Vertex corrections to the shear viscosity critical exponent*

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Mode-coupling equations for the anomalous shear viscosity which incorporate vertex corrections are used to calculate the critical exponent for this transport coefficient. These corrections are shown to have an important effect on the magnitude of this exponent.

In several recent articles the mode-coupling equations for the description of dynamic critical phenomena have been used to calculate the critical indices for transport coefficients at the critical point.[1,2] Such studies are complementary to the calculation of these dynamical critical exponents by the renormalization-group approach[3] and lead to similar results in regions where both are applicable. The recent work of Ohta and Kawasaki[2,4] has shown that the mode-coupling approach can be used to calculate critical exponents even in three dimensions without the use of an ϵ expansion.

Ohta and Kawasaki[4] have presented a detailed study of the effects of frequency and vertex corrections to the diffusion coefficient of a binary mixture at the critical point. Both of these effects were found to be very small. These corrections are also small in the vicinity of the critical point.[5,6] Recently, we have considered the effect of vertex corrections to the shear viscosity near the critical point.[7] In contrast to the case of diffusion, the viscosity vertex corrections were found to yield a contribution to the anomalous shear viscosity which was about 20% of the standard anomalous contribution in the vicinity of the critical point. Hence it is of some interest to see if these corrections are important at the critical point and affect the magnitude of the critical exponent. In this paper we present the results of such a study.

The critical exponents for the diffusion and shear viscosity coefficients are defined by the equations

$$D(k) = A_D k^{z_D}, \tag{1a}$$

$$\eta(k) = A_\eta k^{z_\eta}. \tag{1b}$$

On the basis of the work of Ohta and Kawasaki,[4] we can neglect both vertex and frequency corrections to the diffusion coefficient, so that

$$D(k) = \frac{k_B T}{8\pi^3} \int d\vec{q}\,[1 - (\hat{q}\cdot\hat{k})^2] \frac{\chi_{k-q}}{\chi_k} [q^2 \eta(q)]^{-1}. \tag{2}$$

The static susceptibilities have the form $\chi_k \sim k^{-2+\eta}$. We only consider the three-dimensional case here. Using (1a) and (1b) in (2) and following Ohta,[2] this leads to

$$z_D = 1 - z_\eta \tag{3}$$

and

$$A_D = k_B T C_D / (2\pi)^2 A_\eta, \tag{4}$$

where

$$C_D = \frac{\Gamma(\frac{3}{2})\Gamma(\frac{1}{2} + \frac{1}{2}z_\eta - \frac{1}{2}\eta)\Gamma(\frac{1}{2} - \frac{1}{2}z_\eta)\Gamma(\frac{3}{2} + \frac{1}{2}\eta)}{\Gamma(1 - \frac{1}{2}\eta)\Gamma(2 - \frac{1}{2}z_\eta + \frac{1}{2}\eta)\Gamma(2 + \frac{1}{2}z_\eta)}. \tag{5}$$

From our earlier work[7] the shear viscosity can be written

$$\eta(k) = J_k^1 + J_k^2 \tag{6}$$

with

$$J_k^1 = \frac{k_B T}{16\pi^3 k^2} \int d\vec{q}\,(q^x)^2 \frac{(\chi_{k-q} - \chi_q)^2}{\chi_{k-q}\chi_q} [q^2 D(q) + |\vec{k} - \vec{q}|^2 D(|\vec{k} - \vec{q}|)]^{-1}, \tag{7}$$

$$\begin{aligned} J_k^2 = -\frac{1}{4k^2}\left(\frac{k_B T}{(2\pi)^3}\right)^2 \int d\vec{q} \int d\vec{q}\,' &(\chi_q^{-1} - \chi_{k-q}^{-1})(\chi_q \chi_{k-q-q'} + \chi_{k-q}\chi_{q+q'})(\chi_{q+q'}^{-1} - \chi_{k-q-q'}^{-1}) \\ &\times (\vec{k} - \vec{q})\cdot[\vec{q} - \hat{q}'(\vec{q}\cdot\hat{q}')][\vec{q} - (\vec{q}\cdot\hat{k})\hat{k}]\cdot(\vec{q} + \vec{q}\,')[q'^2 \eta(q')]^{-1}[q^2 D(q) + |\vec{k} - \vec{q}|^2 D(|\vec{k} - \vec{q}|)]^{-1} \\ &\times [|\vec{q} + \vec{q}\,'|^2 D(|\vec{q} + \vec{q}\,'|) + |\vec{k} - \vec{q} - \vec{q}\,'|^2 D(|\vec{k} - \vec{q} - \vec{q}\,'|)]^{-1}. \end{aligned} \tag{8}$$

