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Surface modes in electron plasmas

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Density fluctuations in a semi-infinite collisional electron plasma are studied using "two-stream" distribution functions. Microscopic boundary conditions, imposed separately upon each of the velocity streams, are formulated with the inclusion of both specular reflection and diffuse scattering contributions. The strong damping of surface modes noted by previous investigators is shown to be, in part, a consequence of the use of pure specular reflection boundary conditions. With increasing diffuse scattering contributions the surface modes are seen to become an enhanced and identifiable spectral feature.

I. INTRODUCTION

All natural and laboratory plasmas are in some fashion bounded. In the study of the interaction of charged particles and of electromagnetic radiation with the collective modes of a plasma, one should in principle take into consideration not only the bulk modes, but also those modes localized in the region of the plasma surface. As will be shown here the inclusion of these localized, or surface, modes may lead to predictions significantly different from those of infinite medium theories.

Two quantities of general interest in the study of plasmas are the electron and charge density fluctuation spectra. These spectra pertain to such problems as plasma diagnostics via laser light scattering, the energy loss of charged particles in plasmas, and the rf heating of plasmas. In this context one would expect surface modes to be of importance. First, the initial interaction between the plasma and incident particle occurs in the boundary region. Secondly, for low-energy charged particles and photons the actual penetration into, and therefore interaction with, the bulk is small so that the effects of the surface modes may even become dominant.

The manifestation of surface modes may be viewed, for conceptual convenience, as being either direct or indirect. In their direct manifestation surface modes are exhibited as a distinct feature of the fluctuation spectrum. Experimentally, such phenomena as anomalous plasma heating[1] and the decoupling of source radiation and imploding plasma in fusion experiments[2] may be direct manifestations of surface modes. Indirectly, surface modes may be manifest through a modification of the plasma bulk mode properties. This should be of particular importance in the case of long wavelength surface modes for which the penetration into the bulk of the plasma is large. Additionally, bulk modes may be either enhanced or diminished, depending upon conditions at the boundary, as a result of the reflection of these bulk modes at the boundary.[3] There is also the possibility of interference between bulk and surface modes of the same frequency.[3]

Surface modes have been studied at the macroscopic level by several investigators.[4-6] A number of more recent investigations have dealt with microscopic theories of the dynamics of surface modes in classical fully ionized plasmas.[7-9] These investigations are similar in that they consider electrostatic surface modes in Vlasov plasmas with pure specular reflection of particles at the boundary. The results of these theories are therefore essentially the same. At long wavelengths, the electrostatic surface modes for a semi-infinite plasma propagate at a frequency of approximately $\omega_p/\sqrt{2}$, where ω_p is the plasma frequency. Thermal effects contribute a dispersion correction which is linear in $k_{\parallel}$, the component of the wave vector parallel to the surface. This is to be compared with the quadratic wave number dispersion correction for the bulk modes.[10] A significant feature of these theories is the prediction of strong damping of the surface modes in comparison to those of the bulk, this strong damping persisting to relatively short wavelengths. The solid-state analog of this problem, that of surface plasmons in metals, has also been studied at the microscopic level and has yielded qualitatively similar results.[11-15]

In this paper we report on results of a study of surface modes in a classical semi-infinite plasma which incorporates more realistic boundary conditions and the effects of collisions. We consider specifically an electron plasma sharply bounded by a dielectric, for example, a neutral gas. The microscopic boundary conditions are assumed to contain both specular reflection and diffuse scattering contributions. In addition to the nonspecular reflection of a particular electron incident upon the boundary, the diffuse scattering contribution is included to also simulate such physically realistic effects as the reentry into the plasma of secondary electrons produced by such atomic processes as ionization and charge exchange in which the reentrant electron velocity is virtually uncorrelated with that of the original incident electron. A basic assumption of the model is that all of these boundary processes may be considered as localized in space, i.e., to occur in the $z=0$ plane.

In Sec. II we give a brief formulation of the kinetic theory description of the semi-infinite electron plasma with a full-conservation description of electron-electron collisions. The electron density fluctuation spectrum is calculated by use of two-stream distribution functions which describe particles going towards and away from the boundary and by use of a direct variational method which admits trial functions which are discontinuous in velocity space. This approach is new and is formulated in detail in Sec. III. The choice of variational trial functions and the boundary conditions to be imposed upon each of the two velocity streams are discussed in Sec. IV. Computational details are discussed briefly in Sec. V. Results of the calculations are discussed in Sec. VI. In Sec. VII, we offer some concluding remarks and suggestions for further study.

II. KINETIC THEORY FORMULATION

We consider a uniform electron plasma occupying the region $z>0$. The starting point of our theory is the linearized kinetic equation

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}_r\right)f(\vec{v}\vec{r}t)+\frac{ne}{m}\vec{\nabla}_r\varphi(\vec{r}t)\cdot\frac{\partial f_0(v)}{\partial\vec{v}}=\hat{J}(\vec{v}\,|\,\vec{v}')f(\vec{v}'\vec{r}t) \tag{2.1}$$

for the perturbed distribution function $f(\vec{v}\,\vec{r}t)$, where $\varphi(\vec{r}t)$ is the electric potential, which is a functional of f, and $\hat{J}(\vec{v}\,|\,\vec{v}')$ is the collision operator which accounts for individual particle interactions. Our notation is that an integration over the repeated primed velocity variable is implied. The equilibrium distribution $f_0(v)$ is given in terms of the electron thermal velocity $a=k_BT/m$ according to

$$f_0(v)=n\Phi(v)=n(1/2\pi a^2)^{3/2}e^{-v^2/2a^2}. \tag{2.2}$$

Defining a time and two-dimensional space transform according to

$$f(\vec{v}k_\parallel z\sigma)=\int_0^\infty dt\int d^2\vec{r}_\parallel\, e^{-\sigma t}e^{-i\vec{k}_\parallel\cdot\vec{r}_\parallel}f(\vec{v}\,\vec{r}t), \tag{2.3}$$

Eq. (2.1) may be written

$$\left(\sigma+ik_\parallel v_\parallel+v_z\frac{\partial}{\partial z}\right)f(\vec{v}k_\parallel z\sigma)-\frac{ne}{k_BT}\left(ik_\parallel v_\parallel+v_z\frac{\partial}{\partial z}\right)\varphi(k_\parallel z\sigma)-\hat{J}(\vec{v}\,|\,\vec{v}')f(\vec{v}'k_\parallel z\sigma)=X(\vec{v}k_\parallel z), \tag{2.4}$$

where $X(\vec{v}k_\parallel z)=f(\vec{v}k_\parallel z, t=0)$ and where we've assumed $\hat{J}(\vec{v}\,|\,\vec{v}')$ to be local and Markovian. In terms of the distribution function the transformed potential is given by[7]

$$\varphi(k_\parallel z\sigma)=-2\pi e\int d^3\vec{v}'\int_0^\infty dz'\frac{e^{-k_\parallel|z-z'|}}{k_\parallel}f(\vec{v}'k_\parallel z'\sigma). \tag{2.5}$$

