition of ²⁸Si¹⁶O by Lovas and Johnson³ from a theoretical prediction of the frequency based on Törring's measurement of the $J=0\rightarrow 1$ transition. Since that time, SiO maser emission lines corresponding to the transitions $v=1, J=1\rightarrow 0; v=1, J=2\rightarrow 1; v=1, J=3\rightarrow 2;$ and $v=2, J=1\rightarrow 0$ have been observed in 20 astronomical sources.⁴

With the present work, direct laboratory measurements have been made of the frequencies of all of the transitions which give rise to the important maser signals from outer space, and measurements of the rotational spectrum of ²⁸Si¹⁶O are extended to 303 926.96 GHz. These measurements, shown in Table I, include seven rotational transitions of ²⁸Si¹⁶O in five vibrational states. They allow precise evaluation of the centrifugal distortion effects which influence all rotational and vibrational constants of the molecule and which cannot be evaluated from the $J = 0 \rightarrow 1$ transition alone. These constants make possible the accurate prediction of all transitions that are likely to be of astrophysical interest. Figure 1 shows the laboratory, real-time oscilloscope display of the most commonly observed astrophysical maser, the J=2-1, v=1 transition.

EXPERIMENT

A sufficient concentration of silicon monoxide vapor for microwave absorption measurements was produced by the heating of a homogeneous mixture of silicon and silicon dioxide to $1350 \,^{\circ}$ C in a high-temperature reaction chamber which also served as the microwave absorption cell. This combination of reaction chamber and absorption cell consisted of a Mullite MV30 ceramic tube (9 × 45 cm) concentrically positioned inside a cold-rolled steel pipe (25 × 105 cm) which was connected to a pumping line. For the purpose of mechanical support and thermal insulation, the volume between the ceramic tube and the outer steel jacket was filled with A.P. Green G-26 insulating firebricks. The heating element was constructed from ten graphite welding rods which were electrically connected in series and equally spaced around the inside of the Mullite tube.

A tapered horn combined with a Teflon lens focused the microwave radiation into the reaction chamber and through the vapor above the reactants. The radiation emerging from the reaction zone was then focused onto a sensitive InSb radiation detector operated at 1.6 °K. The detected absorption-line signals, after amplification, were displayed on an oscilloscope for the frequency measurements. Description of the millimeterwave spectrometer and measuring techniques are given in an earlier publication.⁵

DATA AND ANALYSIS

The most accurate formulas for analysis of the rotational spectra of diatomic molecules are provided by Dunham theory.⁶ Dunham solved the Schrödinger wave equation for the diatomic vibrating rotor with a potential function of the form

$$V(\xi) = cha_0\xi^2(1 + a_1\xi + a_2\xi^2 + \cdots), \qquad (1)$$

where $\xi = (r - r_e)/r_e$, $a_0 = \omega_e^2/4B_e$, and a_1, a_2, \ldots are the potential constants. His solution for the rotation-vibration energies is expressed by the

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$J \rightarrow J + 1$	v	Measured frequencies (MHz)	Difference from calculated frequencies (MHz)		
		285	²⁸ Si ¹⁶ O		
$0 \rightarrow 1$	0	13 193 762	0.04		
0 1	1	43 122 032	-0.04		
	2	$42.820.48^{a}$	-0.06		
	3	42 519.34 ^a	0.01		
$1 \rightarrow 2$	0	86846.96	0.07		
	1	86243.37	0.02		
	2	85 640.46	0.09		
$2 \rightarrow 3$	0	130 268.61	0.04		
	1	129363.24	-0.02		
3-+4	0	173688.31	0.16		
	1	$172\ 481.15$	0.09		
	2	171275.28	0.18		
	3	170 070.35	0.07		
$4 \rightarrow 5$	0	217104.98	0.09		
	1	215 595.95	-0.09		
	2	214088.54	-0.06		
	3	212582.60	0.04		
	4	$211\ 077.87$	-0.07		
$5 \rightarrow 6$	0	260 518.02	-0.09		
	1	258707.45	-0.04		
$6 \rightarrow 7$	0	303 926.96	-0.15		
		²⁹ S	i ¹⁶ O		
$0 \rightarrow 1$	0	42 879.82 ^a	0.00		
		$^{30}\mathrm{S}$	i ¹⁶ O		
$0 \rightarrow 1$	0	49 373 34 2	0.00		
J 1		44 010.04	0.00		

TABLE I. Observed transitions of silicon monoxide.

