PHYSICAL REVIEW A VOLUME 15, NUMBER 3 MARCH 1977

Reply to "Transport properties of negative muons in matter"

H. Daniel
Physics Department, Technical University of Munich, Munich, Germany
(Received 18 October 1976)

In the model recently used by me to calculate the Coulomb capture of mesonic particles, the energy spectrum is explicitly shown to be white. Other models, and in particular less appropriate potential shapes, yield other energy spectra. Experiments of the type suggested in the preceding Comment have already been performed.

In the preceding Comment[1] on a recent paper of mine[2] the authors emphasize the fact well known in transport theory[3] that slowing-down and capture cross sections of mesonic particles must be combined in a consistent way. This has been done indeed in my paper as is easily seen when calculating the energy spectrum of the mesonic particles in matter explicitly. The notation used is that of Fermi and Teller[4] and my recent paper.[2]

Consider slow mesonic particles entering an atom with impact parameters $q<r_0$, where r_0 is the cut-off radius of the atomic potential U ($U=0$ for radii $r>r_0$). The energy loss ΔW of the particles has been calculated, in the approximation used, to depend only on the atomic number Z. In particular, it is energy independent. After traversal of the atom all mesonic particles are shifted by the same amount of energy ΔW to smaller energies. In particular, mesonic particles which entered with an energy W_0 in the range $W_1 \leq W_0 < W_1+\epsilon$ are now at $W_1+\Delta W \leq W < W_1+\Delta W+\epsilon$, where W_1 is a fixed value which may satisfy $W_1 > -\Delta W$ (i.e., these particles are not captured) and ϵ is a small quantity. The flux of entering particles was

$$n(W_1,\epsilon)=P(W_1)\epsilon\ ,$$

where $P(W)$ is the spectral distribution function. As none of the considered mesonic particles are captured, the flux of outgoing particles is again $n(W_1,\epsilon)$. On the other hand, this flux is

$$n(W_1+\Delta W,\epsilon)=P(W_1+\Delta W)\epsilon\ .$$

This means

$$P(W_1+\Delta W)=P(W_1)\ ,$$

i.e., a white energy spectrum from zero up to an energy W_w where the approximation breaks down. In the model W_w can be made arbitrarily large by decreasing r_0 (not arbitrarily small as r_0 is limited by half the distance between two adjacent nuclei).

The difference between the spectral shape derived above and that of Vogel *et al.*[5] clearly results mainly from the different potentials used, particularly in the outer region of the atom. The fair agreement of Eq. (6) of Ref. 2 with the experiment[2,6] and the failure of formulas based on $P(W)$ approximately proportional[1,5] to W (E in the notation of Ref. 1) then indicates that the potential U is rather steep in its outer region and not flat, as it would result just from a superposition of free-atom potentials.[5] The failure of basically a free-atom potential in the case of a lattice is not surprising in view of the observed large differences in outer-electron binding energies between free atoms[7,8] and solids.[8]

The difference of the numerical factors in the logarithmic terms in Eq. (6) of my paper[2] and Eq. (7) of the paper by Haff and Vogel[1] is of minor importance; both factors are, of course, approximations.

At the end of the preceding Comment, the authors suggest some experiments. Experiments of this kind have already been performed. Measurements on dilute solutions were published one year ago.[9] Measurements on alloys were recently performed and have just been published.[6] The most important point in the case of alloys is missing in the preceding comment.[1] It is essential to use single-phase alloys.[6] Inhomogeneous matter may yield completely different results.[10] Experiments on gas mixtures were performed some years ago.[11] The results on the atomic capture ratios show no strong Z dependence, in agreement with a recent calculation for gases[12] and in disagreement with various proposed laws of a more pronounced Z dependence.[1,4,5] Finally, direct measurements of energy spectrum and capture cross section for very slow muons have recently been proposed, and a possible experimental set-up sketched.[13]

[1]P. K. Haff and P. Vogel, preceding Comment, Phys. Rev. A 15, 1336 (1977).
[2]H. Daniel, Phys. Rev. Lett. 35, 1649 (1975).
[3]D. T. Goldman and C. O. Muehlhause, in *Handbook of Physics*, edited by E. U. Condon and H. Odishaw (McGraw-Hill, New York, 1967), 2nd ed., p. 9–200ff.
[4]E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).
[5]P. Vogel, P. K. Haff, V. Akylas, and A. Winther, Nucl. Phys. A 254, 445 (1975).
[6]R. Bergmann, H. Daniel, T. von Egidy, F. J. Hartmann, and H.-J. Pfeiffer, Z. Phys. A 280, 27 (1977).
[7]*Handbook of Chemistry and Physics*, edited by R. C. Weast (CRC, Cleveland, 1973), 54th ed., p. E67.
[8]K. D. Sevier, *Low Energy Electron Spectrometry* (Wiley-Interscience, New York, 1972), p. 242ff and p. 356ff.
[9]J. D. Knight, C. J. Orth, M. E. Schillaci, R. A. Naumann, H. Daniel, K. Springer, and H. B. Knowles, Phys. Rev. A 13, 43 (1976).
[10]H. Daniel (unpublished).
[11]H.-J. Pfeiffer, K. Springer, and H. Daniel, Nucl. Phys. A 254, 433 (1975).
[12]H. Daniel, Radiat. Eff. 28, 189 (1976).
[13]R. Bergmann, H. Daniel, V. Dornow, T. von Egidy, H. Hagn, F. J. Hartmann, H.-J. Pfeiffer, and K. Springer, Vorschlag für ein Experiment am SIN: Mesonische Atome in Atom- und Festkörperphysik, Technische Universität München, Munich, 1975 (unpublished).