

Comments and Addenda

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Effect of laser-cyclotron resonance on the Landau damping of plasma waves

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The plasmon scattering by electrons in the simultaneous presence of laser and magnetic fields is discussed. A kinetic equation is derived, and the rate of change of the plasmon population is calculated. For laser radiation propagating parallel to the magnetic field it is found that multiphoton processes are dominant when the laser frequency is near the electron cyclotron frequency. Furthermore, the damping rate is found to decrease exponentially as the laser frequency approaches the electron cyclotron frequency.

In a recent paper¹ the cyclotron resonance absorption of laser radiation due to inverse bremsstrahlung was discussed. Although the laser frequency is usually much greater than the electron cyclotron frequency, a resonance condition, where the laser frequency is equal to the cyclotron frequency, may be approached by increasing either the magnetic field strength or by using longer-wavelength lasers. Since intense submillimeter lasers are becoming available,² it is therefore important to consider the effects of this radiation on the several wave-particle processes in a magnetized plasma.

Here we extend the theory of Ref. 1 by considering the modification of the Landau damping of electron plasma waves by a laser field and include the effect of an external uniform magnetic field.

The laser beam is treated as a classical plane electromagnetic wave in the dipole approximation. The plasma electrons are described by the solution to the Schrödinger equation for an electron in the simultaneous presence of the laser field and a uniform static magnetic field.³ The scattering of plasma waves by the electrons is treated using first-order perturbation theory.⁴ The kinetic equation for the plasmons is then derived using the quantum approach of Harris⁴ based upon the transition probabilities. For the case of a right-hand circularly polarized plane wave propagating parallel to the magnetic field (assumed to be along the +z direction), the damping rate is found to de-

crease as the laser frequency approaches the electron cyclotron frequency. A simple physical explanation of this result is also given.

Assuming a right-hand circularly polarized laser beam propagating along the z axis and proceeding as in Refs. 1 and 4, the transition probability per unit time for a transition from a state 1 ($n+l, p_x + \hbar k_x, p_z + \hbar k_z$) to a state 2 (n, p_x, p_z) due to a collision with a plasmon \vec{k} with absorption ($\nu < 0$) or emission ($\nu > 0$) of $|\nu|$ photons may be written as

$$T_\nu(1 \rightarrow 2; \vec{k}) = (2\pi/\hbar) J_\nu^2(\lambda/h\omega) V_k^2 |\chi(n+l, n, \rho)|^2 \times \delta(E_1 - E_2 - \hbar\omega_k - \nu\hbar\omega), \quad (1)$$

where $E_1 = \hbar\omega_c(n+l + \frac{1}{2}) + (p_x + \hbar k_x)^2/2m$, $E_2 = \hbar\omega_c(n + \frac{1}{2}) + p_x^2/2m$, $V_k^2 = 2\pi e^2 \hbar\omega_k / V k^2$ is the electron-plasmon vertex,^{5,6} J_ν is the Bessel function of order ν , $\lambda = e\hbar k_x E_0 / m(\omega - \omega_c)$ is the field parameter, $\omega_k = \omega_p |k_x| / k$ is the plasmon dispersion relation, $\rho = \hbar k_x^2 / 2m\omega_c$, and $\chi(n+l, n, \rho)$ is the overlap of the Landau harmonic oscillators as defined, for instance, in Ref. 7.

The change in N_k , the number of plasmons of wave number \vec{k} , may be written schematically as in Fig. 1. As usual,^{1,4} we may convert this schematic equation into a mathematical one by substituting the transition probability. One has

$$\frac{dN_k}{dt} = \gamma_k N_k, \quad (2)$$

where

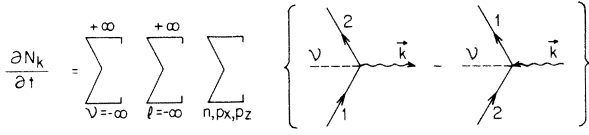


FIG. 1. Change in N_k , the number of plasmons of wave number k . The wavy lines represent plasmons; the dashed lines, photons.

$$\gamma_k = \frac{2\pi}{\hbar} \sum_{n,l} \sum_{\nu=-\infty}^{+\infty} J_\nu^2(\lambda/\hbar\omega) V_k^2 |\chi(n+l, n, \rho)|^2 \times [f(E_1) - f(E_2)] \delta(E_1 - E_2 - \hbar\omega_k - \nu\hbar\omega). \quad (3)$$

$$\gamma_k = \frac{\pi}{\hbar} \sum_{np_x p_z} \sum_{l=-\infty}^{+\infty} V_k^2 |\chi(n+l, n, \rho)|^2 [f(E_2)(e^{-(\lambda+\hbar\omega_k)/k_B T} - 1) \delta(E_1 - E_2 - \hbar\omega_k - \lambda) + f(E_2)(e^{(\lambda-\hbar\omega_k)/k_B T} - 1) \delta(E_1 - E_2 - \hbar\omega_k + \lambda)]. \quad (4)$$