^aReference 1.

series

$$E_{v, J} = h \sum_{l, m} Y_{l, m} (v + \frac{1}{2})^{l} [J(J+1)]^{m}, \qquad (2)$$

in which the coefficients $Y_{l,m}$ are now known as



FIG. 1. Video display of the $J=1\rightarrow 2$, v=1 transition of ${}^{28}\text{Si}{}^{16}\text{O}$.

Dunham's constants. Each $Y_{l,m}$ is a function of the molecular parameters $r_e, a_0, a_1, a_2, \ldots$ which appear in Eq. (1). In terms of the familiar spectroscopic constants,

$$\begin{aligned} Y_{10} &\approx \omega_e , \quad Y_{01} \approx B_e , \quad Y_{11} \approx -\alpha_e , \quad Y_{21} \approx \gamma_e , \\ Y_{20} &\approx -\omega_e x_e , \quad Y_{02} \approx -D_e , \quad Y_{12} \approx -\beta_e , \quad Y_{03} \approx H_e . \end{aligned}$$

With the selection rules $J \rightarrow J + 1$ and $v \rightarrow v$ for pure rotational transitions, Eq. (2) predicts the rotational frequencies to be

$$\nu_{v, J \star v, J+1} = 2[Y_{01} + Y_{11}(v + \frac{1}{2}) + Y_{21}(v + \frac{1}{2})^2 + \cdots](J+1) + 4Y_{02}(J+1)^3 + \cdots,$$
(4)

where only terms which give a detectable contribution to the spectra reported in this paper are retained. For the less abundant isotopic species the Dunham isotope relation,

$$Y'_{l,m} = (\mu/\mu')^{(l+2m)/2} Y_{l,m}, \qquad (5)$$

is accurate to within experimental error for all $Y_{I,m}$ except $Y_{0,1}$. In Eq. (5) μ' denotes the reduced mass of the less abundant species, and the unprimed parameters, $Y_{I,m}$ and μ , refer to the parent species (²⁸Si¹⁶O).

The silicon monoxide microwave spectrum was analyzed by substitution of each measured absorption frequency and the appropriate J and vquantum numbers into Eq. (4). In this manner a

TABLE II. Comparison of molecular constants measured in this work with previous values.

	²⁸ Si ¹⁶ O		²⁹ Si ¹⁶ O		³⁰ Si ¹⁶ O	
Constants	This work ^a	Others	This work	Others	This work	Others
Y ₀₁ (MHz)	21787.453(11)	21 787.462(25) ^b	21 514.046(13)	21 514.074(25) ^b	21259.494(13)	21 259,425(25) ^b
Y ₁₁ (MHz)	-151.026(11)	-151.05(4) ^b	-148.193(11)	-148.22(4) ^b	-145.570(11)	-145.59(4) b
Y_{21} (kHz)	70.5(24)	76.(1) ^b	68.7(24)	74.(1) ^b	67.1(24)	72.(1) ^b
Y_{02} (kHz)	-29.38(13)	$-29.9(1)^{b,c}$	-28.64(13)	$-29.2(1)^{b,c}$	-27.97(13)	-28.5(1) ^{b,c}
B_e (MHz)	21787.5(10)		21 514.1(10)	· ·.	21 259.5(10)	
$\omega_e \ (\mathrm{cm}^{-1})$	1 252.(3)	1241.4(1)	1244.(3)		1237.(3)	
$\omega_e x_e \ (\mathrm{cm}^{-1})$	5.96(71)	5.9(1)	5.89(71)		5.82(71)	

^aNumbers in parentheses represent one standard deviation.