Equations (2.5) and (2.4) may be combined to give

$$\left(\sigma+ik_\parallel v_\parallel+v_z\frac{\partial}{\partial z}\right)f(\vec{v}k_\parallel z\sigma)+\tfrac{1}{2}k_D^2\Phi(v)\int d^3\vec{v}'\times\int_0^\infty dz'[iv_\parallel+v_z\,\mathrm{sgn}(z-z')]e^{-k_\parallel|z-z'|}f(\vec{v}'k_\parallel z'\sigma)-\hat{J}(\vec{v}\,|\,\vec{v}')f(\vec{v}'k_\parallel z\sigma)=X(\vec{v}k_\parallel z). \tag{2.6}$$

The initial value appropriate to the calculation of the fluctuation spectrum is[16]

$$X(\vec{v}k_\parallel z)=\Phi(v)\left[\delta(z)-\frac{4\pi e^{-(k_D^2+k_\parallel^2)^{1/2}}}{k_\parallel+(k_D^2+k_\parallel^2)^{1/2}}\right]\equiv\Phi(v)\overline{X}(z). \tag{2.7}$$

The density fluctuations may be obtained from the solution of (2.6) via the relation

$$n(k_\parallel k_\perp\omega)=\mathrm{Re}\int_0^\infty dz\,e^{-ik_\perp z}\int d^3\vec{v}f(\vec{v}k_\parallel z,\sigma=-i\omega). \tag{2.8}$$

It should be noted that for the bounded system under consideration $n(k_\parallel k_\perp\omega)$ is not identical to the dynamic structure function $S(k,\omega)$.

For the description of collisions we shall employ the full-conservation, or generalized, Fokker-Planck operator[17,18]

$$\hat{J}(\vec{v}\,|\,\vec{v}')=\nu[J_B(\vec{v})\delta(\vec{v}'-\vec{v})+\Phi(v)K(\vec{v}\,|\,\vec{v}')] \tag{2.9}$$

where ν is the collision frequency and

$$J_B(v)=\frac{\partial}{\partial\vec{v}}\cdot\left(\vec{v}+a^2\frac{\partial}{\partial\vec{v}}\right) \tag{2.10}$$

is the familiar Brownian motion operator. The remaining operator

$$K(\vec{v}\,|\,\vec{v}')=\vec{v}\cdot\vec{v}'/a^2+\tfrac{1}{2}(v^2/a^2-3)(v'^2/a^2-3) \tag{2.11}$$

gives the momentum and energy conserving backflow terms. Finally, to simplify notation, we write (2.6) in the compact form

$$\hat{L}(\vec{v}z\,|\,\vec{v}'z')f(\vec{v}'k_\parallel z'\sigma)=X(\vec{v}k_\parallel z), \tag{2.12}$$

where the definition of $\hat{L}(\vec{v}z\,|\,\vec{v}'z')$ is obvious.

III. DIRECT VARIATIONAL FORMULATION

In addition to the inclusion of nonspecular contributions to the scattering at the boundary, the present study departs significantly from previous investigations in terms of mathematical and intuitive approach. First, we develop our theory in terms of two-stream distribution functions.[19,20] We introduce the decomposition

$$f(\vec{v}k_{\parallel}z\sigma)=f_{+}(\vec{v}k_{\parallel}z\sigma)H(v_z)+f_{-}(\vec{v}k_{\parallel}z\sigma)H(-v_z)\,, \quad (3.1)$$

where

$$\begin{aligned} f_{+}(\vec{v}k_{\parallel}z\sigma)&=f(v_z>0, v_{\parallel}k_{\parallel}z\sigma)\,,\\ f_{-}(\vec{v}k_{\parallel}z\sigma)&=f(v_z<0, v_{\parallel}k_{\parallel}z\sigma)\,, \end{aligned} \quad (3.2)$$

and $H(v_z)$ is the Heaviside function. Such a decomposition gives us a direct and efficient means of incorporating the effects of the discontinuity at $z=0$.[19,21] We note that the distribution functions f_+ and f_- satisfy a coupled set of equations of the general form (2.12), the coupling arising of course from the interaction of particles in the separate streams.

Secondly, we replace the problem of solving the integro-differential equation (2.12) by an equivalent variational problem. We introduce the functional[22]

$$I[\tilde{f},\tilde{h}]=\langle\!\langle Y,\tilde{f}\rangle\!\rangle+\langle\!\langle \tilde{h},X\rangle\!\rangle-\langle\!\langle \tilde{h},\hat{L}\tilde{f}\rangle\!\rangle + \langle v_z\tilde{h}^s_{+}(\tilde{f}^s_{+}-F^s_{+})\rangle_{v_z>0}+\langle v_z\tilde{h}^s_{-}(\tilde{f}^s_{-}-F^s_{-})\rangle_{v_z<0}\,, \quad (3.3)$$

where we have introduced the compact notation

$$\langle\!\langle a,b\rangle\!\rangle\equiv\int d^3\vec{v}\int_0^{\infty}dz\,a^*b \quad (3.4)$$

and

$$\langle a,b\rangle_{v_z>0}\equiv\int_{v_z>0}d^3\vec{v}\,a^*b\,. \quad (3.5)$$

The function $\tilde{f}=\tilde{f}_{+}H(v_z)+\tilde{f}_{-}H(-v_z)$ is a trial approximation to the solution f of (2.12), while the functions $\tilde{f}^s_{+}$ and $\tilde{f}^s_{-}$ are the values assumed by the two-stream components on the boundary. The quantities F^s_{+} and F^s_{-} are the "physically imposed" microscopic boundary conditions for the system. The first observation to be made in connection with (3.3) is that for $\tilde{f}$ exactly satisfying (2.12) and the physically imposed boundary conditions, then the functional assumes the value