If we use (1), (3), and (4) in Eq. (6), we find that z_η is given by the solution of the equation

$$1 = \frac{I_1(z_\eta, \eta)}{4C_D} - \frac{I_2(z_\eta, \eta)}{16\pi^2 C_D^2}. \tag{9}$$

In this equation, I_1 is given by

$$I_1(z_\eta,\eta)=\int_0^\infty dx\,x^4\int_0^\pi d\theta\,\sin^3\theta\,\frac{(x^{2-\eta}-|\hat{k}-\vec{x}|^{2-\eta})^2}{x^{2-\eta}|\hat{k}-\vec{x}|^{2-\eta}}[x^{3-z_\eta}+|\hat{k}-\vec{x}|^{3-z_\eta}]^{-1}, \tag{10}$$

where $x=q/k$. After using the approximate result given in Ref. 2, we can write I_1 as

$$I_1(z_\eta,\eta)\simeq\frac{-2(2-\eta)^2}{15z_\eta}+\frac{4}{3(3+\eta)}. \tag{11}$$

The expression for I_2 is considerably more complicated:

$$\begin{aligned}I_2(z_\eta,\eta)=\int d\vec{x}\int d\vec{y}\,&(x^{2-\eta}-|\hat{k}-\vec{x}|^{2-\eta})(x^{\eta-2}|\hat{k}-\vec{y}|^{\eta-2}+|\hat{k}-\vec{x}|^{\eta-2}y^{\eta-2})(y^{2-\eta}-|\hat{k}-\vec{y}|^{2-\eta})\\&\times[x^{3-z_\eta}+|\hat{k}-\vec{x}|^{3-z_\eta}]^{-1}[y^{3-z_\eta}+|\hat{k}-\vec{y}|^{3-z_\eta}]^{-1}\\&\times|\vec{y}-\vec{x}|^{-2-z_\eta}(\hat{k}-\vec{x})\cdot[\vec{x}-(\vec{y}-\vec{x})\vec{x}\cdot(\vec{y}-\vec{x})/|\vec{y}-\vec{x}|^2][\vec{x}-(\vec{x}\cdot\hat{k})\hat{k}]\cdot\vec{y}.\end{aligned} \tag{12}$$

In writing this equation we have made a change of variable $\vec{q}''=\vec{q}+\vec{q}'$ in Eq. (8) and let $y=q''/k$. In contrast to I_1 which diverges for $z_\eta=0$, the integral I_2 is well behaved even for $z_\eta=0$. Hence we do not expect any qualitative change in the form of the shear viscosity divergence but only a change in the magnitude of the critical exponent. In Refs. 2 and 4, z_η has been calculated with I_2 neglected and has been found to be quite small, $z_\eta\simeq-8/15\pi^2$. Since z_η is small and the integral I_2 converges even with $z_\eta=0$, we might expect that to a first approximation we can consider $I_2(0,\eta)$. This is convenient since for the case $z_\eta=0$ the fivefold integral in Eq. (12) can be reduced to a fourfold integral which is much easier to compute numerically.

We have carried out such a calculation with the static susceptibility given by the Ornstein-Zernike form, $\eta=0$. The result is $I_2(0,0)=-182.6$. Using this value in Eq. (9) and solving for z_η, we find $z_\eta=-0.070$. This should be compared with $z_\eta=-0.054$ which is obtained when I_2 is neglected. Hence the viscosity vertex corrections do have an important effect on the magnitude of the critical exponent in contrast to the analogous corrections to the linewidth.

A few remarks can be made about a more complete study of this correction. As mentioned above, the fivefold integral is quite difficult to evaluate. However, we can estimate the size of the error introduced when z_η is neglected in Eq. (12) by retaining $z_\eta\neq0$ in the diffusive propagators in Eq. (12) while setting $z_\eta=0$ in the viscous propagator. Such a procedure will greatly *underestimate* the magnitude of the change in the exponent since a nonzero z_η in the $|\vec{y}-\vec{x}|^{-2-z_\eta}$ factor will tend to compensate for the nonzero value of z_η in the diffusive terms. The resulting integral is, however, still four dimensional and is much easier to evaluate. The value of z_η obtained in this way is $z_\eta=-0.062$. Hence we believe the approximation using $I_2(0,\eta)$ is quite accurate.

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