^bReference 1.

^cReference 7.

TABLE III. Potential constants and equilibrium bond length for silicon monoxide.

	This work ^a	Ref. 1
a ₀	$5.390(22) \times 10^5$ cm ⁻¹	$5.298(1) \times 10^5$ cm ⁻¹
a_1	-2.9899(41)	-2.973(2)
a_2	5.7(7)	5.58(10)
a_3	-9.0(5)	-8.32(75)
r_{e}	1.509733(40) Å	1.509732(40) Å

^aErrors are in standard deviation.

set of N simultaneous equations in six unknowns was constructed, where N is the number of experimentally measured frequencies and where the six unknown constants are Y_{01} , Y_{11} , Y_{21} , and Y_{02} for ²⁸Si¹⁶O, plus the Y_{01} values for ²⁹Si¹⁶O and ³⁰Si¹⁶O. By the method of least squares, this set of equations was solved for these constants. The vibrational constants Y_{10} and Y_{20} were calculated for each isotopic species by use of the following relations:

$$Y_{10} = (4Y_{01}^3 / - Y_{02})^{1/2}$$
(6)

and

$$Y_{20} = \frac{3}{2} Y_{01} \left(a_2 - \frac{5}{4} a_1^2 \right) , \tag{7}$$

where a_1 and a_2 are the Dunham potential constants which appear in Eq. (1).

RESULTS AND COMMENTS

Table II summarizes the results of this analysis and gives a comparison of the rotational, vibrational, and potential constants derived from our measurements with those obtained by others.

TABLE IV. Predicted rotational spectrum of silicon monoxide.

		²⁸ Si	²⁸ Si ¹⁶ O		²⁹ Si ¹⁶ O		³⁰ Si ¹⁶ O	
	Transition	Predicted		Predicted		Predicted		
2	$J \rightarrow J'$	frequency	Uncertainty	frequency	Uncertainty	frequency	Uncertainty	
($0 0 \rightarrow 1$	43423.798	0.015	42879.820	0.025	$42\ 373.340$	0.025	
]	$1 0 \rightarrow 1$	43122.027	0.015	42583.710	0.028	42 082.469	0.028	
2	$2 0 \rightarrow 1$	42820.539	0.015	42287.874	0.030	41791.866	0.030	
i	$3 0 \rightarrow 1$	$42\ 519.332$	0.015	41992.314	0.030	$41\ 501.531$	0.030	
4	$4 0 \rightarrow 1$	42218.407	0.026	$41\ 697.028$	0.035	41211.465	0.034	
5	$5 0 \rightarrow 1$	41917.764	0.052	41402.018	0.053	$40\ 921.667$	0.052	
($1 \rightarrow 2$	86846.891	0.028	85758.953	0.050	84746.009	0.050	
-	$1 1 \rightarrow 2$	86243.350	0.020	85166.732	0.056	84164.266	0.056	
2	$1 \rightarrow 2$	85640.373	0.022	84575.061	0.060	83 583.060	0.060	
ć	$1 \rightarrow 2$	85037.959	0.025	83983.940	0.060	83 002.391	0.060	
4	$4 1 \rightarrow 2$	84436.110	0.052	83 393.369	0.070	$82\ 422.259$	0.069	
ŧ	$5 1 \rightarrow 2$	83834.824	0.103	82803.348	0.106	81842.663	0.103	
($2 \rightarrow 3$	130268.574	0.037	$128\ 636.710$	0.076	127117.335	0.076	
]	$1 2 \rightarrow 3$	129363.262	0.027	127748.379	0.085	126244.721	0.085	
2	$2 2 \rightarrow 3$	128458.796	0.030	126860.873	0.092	125372.912	0.092	
3	$3 2 \rightarrow 3$	127555.176	0.034	125974.192	0.092	$124\ 501.908$	0.092	
4	$4 2 \rightarrow 3$	126652.402	0.075	125088.335	0.105	123631.710	0.104	
Ę	$5 2 \rightarrow 3$	125750.473	0,153	124203.303	0.158	122762.317	0.154	
($3 \rightarrow 4$	173688.142	0.046	171512.406	0.104	169486.647	0.104	
1	$1 3 \rightarrow 4$	172481.060	0.035	170327.964	0.118	$168\ 323.162$	0.117	
2	$2 3 \rightarrow 4$	171275.105	0.041	169144.623	0.127	167160.750	0.127	
ŝ	$3 3 \rightarrow 4$	170070.278	0.045	167 962.381	0.126	165999.411	0.126	
4	$4 3 \rightarrow 4$	168866.579	0.098	166781.239	0.142	164839.147	0.140	
5	$5 3 \rightarrow 4$	167664.008	0.202	165601.196	0.210	163679.957	0.205	
($4 \rightarrow 5$	217 104.891	0.058	214 385.351	0.138	211 853.275	0.137	
1	$1 4 \rightarrow 5$	215596.037	0.052	212 904,800	0.157	210 398,918	0.156	
2	$2 4 \rightarrow 5$	214 088.594	0.061	211 425.623	0.168	208 945,902	0.167	
	$3 4 \rightarrow 5$	212 582.560	0.062	209 947 .820	0.166	207494.230	0.165	
2	$4 \rightarrow 5$	211 077,936	0.122	208 471 .392	0.183	206 043.899	0.181	
5	5 4 → 5	209 574 722	0.249	206 996 339	0.264	204 594.912	0.258	
ĺ	$5 \rightarrow 6$	260 518 114	0.084	257 254 860	0.181	254 216.546	0.180	
1	$5 \rightarrow 6$	258 707.489	0.086	255 478,198	0.206	252 471,317	0.204	
2	$2 5 \rightarrow 6$	256 898.557	0.095	253 703.185	0.219	250 727,699	0.217	
-	$5 \rightarrow 6$	255 091.317	0.094	251 929,823	0.216	248 985,692	0.214	
4	$4 5 \rightarrow 6$	253 285 768	0.152	250 158 109	0.232	247 245 296	0.228	
ŗ	$5 5 \rightarrow 6$	251 481 912	0.298	248 388 046	0.322	245 506 510	0.315	
,		TOT JOT 914	0.000	210 000.010	0.000	210 000.010	0.010	