$$I[f,\tilde{h}]=\langle\!\langle Y,f\rangle\!\rangle\,. \quad (3.6)$$

More specifically for $Y=e^{ik_{\perp}z}$, then

$$I[f,\tilde{h}]=\int_0^{\infty}dz\,e^{-ik_{\perp}z}\int d^3\vec{v}f(\vec{v}k_{\parallel}z\sigma)=n(k_{\parallel}k_{\perp}\sigma)\,, \quad (3.7)$$

which in the limit $\sigma=-i\omega$ is the desired density fluctuation spectrum. This result is independent of $\tilde{h}$ which is the trial approximation to the solution of the adjoint equation

$$\left(\sigma^*-ik_{\parallel}v_{\parallel}-v_z\frac{\partial}{\partial z}\right)h(\vec{v}z)-\tfrac{1}{2}k_D^2\int d^3\vec{v}' \times\int_0^{\infty}dz'\,\Phi(v')[iv'_{\parallel}+v'_z\,\mathrm{sgn}(z'-z)]e^{-k_{\parallel}|z'-z|}h(\vec{v}'z') - J(\vec{v}\,|\,\vec{v}')h(\vec{v}'z)=Y(z) \quad (3.8)$$

where we use the fact that the generalized Fokker-Planck operator is self-adjoint. In the case that neither $\tilde{f}$ nor $\tilde{h}$ is exact, the functional I is seen to constitute a variational principle. Its stationary value is $n(k_{\parallel}k_{\perp}\sigma)$ and its first-order variations vanish subject to the Euler-Lagrange equations (2.12) and (3.8) and to the physically imposed discontinuous boundary conditions.[22] In the general case we find that

$$I[\tilde{f},\tilde{h}]=n(k_{\parallel}k_{\perp}\sigma)+(\text{second-order terms in }\delta f,\delta h)\,, \quad (3.9)$$

where δf and δh are the errors in the variational trial approximations. It should be noted that the variational trial functions need not be constrained to exactly satisfy the boundary conditions since these will be built into the results through the evaluation of the variational parameters of the problem. Finally, we note that the boundary terms in the functional I are actually boundary conditions on the transverse currents.

IV. TRIAL FUNCTIONS AND BOUNDARY CONDITIONS

To proceed with the variational calculation, we must choose the forms of the required trial functions and specify the microscopic boundary conditions. We consider first the choice of the trial distribution functions for the two velocity streams. For the trial approximations to (2.6) we use the general form, suppressing the $\vec{k}$ and σ dependences,

$$\tilde{f}_{+}(\vec{v}z)=A_1\Psi_1(\vec{v}z)+\sum_{j=2}A^{+}_{j}\Psi^{+}_{j}(\vec{v}z) \quad (4.1)$$

and

$$\tilde{f}_{-}(\vec{v}z)=A_1\Psi_1(\vec{v}z)+\sum_{j=2}A^{-}_{j}\Psi^{-}_{j}(\vec{v}z)\,. \quad (4.2)$$

where the A's are wave-number- and frequency-dependent variational parameters. We've chosen the first term in (4.1) and (4.2) to be identical. These terms will be chosen specifically to represent the behavior of the distribution functions far from the boundary. Physically we expect no intrinsic difference in the distribution of speeds in the two streams well beyond the boundary region. It follows that the functions Ψ_j^+ and Ψ_j^-, for $j \geq 2$, describe the distinctive nature of the two streams in the vicinity of the boundary. Given the choice of the physical character of the first terms, we see that Ψ_j^+ and Ψ_j^- should vanish well outside of the boundary region.

We shall base our choice of the Ψ's on the collisionless form of (2.6). The effects of collisions will be automatically incorporated into the results through the variationally determined set of parameters A. An obvious choice for Ψ_1 is just the solution of the infinite medium Vlasov equation. This standard result is

$$\Psi_1(\vec{v}) = \frac{X_0(k)\Phi(v)}{\sigma + i\vec{k}\cdot\vec{v}} - \frac{k_0^2}{k^2}\frac{X_0(k)}{\epsilon(k\sigma)}\frac{i\vec{k}\cdot\vec{v}\Phi(v)}{\sigma + ik\cdot\vec{v}}\frac{W(\zeta)}{\theta}, \tag{4.3}$$

where W is the plasma dispersion function[23]

$$X_0(k) = 1 - k_D^2/(k^2 + k_D^2) = k^2/(k^2 + k_D^2) \tag{4.4}$$

and the dielectric function $\epsilon(k\sigma)$ is given by

$$\epsilon(k\sigma) = 1 + (k_D^2/k^2)[1 + \zeta W(\zeta)] \ . \tag{4.5}$$

We have also introduced the quantities $\theta = \sqrt{2}\ ika$ and the dimensionless frequency $\zeta = -\sigma/\theta$, where $\zeta = x = \omega/\sqrt{2}\ ka$ at $\sigma \to -i\omega$

The choice of the functions Ψ_j^+ and Ψ_j^- is based upon a somewhat more formal analysis, though it is a motivation rather than a derivation. We note that the transformation

$$f(\vec{v}z) = e^{-\alpha(v)z} g(\vec{v}z)\ , \tag{4.6}$$

where

$$\alpha(v) = (\sigma + ik_\parallel v_\parallel)/v_z \tag{4.7}$$

allows for the formal integration of the collisionless equation. We find first that

$$g(\vec{v}z) = g(\vec{v},0) + \frac{\Phi(v)}{v_z}\int_0^z d\bar{z}\,\bar{X}(\bar{z})e^{\alpha(v)\bar{z}} - \frac{1}{2}k_D^2\frac{\Phi(v)}{v_z}\int_0^z d\bar{z}\int d^3\vec{v}' \times \left\{(iv_\parallel - v_z)e^{(\alpha - k_\parallel)\bar{z}}\int_0^{\bar{z}} dz' e^{-(\alpha' - k_\parallel)z'} + (iv_\parallel + v_2)e^{(\alpha + k_\parallel)\bar{z}}\int_{\bar{z}}^\infty dz'\, e^{-(\alpha' + k_\parallel)z'}\right\} g(\vec{v}'z')\ , \tag{4.8}$$

where $\alpha' = \alpha(v')$. An approximate form for $g(\vec{v}z)$ can be obtained by iteration. Using the first two terms on the right side of (4.8) in the integrands of the last terms and taking the limiting form of the results for $z \to 0$ suggests the following choices of the functions $\Psi_j^\pm$ for $j \geq 2$:

$$\Psi_2^\pm = \frac{4\pi e^{-\lambda z}}{k_\parallel + \lambda}\frac{\Phi(v)}{\sigma + ik_\parallel v_\parallel + \lambda v_z}\ , \tag{4.9}$$

