

Quadrupole antishielding factors for actinide ions

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Perturbation-numerical calculations have been performed using nonrelativistic Hartree-Fock-Slater wave functions to obtain the Sternheimer quadrupole antishielding factor γ_∞ for all the tetrapositive ions in the actinide series ($90 \leq Z \leq 103$). For these ions, the nonrelativistic γ_∞ values seem to be constant at ~ -95 . It is estimated that the relativistic effects would increase the $|\gamma_\infty|$ value by $\sim 65\%$.

The repercussions of the induced effects in the core electrons, respectively due to the external point charges and the valence electrons on the total electric field gradient q_{total} acting at the nuclear site, were first represented by Sternheimer¹ as

$$q_{\text{total}} = (1 - \gamma_\infty)q_{\text{ion}} + (1 - R)q_{\text{valence}}, \quad (1)$$

where q_{ion} and q_{valence} define the electric field gradient due to the external point charges and the valence electrons respectively. The constants γ_∞ and R are commonly known as Sternheimer antishielding (or shielding) factors.

During recent years a few nuclear quadrupole coupling data on actinide nuclei ($90 \leq Z \leq 103$) have become available, mainly through Mössbauer-effect² and atomic-beam magnetic-resonance³ experiments. In the interpretation of such data it is essential to have a knowledge of reliable theoretical values of Sternheimer antishielding factors, corresponding to the atom or ion under investigation. In this paper we report the results of our perturbation-numerical calculations of γ_∞ for all tetrapositive actinide ions using nonrelativistic Hartree-Fock-Slater⁴ (HFS) wave functions for the unperturbed state. In view of the fact that the elements in the actinide series occur in several valence states we have also calculated γ_∞ values for U^{3+} , U^{5+} , U^{6+} , and Am^{2+} ions.

The perturbed radial functions $u'_i(nl \rightarrow l')$ were obtained by directly solving the following nuclear moment-perturbed Schrödinger equation¹:

$$\left(-\frac{d^2}{dr^2} + \frac{l'(l'+1)}{r^2} + V_0(r) - E \right) u'_i(nl \rightarrow l') = u'_0(nl) \left(\frac{1}{r^3} - \left\langle \frac{1}{r^3} \right\rangle_{nl} \right) \delta_{il'}. \quad (2)$$

Excluding the consistency and correlation effects,⁵ the quadrupole antishielding factor γ_∞ is given to zeroth order by

$$\gamma_\infty = \sum_{nl} c(nl \rightarrow l') \int_0^\infty u'_0(nl) u'_i(nl \rightarrow l') r^2 dr. \quad (3)$$

The constants $c(nl \rightarrow l')$ involving angular integrals have been tabulated by Sternheimer.⁶ For each ion, the coefficients corresponding to the perturbations of the $5f$ orbital were multiplied by the fraction to which the $5f$ orbital is occupied. All calculations were carried out by a method reported earlier⁷ using the IBM 7044/1401 system at the Indian Institute of Technology, Kanpur.

TABLE I. Nonrelativistic total γ_∞ values to zeroth order for the ions in actinide series ($90 \leq Z \leq 103$). The electronic configuration considered in each case is shown in column 2.

Ion	Config. (Rn)–	Present calc.	Other calcs.
$_{90}\text{Th}^{4+}$	$5f^0$	–107.170	–177.5 ^a
$_{91}\text{Pa}^{4+}$	$5f^1$	–105.325	
$_{92}\text{U}^{3+}$	$5f^3$	–117.987	
$_{92}\text{U}^{4+}$	$5f^2$	–103.805	
$_{92}\text{U}^{5+}$	$5f^1$	–95.530	
$_{92}\text{U}^{6+}$	$5f^0$	–85.155	–143.9 ^a
$_{93}\text{Np}^{4+}$	$5f^3$	–102.170	
$_{94}\text{Pu}^{4+}$	$5f^4$	–100.970	
$_{95}\text{Am}^{2+}$	$5f^7$	–132.867	–137.3 ^b
$_{95}\text{Am}^{4+}$	$5f^5$	–99.786	
$_{96}\text{Cm}^{4+}$	$5f^6$	–98.842	
$_{97}\text{Bk}^{4+}$	$5f^7$	–97.985	
$_{98}\text{Cf}^{4+}$	$5f^8$	–96.890	
$_{99}\text{E}^{4+}$	$5f^9$	–96.47	
$_{100}\text{Fm}^{4+}$	$5f^{10}$	–96.40	
$_{101}\text{Md}^{4+}$	$5f^{11}$	–95.417	
$_{102}\text{No}^{4+}$	$5f^{12}$	–95.210	
$_{103}\text{Lw}^{4+}$	$5f^{13}$	–94.211	

^a Reference 9.

^b Reference 8.

Our results are given in Table I along with the earlier available^{8,9} results for purposes of comparison. Sternheimer⁸ calculated $\gamma_\infty = -137$ for Am^{2+} by using HFS wave functions corresponding to the neutral atom. He had predicted that the use of actual ionic wave functions for Am^{2+} should reduce $|\gamma_\infty|$ by $\sim 5\%$. The present value of $\gamma_\infty = -133$ confirms this prediction. Feiok and Johnson⁹ have performed uncoupled calculations similar to those of ours using relativistic HFS wave functions and obtained γ_∞ as -177.5 and -143.9 for Th^{4+} and

U^{6+} ions, respectively. A comparison of these values with the present nonrelativistic values show that the relativistic effects increase $|\gamma_\infty|$ by $\sim 65\%$ and $\sim 69\%$ for Th^{4+} and U^{6+} ions, respectively. The present nonrelativistic γ_∞ values, however, appear to be constant and ~ -95 for the tetrapositive ions in the actinide series.

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