Törring¹ could not determine the effects of centrifugal distortion from his microwave measurements alone; but, by combining his accurate measurements of the J = 0 - 1 transition with the earlier measurements of the vibrational spectra of SiO by Lagerquist and Uhler,⁷ he was able to make corrections for these effects in his derived constants. We believe that the vibrational constant ω_e , which we have derived entirely from microwave measurements (see Table II), is more accurate than the values obtained more directly from vibrational spectra. The inclusion of Törring's frequencies for the $J = 0 \rightarrow 1$ transitions with our results did not alter the constants derived from our results alone. We conclude that his measurements are at least as accurate as ours.

Table III gives the Dunham potential constants a_0, a_1, a_2, a_3 which were calculated from the $Y_{l,m}$'s of Table II. The Y_{12} constant, which is necessary for the computation of the constants a_2 and a_3 but not recorded in Table II, was found

to be zero within the limits of experimental error. A contribution to the uncertainty in the a_i from a calculated upper limit for Y_{12} is included in our estimates of the uncertainty in the a_i . This contribution, as well as other contributions, was neglected in Törring's uncertainties. Although our listed uncertainties are somewhat larger than those quoted for previous work, our a_i values are, in fact, substantially more accurate. The values for B_e which appear in Table II were obtained from Y_{01} by application of the Dunham correction. The wobble-stretch correction is zero to within experimental error, but a calculated upper limit on it is included in the uncertainty.⁵

Table IV shows a set of predictions of all transitions that appear to be of possible astrophysical interest as well as the uncertainties in these predictions. For the higher v, J states, these predictions are comparable in accuracy with the experimental observations, but at low v, J they are substantially better.

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FIG. 1. Video display of the $J=1\rightarrow 2$, v=1 transition of $^{28}{\rm Si}^{16}{\rm O}$.