$$\Psi_3^\pm = \frac{4\pi\Phi(v)}{k_\parallel + \lambda}\left(\frac{ik_\parallel v_\parallel - \lambda v_z}{\sigma + ik_\parallel v_\parallel - \lambda v_z}\right)(e^{-\lambda z} - e^{-\alpha z})\ , \tag{4.10}$$

$$\Psi_4^\pm = \frac{1}{k_\parallel}\left(\frac{ik_\parallel v_\parallel - k_\parallel v_z}{\sigma + ik_\parallel v_\parallel - k_\parallel v_z}\right)(e^{-k_\parallel z} - e^{-\alpha z})\ , \tag{4.11}$$

where $\lambda = (k^2 + k_D^2)^{1/2}$. In arriving at Eqs. (4.9)–(4.12) we've simply retained the essential functional forms from the $g(\vec{v}z)$ iteration, using the variational parameters A to effectively replace wave-number and frequency-dependent factors resulting from integrations over $\vec{v}'$. It should be pointed out that although the Ψ_j's are identical for each stream the f's will not be the same by virtue of the differences in A_j^+ and A_j^- resulting both from the half-range integrations and the different boundary conditions imposed upon the two streams.

For the trial functions for the adjoint equation we use the forms

$$\tilde{h}_+(\vec{v}z) = A_1\bar{\Psi}_1 + \sum_{j=2} A_j^+\bar{\Psi}_j^+(\vec{v}z) \tag{4.12}$$

and

$$\tilde{h}_-(\vec{v}z) = A_1\bar{\Psi}_1 + \sum_{j=2} A_j^-\bar{\Psi}_j^-(\vec{v}z)\ . \tag{4.13}$$

The functions $\bar{\Psi}_j$ for the adjoint equation are determined in the same spirit as the functions Ψ for the direct equation. Thus we take $\bar{\Psi}_1$ as the solution to the adjoint infinite-medium Vlasov equation. The specific form is

$$\bar{\Psi}_1(\vec{v}) = -\frac{1}{\sigma^* - i\vec{k}\cdot\vec{v}}\left(1 - \frac{k_D^2}{k^2}\frac{1 + \zeta\bar{W}(\zeta)}{\bar{\epsilon}(k\sigma)}\right), \tag{4.14}$$

where $\bar{W}$ and $\bar{\epsilon}$ are the analytic continuations of W and ϵ into $\mathrm{Im}\,\zeta < 0$. The initial value appearing in the infinite-medium adjoint equation appropriate to the variational calculation of the density fluctuations is unity.[24] The motivation of the $\bar{\Psi}_j^\pm$'s follows the lines of the previous development for the $\Psi_j^{\pm}$'s. Here the transformation required for formal integration is

$$h(\vec{v}z) = e^{+\alpha^*(v)z} q(\vec{v}z) \tag{4.15}$$

yielding

$$q(\vec{v}z) = q(\vec{v}, 0) - \frac{1}{v_z}\int_0^z d\bar{z}\, e^{-\alpha^*\bar{z}}\, Y(\bar{z}) - \frac{1}{2}\frac{k_D^2}{v_z}\int_0^z d\bar{z} \times \left\{ e^{-(k_\parallel+\alpha^*)\bar{z}} \int d^3\vec{v}'(iv'_\parallel - v'_z)\int_0^{\bar{z}} dz' e^{(k_\parallel+\alpha'^*)\bar{z}} q(\vec{v}'z') + e^{(k_\parallel-\alpha^*)\bar{z}} \int d^3\vec{v}'(iv'_\parallel + v'_z)\int_{\bar{z}}^\infty dz' e^{-(k_\parallel-\alpha'^*)z'} q(\vec{v}'z') \right\}. \tag{4.16}$$

The analogous first iteration based upon the two leading terms on the right side of (4.16) leads, for small z, to the choices

$$\Psi_2^{\pm} = \frac{e^{ik_\perp z} - e^{\alpha^* z}}{\sigma^* - ik_\parallel v_\parallel - ik_\perp v_z}\ , \tag{4.17}$$

$$\Psi_3^{\pm} = -\tfrac{1}{2}k_D^2 z\, e^{\alpha^* z}/v_z\ , \tag{4.18}$$

$$\Psi_4^{\pm} = -\frac{1}{2}\frac{k_D^2}{k_\parallel - ik_\perp}\frac{z e^{\alpha^* z}}{v_z}\ . \tag{4.19}$$

We now turn to the consideration of boundary conditions. We want to allow for the nonspecular contributions to scattering at the boundary. In general we relate the exact microscopic distribution of particles at $z=0$ leaving the boundary to those incident upon the boundary from within the plasma according to [25-27]

$$v_z F_+^s = \int_{v_z<0} d^3\vec{v}'\, P(\vec{v}\,|\,\vec{v}') F_-^s(\vec{v}') v'_z\ , \tag{4.20}$$

where $P(\vec{v}\,|\,\vec{v})$ is the probability that an incident particle of velocity $\vec{v}'$ is scattered with velocity $\vec{v}$. We take $P(\vec{v}\,|\,\vec{v})$ to have the general form

$$P(\vec{v}\,|\,\vec{v}') = \mu\delta(v'_\parallel - v_\parallel)\delta(v'_z + v_z) + (1-\mu)P_d(\vec{v})\ . \tag{4.21}$$

The first term clearly represents specular reflection, $v'_\parallel \to v_\parallel$ and $v'_z \to -v_z$, in which there is complete memory by the scattered particle of incident velocity. The second term represents the opposite limit, diffuse scattering, in which there is a complete loss of memory by the particle of its incident velocity. Intuitively then we would expect the probability $P_d(\vec{v})$ to be a function characteristic of the boundary itself. The parameter μ may vary from 0 to 1 and describes the relative contribution of specular reflection and diffuse scattering. We choose the diffuse scattering probability as

$$P_d(\vec{v}) = (2\pi)^{1/2}(v_z/a_w)\Phi_w(v)\ , \tag{4.22}$$

where $\Phi_w(v)$ and a_w are the Maxwell distribution and electron thermal velocity respectively, evaluated at the wall temperature T_w.

It remains to specify F_-^s. We base our specification upon the physical notion that for the distribution of incident particles within a free path of the boundary, collision effects may be ignored. One of the motivations for the present approach is that although we can not specify the exact full range microscopic boundary condition, reasonable forms for F_-^s should be obtainable. Physically we would expect F_-^s to be primarily characteristic of the plasma itself, since these particles are incident from within the plasma. In F_-^s we are in the main concerned with particles other than the initial excess thermalized particle at the boundary. Information about the initial condition at the boundary is, however, transmitted to such a typical incident particle through the field particles near the boundary with which it interacts. With this motivation we take F_-^s to satisfy an equation of the form

$$(\sigma + ik_\parallel v_\parallel)F_-^s(\vec{v}) + \tfrac{1}{2}k_D^2\Phi(v)(iv_\parallel + v_z) \times \int d^3\vec{v}' \int_0^\infty dz' e^{-k_\parallel z'} f(\vec{v}'z') = X(0)\ , \tag{4.23}$$

where the distribution $f(\vec{v}z)$ in the integrand describes field particles containing information about the initial disturbance. For the field particle distribution we use a form derived from our small z iteration of (4.8). Specifically (see Appendix) we take

$$F_-^s(\vec{v}) = \frac{X(0)}{\sigma + ik_\parallel v_\parallel} - \frac{1}{2}\frac{k_D^2\Phi(v)(iv_\parallel + v_z)F_0}{\sigma + ik_\parallel v_\parallel}\ , \tag{4.24}$$

where a transverse momentum-transfer balance has been incorporated and where

$$F_0 = -\frac{1}{\theta_\parallel}\frac{8\pi}{k_D^2}\frac{k_\parallel}{k_\parallel + \lambda}\frac{1}{\zeta_\parallel}\ . \tag{4.25}$$

V. DETAILS OF CALCULATION

The computational procedure[28] consists of substituting the forms (4.1), (4.2), (4.12), and (4.13) into (3.3). This gives the result

$$
\begin{aligned}
I[\tilde{f},\tilde{h}] = {} & \sum_{\alpha}\sum_{j}\left[A_j^{\alpha}\langle\langle Y,\Psi_j^{\alpha}\rangle\rangle + (A_j^{\alpha})^{*}\langle\langle\bar{\Psi}_j^{\alpha},X\rangle\rangle\right] \\
& - \sum_{\alpha}\sum_{\beta}\sum_{j}\sum_{l}(A_l^{\beta})^{*}A_j^{\alpha}\langle\langle\bar{\Psi}_l^{\beta},\hat{L}\Psi_j^{\alpha}\rangle\rangle \\
& + \sum_{\alpha}\sum_{\beta}\sum_{j}\sum_{l}(A_l^{\beta})^{*}A_l^{\alpha}\langle v_z\bar{\Psi}_l^{s,\beta},\Psi_l^{s,\alpha}\rangle\delta_{\alpha\beta} \\
& - \sum_{\alpha}\sum_{j}(A_j^{\alpha})^{*}\langle v_z\bar{\Psi}_j^{s,\alpha},F_{\alpha}^{s}\rangle\,, \qquad (5.1)
\end{aligned}
$$

where $j,l=1,2,3,4$ and $\alpha,\beta=\{+,-\}$. We now introduce

$$\Gamma_{lj}^{\beta\alpha} \equiv \langle\langle\bar{\Psi}_l^{\beta},\hat{L}\Psi_j^{\alpha}\rangle\rangle - \langle v_z\bar{\Psi}_l^{s,\beta},\Psi_j^{s,\alpha}\rangle\delta_{\alpha\beta}\,, \qquad (5.2)$$

so that

$$
\begin{aligned}
I[\tilde{f},\tilde{h}] = \sum_{\alpha}\sum_{j}\Big\{ & A_j^{\alpha}\langle\langle Y,\Psi_j^{*}\rangle\rangle \\
& + (A_j^{\alpha})^{*}\left[\langle\langle\bar{\Psi}_j^{*},X\rangle\rangle - \langle v_z\bar{\Psi}_j^{s,\alpha}F_{\alpha}^{s}\rangle\right] \\
& - \sum_{\beta}\sum_{l}(A_l^{\beta})^{*}A_j^{\alpha}\Gamma_{lj}^{\beta\alpha}\Big\}. \qquad (5.3)
\end{aligned}
$$

The coefficients A_j^{α} may be written in terms of their real and imaginary parts as

$$A_j^{\alpha} = B_j^{\alpha} + D_j^{\alpha} \qquad (5.4)$$

where $(D_j^{\alpha})^{*} = -D_j^{\alpha}$. In terms of the B_j^{α}'s and D_j^{α}'s the functional reduces to

$$
\begin{aligned}
I[\tilde{f},\tilde{h}] = \sum_{\alpha}\sum_{j}\Big(& 2(B_j^{\alpha}\gamma_j^{\alpha} + D_j^{\alpha}\bar{\gamma}_j^{\alpha}) \\
& - \sum_{\beta}\sum_{l}(B_j^{\alpha}B_l^{\beta} + D_j^{\alpha}B_l^{\beta} \\
& \qquad - B_j^{\alpha}D_l^{\beta} - D_j^{\alpha}D_l^{\beta})\Gamma_{lj}^{\beta\alpha}\Big)\,, \qquad (5.5)
\end{aligned}
$$

where

$$2\gamma_j^{\alpha} = \langle\langle Y,\Psi_j^{\alpha}\rangle\rangle + \langle\langle\bar{\Psi}_j^{\alpha},X\rangle\rangle - \langle v_z\bar{\Psi}_j^{s,\alpha}F_{\alpha}^{s}\rangle\,, \qquad (5.6)$$

$$2\bar{\gamma}_j^{\alpha} = \langle\langle Y,\Psi_j^{\alpha}\rangle\rangle - \langle\langle\bar{\Psi}_j^{\alpha},X\rangle\rangle + \langle v_z\bar{\Psi}_j^{s,\alpha},F_{\alpha}^{s}\rangle\,. \qquad (5.7)$$

Our actual variation is now performed by the parameter variation scheme of Ritz. We take

$$\frac{\partial I[\tilde{f},\tilde{h}]}{\partial B_j^{\alpha}} = 0\,, \quad \frac{\partial I[\tilde{f},\tilde{h}]}{\partial D_j^{\alpha}} = 0\,, \qquad (5.8)$$

which yields the set of algebraic equations

$$\sum_{\beta}\sum_{l}(B_l^{\beta}\Lambda_{jl}^{\alpha\beta} + D_l^{\beta}\Delta_{jl}^{\alpha\beta}) = \gamma_j^{\alpha}$$

$$(j=1,2,3,4;\ \alpha=+,-)\,, \qquad (5.9)$$

$$\sum_{\beta}\sum_{l}(B_l^{\beta}\Delta_{jl}^{\alpha\beta} + D_l^{\beta}\Lambda_{jl}^{\alpha\beta}) = -\bar{\gamma}_j^{\alpha}$$

$$(j=1,2,3,4;\ \alpha=+,-)\,, \qquad (5.10)$$

where

$$2\Lambda_{jl}^{\alpha\beta} = \Gamma_{jl}^{\alpha\beta} + \Gamma_{lj}^{\beta\alpha}\,, \qquad (5.11)$$

$$2\Delta_{jl}^{\alpha\beta} = \Gamma_{jl}^{\alpha\beta} - \Gamma_{lj}^{\beta\alpha}\,. \qquad (5.12)$$

With the B_j^{α}'s and D_j^{α}'s determined by (5.9) and (5.10) it is easy to show that (5.5) reduces to the relatively simple form

$$I[\tilde{f},\tilde{h}] = \sum_{\alpha}\sum_{j}(B_j^{\alpha}\gamma_j^{\alpha} + D_j^{\alpha}\bar{\gamma}_j^{\alpha})\,. \qquad (5.13)$$

The calculations of the quantities $\Gamma_{lj}^{\beta\alpha}$, γ_j^{α}, and $\bar{\gamma}_j^{\alpha}$ require the evaluation of numerous integrals which, in the limits $k_{\perp}/k_{\parallel} \ll 1$ or $k_{\perp}/k_{\parallel} \gg 1$, can be either explicitly evaluated or expressed in terms of plasma dispersion functions of appropriate arguments.[29] In the present calculation we've used a collision frequency whose dimensionless value y is given by[30]

$$y = i\nu/\theta = 0.00748(k_D/k)g\ln(37.7/g)\,, \qquad (5.14)$$

where $g = k_D^3/n$ is the plasma parameter.

VI. DISCUSSION OF RESULTS

Our basic results are given in terms of the dimensionless density-fluctuation spectrum

$$R(x,y) = (1/2\pi)(1/\sqrt{2}ka)\,\mathrm{Re}\,n(k_{\parallel}k_{\perp},\sigma=-i\omega)\,, \qquad (6.1)$$

where $x=\omega/\sqrt{2}ka$. There are several relevant dimensionless parameters to the problem. In

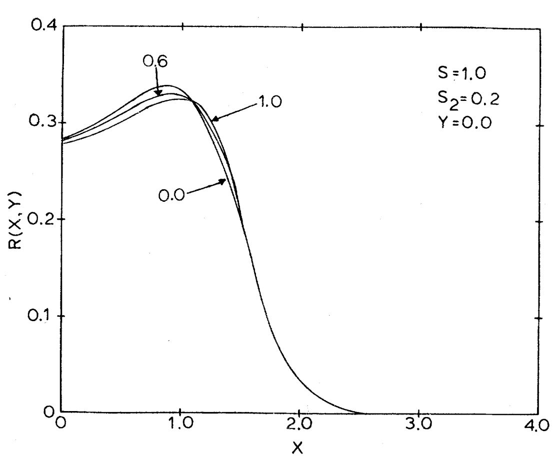

FIG. 1. Dimensionless density fluctuation spectrum for bounded electron plasma as a function of dimensionless frequency $x=\omega/\sqrt{2}ka$ for $\mu=1.0$, 0.6, and 0.0 with μ the fraction of specular reflection relative to diffuse scattering. Other parameters are $s=k_D/k$, $s_2=k_{\perp}/k_{\parallel}$ and the dimensionless collision frequency $y=\nu/\sqrt{2}ka$.

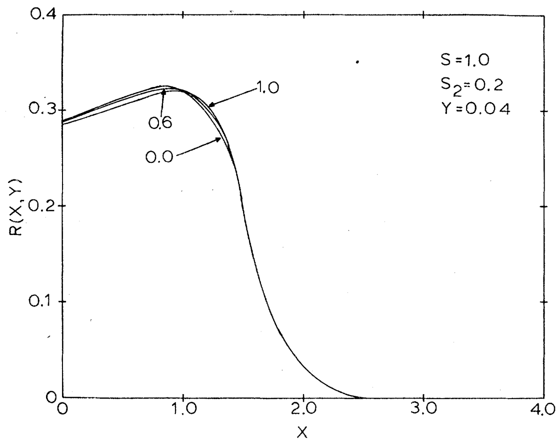

FIG. 2. Dimensionless density fluctuation spectrum for bounded electron plasma as a function of dimensionless frequency $x=\omega/\sqrt{2}ka$ for $\mu=1.0$, 0.6, and 0.0 with μ the fraction of specular reflection relative to diffuse scattering. Other parameters are $s=k_D/k$, $s_2=k_\perp/k_\parallel$ and the dimensionless collision frequency $y=\nu/\sqrt{2}ka$.

addition to μ, x, y, and g previously introduced, we introduce the new dimensionless parameters

$$s=k_D/k\ , \quad s_1=k_D/k_\parallel\ , \quad s_2=k_\perp/k_\parallel\ . \tag{6.2}$$

The collective regime corresponds to s, $s_1 \gg 1$. The nature of the results is indicated in the figures.

Figures 1 and 2 show the spectrum for $s=1$, corresponding to the transitional regime between the free particle and collective regimes for the case $s_2=0.2$ with y values of 0 and 0.04. In each figure, curves are given for $\mu=1.0$, 0.6, and 0.0, corresponding to a progression from pure specular reflection to pure diffuse scattering. In all cases the curves show a single peak with a shape similar to that obtained for the infinite plasma. We

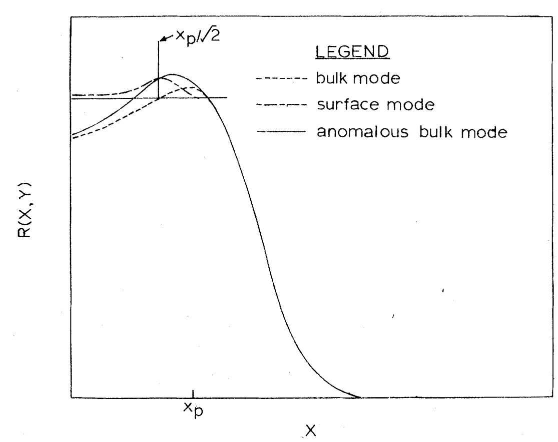

FIG. 3. Illustration of the production of anomalous or shifted bulk-mode spectra as a result of the superposition of surface modes.

note, however, that in Figs. 1 and 2 the curves shift towards lower frequencies with increasing diffuse scattering contributions. This is a result of the superposition of the diminished bulk mode and enhanced surface modes occurring as a consequence of diffuse scattering.

Figure 3 schematically illustrates how such a shift arises. The sharper the surface-mode contribution, the greater the shift. Physically this enhancement is due to the possibility of charge density accumulation at the surface as a result of the diffuse scattering. Note that the boundary condition (4.20) is a condition on the "currents" of the two streams at the boundary rather than on the actual distributions of particles in each stream. The shift is more dramatic in the collisionless case where the lines are broadened only by Landau damping. It appears then that the strong Landau damping of the high-frequency surface mode in the case of specular reflection is not as severe for a boundary with diffuse scattering. The downward shift of the single peak due to the enhanced surface modes has a very particular significance in terms of plasma diagnotics via laser light scattering. In the collisionless case we observe a shift, between the limiting scattering regimes, of in excess of 15%. We note that the curves may be fitted by an infinite medium theory with readjusted parameters. This would lead, however, to errors in the characterization of such parameters as the plasma frequency and electron density. The similarity in line shape of this "anomalous bulk mode" and that of the true infinite medium is perhaps the reason why manifestations of the effects of surface modes have seldom been reported by experimen-

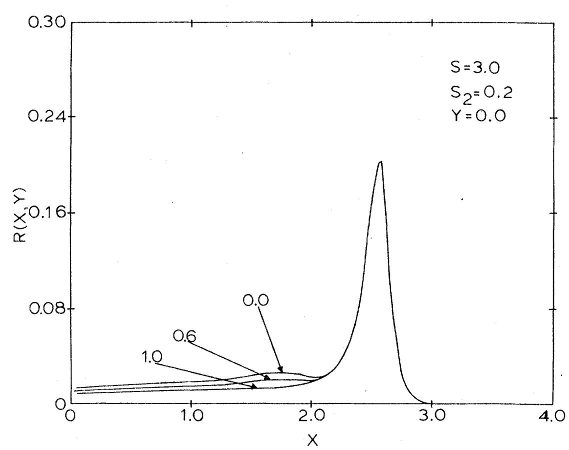

FIG. 4. Dimensionless density fluctuation spectrum for bounded electron plasma as a function of dimensionless frequency $x=\omega/\sqrt{2}ka$ for $\mu=1.0$, 0.6, and 0.0 with μ the fraction of specular reflection relative to diffuse scattering. Other parameters are $s=k_D/k$, $k_\perp/k_\parallel$ and the dimensionless collision frequency $y=\nu/\sqrt{2}ka$.

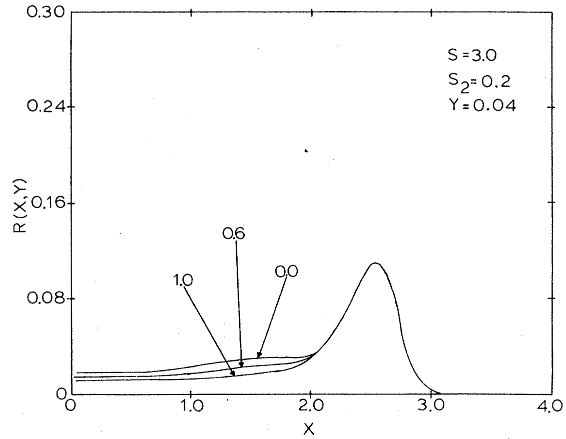

FIG. 5. Dimensionless density fluctuation spectrum for bounded electron plasma as a function of dimensionless frequency $x=\omega/\sqrt{2}ka$ for $\mu=1.0$, 0.6, and 0.0 with μ the fraction of specular reflection relative to diffuse scattering. Other parameters are $s=k_D/k$, $k_\perp/k_\parallel$, and the dimensionless collision frequency $y=\nu/\sqrt{2}ka$.

talists.

Figures 4 and 5 show the corresponding spectra for the case $s=3.0$ which corresponds to the collective regime. These are of interest in that with increasing diffuse scattering the surface modes become a distinct spectral feature although they remain rather broad. It is not clear to us as theorists whether these are well enough above noise level to be reliably measured. Data for short-wavelength surface modes are not presented since these tended to remain highly damped even with the inclusion of diffuse scattering.

VII. CONCLUDING REMARKS

We have shown that the predictions of the dynamics of the surface modes of bounded plasmas from microscopic theory are in some regimes quite sensitive to the choice of boundary conditions. We have studied density fluctuations in an electron plasma with varying relative contributions of specular reflection and diffuse scattering of particles at the boundary and have found that diffuse scattering tends to sharpen or enhance the surface mode feature.

The calculations performed have been done within a direct variational framework in which the admissable trial functions are discontinuous in velocity space, allowing one to incorporate in intrinsic fashion the effects of the discontinuity at the boundary. Though we make no claims as to the validity of our model of the microscopic boundary conditions, we emphasize that the basic formalism employed is quite general and can be used for arbitrary forms of physically imposed microscopic boundary conditions. In this context it would appear that more in depth studies of the microscopic boundary conditions for realistic plasma confinement situations would be warranted. It may also be noted that the formalism as given in the present paper is directly applicable to the study of the dynamics of liquid surfaces.

Our direct variational formulation, which is already nonadjoint, can be trivially extended to the case of magnetized plasmas. The extension of the formulation to two component plasmas, with and without applied magnetic fields, is also straightforward and would allow for study at the microscopic level of low-frequency electrostatic and electromagnetic surface modes. Such a formulation, for the case of an infinite fully ionized plasma, has already been developed and utilized for the calculation of the dynamic structure function and the energy loss function of a charged particle in the plasma.[24,31]

The foregoing remarks clearly indicate areas for further study and extensions of the present work. It is anticipated that some of these will be studied by one of the present investigators (J.H.H.).

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We acknowledge our gratitude to Professor J. Korringa and Professor J. Tough, whose questions raised during the thesis defense of one of us (W.L.W.) pointed out the need for a more detailed clarification of certain concepts and notation which we feel resulted in a significant improvement of this paper.

APPENDIX: SPECIFICATION OF F_-^s

From (4.23) we have

$$(\sigma+ik_\parallel v_\parallel)F_-^s+\tfrac{1}{2}k_D^2\Phi(v)(iv_\parallel+v_z)\times\int d^3\vec{v}'\int_0^\infty dz' e^{-k_\parallel z'}f(\vec{v}'z')=X(0)\,,\quad \text{(A1)}$$

where we take

$$X(0)\equiv-4\pi\Phi(v)/(k_\parallel+\lambda) \quad \text{(A2)}$$

with $v_z<0$ implied in $\Phi(v)$. The assumed form of $X(0)$ is motivated by the notion that we are concerned with that set of particles incident upon the boundary from within the plasma that was not at the origin at the initial time. These are assumed to interact with field particles whose distribution is however determined by the presence of the excess thermalized particle at $t=0$. We note from (4.3) and (4.9)–(4.11) that the integration over the field particles in (A1) gives a term that is independent of v and z. Thus we may write

$$F_-^s = \frac{X(0)}{\sigma + ik_\parallel v_\parallel} - \frac{k_D^2}{2}\frac{\Phi(v)(iv_\parallel + v_z)F_0}{\sigma + ik_\parallel v_\parallel}. \tag{A3}$$

Since the field-particle distribution is only approximately known, we shall not use its explicit form, as previously developed, to determine F_0. Rather, we determine F_0 from a momentum-transfer balance condition, i.e., we require that

$$\int_{v_z<0} d^3\vec{v}\, v_z v_\parallel F_-^s + \int_{v_z>0} d^3\vec{v}\, v_z v_\parallel F_+^s = 0. \tag{A4}$$

This gives, using (A3), (4.20) and (4.21),

$$-\frac{4\pi}{k_\parallel + \lambda}\int_{v_z<0} d^3\vec{v}\frac{v_z v_\parallel \Phi(v)}{\sigma + ik_\parallel v_\parallel} - \tfrac{1}{2}F_0 k_D^2 \int_{v_z<0} d^3\vec{v}\,\frac{v_z v_\parallel (iv_\parallel + v_z)\Phi(v)}{\sigma + ik_\parallel v_\parallel}$$
$$+\int_{v_z>0} d^3\vec{v}\, v_z v_\parallel \frac{1}{v_z}\int_{v_z'<0} d^3\vec{v}'\, v_z'[\mu\delta(v_\parallel' - v_\parallel)\delta(v_z' + v_z) + (1-\mu)P_d(v)]$$
$$\times\left(\frac{-4\pi\Phi(v')/(k_\parallel + \lambda) - \tfrac{1}{2}F_0 k_D^2 (iv_\parallel' + v_z')\Phi(v')}{\sigma + ik_\parallel v_\parallel'}\right) = 0. \tag{A5}$$

Performing the integrations, noting that $\int_{-\infty}^{+\infty} d^2 v_\parallel\, v_\parallel\, P_d(v) = 0$, we obtain

$$\frac{4\pi}{k_\parallel + \lambda}\frac{\theta_\parallel}{k_\parallel^2}[1 + \zeta_\parallel W(\zeta_\parallel)](\eta_- - \eta_+)$$
$$+\tfrac{1}{2}F_0 k_D^2 \frac{\theta_\parallel^2}{k_\parallel^3}\zeta_\parallel[1 + \zeta_\parallel W(\zeta_\parallel)](\eta_- - \eta_+) = 0, \tag{A6}$$

where, e.g., $\eta_+ = \int_{v_z<0} dv_z\, v_z \Phi(v_z)$. Thus

$$F_0 = -\frac{1}{\theta_\parallel}\frac{8\pi}{k_D^2}\frac{k_\parallel}{k_\parallel + \lambda}\frac{1}{\zeta_\parallel} \tag{A7}$$

so that

$$F_-^s = -\frac{4\pi\Phi(v)}{\sigma + ik_\parallel v_\parallel}\left(1 + \frac{k_\parallel}{\sigma}(iv_\parallel + v_z)\right). \tag{A8}$$

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[1]K. F. Sergeĭchev, Zh. Eksp. Teor. Fiz. 58, 1157 (1970) [Sov. Phys.-JETP 31, 620 (1970)].

[2]K. A. Brueckner and S. Jorna, Rev. Mod. Phys. 46, 325 (1974).

[3]A. G. Sitenko, in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Wiley, New York, 1974), Vol. 5, p. 15.

[4]A. W. Trivelpiece and R. W. Gould, J. Appl. Phys. 30, 1784 (1959).

[5]R. H. Ritchie, Progr. Theor. Phys. (Kyoto) 29, 607 (1963).

[6]P. W. Kaw and J. B. McBride, Phys. Fluids 13, 1784 (1970).

[7]R. L. Guernsey, Phys. Fluids 12, 1852 (1969).

[8]N. Van Trong, Ukr. Fiz. Zh. 17, 1902 (1972).

[9]A. P. Shivarova, J. Phys. A 7, 881 (1974).

[10]D. Bohm and E. P. Gross, Phys. Rev. 75, 1851 (1949).

[11]J. Harris and A. Griffin, Can. J. Phys. 48, 2592 (1970).

[12]J. Harris and A. Griffin, Phys. Rev. B 3, 749 (1971).

[13]L. Kleinman, Phys. Rev. B 7, 2288 (1973).

[14]J. E. Inglesfield and E. Wikborg, J. Phys. C 6, L158 (1973); J. Phys. F 5, 1706 (1975).

[15]J. E. Inglesfield and E. Wikborg, J. Phys. F 5, 1706 (1975).

[16]R. L. Guernsey, Phys. Fluids 13, 2089 (1970).

[17]E. P. Gross, Phys. Rev. 158, 146 (1967).

[18]S. Yip and S. Ranganathan, Phys. Fluids 8, 1956 (1965).

[19]E. P. Gross, E. A. Jackson, and S. Ziering, Ann. Phys. (N.Y.) 1, 141 (1957).

[20]S. Chandrasekhar, *Radiative Transfer* (Oxford, New York, 1950).

[21]E. P. Gross and S. Ziering, Phys. Fluids 2, 701 (1959).

[22]J. H. Hopps, Ph.D. thesis (Brandeis, 1971) (unpublished).

[23]B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic, New York, 1961).

[24]J. H. Hopps (unpublished).

[25]D. Gorman, Phys. Fluids 10, 2502 (1967).

[26]R. I. Mason, J. Math. Phys. 9, 868 (1968).

[27]V. I. Romanova, Zh. Tekh. Fiz. 43, 2281 (1973) [Sov. Phys.-Tech. Phys. 18, 1443 (1974)].

[28]J. H. Hopps, Phys. Rev. A 13, 1226 (1976).

[29]W. L. Waldron, M.S. thesis (Ohio State Univ., 1976) (unpublished).

[30]M. G. Kivelson and D. F. DuBois, Phys. Fluids 7, 1578 (1964).

[31]J. H. Hopps, Bull. Am. Phys. Soc. 21, 647 (1976).