Furthermore, assuming that $\lambda \gg k_B T$ for ω near ω_c , the contribution of processes in which photons are emitted is negligible compared to the contributions of processes in which photons are absorbed. Under these circumstances Eq. (4) becomes

$$\gamma_k = \frac{\pi}{\hbar} \sum_{np_x p_z} \sum_{l=-\infty}^{+\infty} V_k^2 |\chi(n+l, n, \rho)|^2 f(E_2) \times (e^{(\lambda-\hbar\omega_k)/k_B T} - 1) \delta(E_1 - E_2 - \hbar\omega_k + \lambda). \quad (5)$$

We now take the classical limit of Eq. (5) by letting^{1,4,6,8}

$$\hbar \rightarrow 0 \quad \text{and} \quad n \rightarrow \infty, \quad (6)$$

such that

$$n\hbar\omega_c \rightarrow \frac{1}{2} m v_0^2, \quad (7)$$

$$\sum_{np_x p_z} (\dots) f(E_{np_x p_z}) \rightarrow V \int d^3v (\dots) f(\vec{v}). \quad (8)$$

Hence expanding Eq. (5) in powers of \hbar and retaining only the lowest-order terms, one has

$$\gamma_k = \frac{2\pi^2 e^2 \omega_k}{k^2 k_B T} (k_1 v_0 - \omega_k) \times \sum_{l=-\infty}^{+\infty} \int d^3v J_l^2\left(\frac{k_1 v_\perp}{\omega_c}\right) f(\vec{v}) \times \delta(l\omega_c + k_z v_z - \omega_k + k_1 v_0), \quad (9)$$

where we have written λ as $\hbar k_1 v_0$, with $v_0 e E_0 / m(\omega - \omega_c)$, and replaced $|\chi(n+l, n, \rho)|^2$ by its classical limit,⁶ namely, $|\chi(n+l, n, \rho)|_{\text{CL}}^2 \rightarrow J_l^2(k_1 v_\perp / \omega_c)$. Replacing $f(\vec{v})$ by $n_0 (\pi v_T^2)^{-3/2} \exp(-v^2/v_T^2)$, where v_T^2

We now assume a Maxwellian distribution and consider only the case of $\omega \sim \omega_c$. Then $\lambda \gg \hbar\omega$ and the argument of the Bessel function is large. For large values of argument, the Bessel function is small except when the order ν is equal to the argument. The sum over ν in Eq. (3) may be written approximately^{1,8}

$$\sum_{\nu=-\infty}^{+\infty} J_\nu^2(\lambda/\hbar\omega) \delta(E - \nu\hbar\omega) \simeq \frac{1}{2} [\delta(E - \lambda) + \delta(E + \lambda)],$$

where $E = E_1 - E_2 - \hbar\omega_k$. The first δ function corresponds to the emission and the second to the absorption of $\lambda/\hbar\omega$ photons. Since $\lambda \gg \hbar\omega$, only multiphoton processes are significant. The damping rate then becomes

$= 2k_B T / m$, and performing the integration over v_z using the δ function, Eq. (9) reduces to

$$\gamma_k = \frac{\pi^{1/2} \omega_k^3}{k^3 v_T^3} (k_1 v_0 - \omega_k) \times \sum_{l=-\infty}^{+\infty} \exp\left(-\frac{(k_1 v_0 - \omega_k + l\omega_c)}{k_z^2 v_T^2}\right) F_l(k_1 v_T / \omega_c) \quad (10)$$

with

$$F_l(k_1 v_T / \omega_c) = \int_0^\infty dx e^{-x} J_l^2(k_1 v_T \sqrt{x} / \omega_c).$$

In arriving at Eq. (10) we have the dispersion relation for the plasma waves in a magnetic field, namely, $\omega_k = \omega_p |k_z| / k$, where ω_p is the plasma frequency.

For $E_0 = 0$ ($v_0 = 0$), Eq. (10) reduces to the well-known expression of the Landau damping of plasma waves in a uniform magnetic field.⁶ On the other hand, for $B \rightarrow 0$ the argument of the Bessel function in Eq. (9) is large, so that as before we may approximate the expression for γ_k by

$$\gamma_k \simeq \frac{2\pi^2 e^2 \omega_k}{k^2 k_B T} (k_1 v_0 - \omega_k) \int d^3v f(\vec{v}) \delta(\vec{k} \cdot \vec{v} - \omega_k - k_1 v_0). \quad (11)$$

Equation (11) is essentially the same one would get for the plasmon damping if we had only the radiation field, in the limit of either $\omega \rightarrow 0$ or $E_0 \rightarrow \infty$. This can easily be worked out from the electron wave function in the radiation field as given by See-

ly and Harris.⁸ The fact that Eq. (9) is valid for ω near resonance means that in the absence of the uniform magnetic field we must have either $\omega \rightarrow 0$ or $E_0 \rightarrow \infty$.

We now go back to Eq. (10). The first point one notes is that for $\omega = \omega_c$ (i.e., $v_0 \rightarrow \infty$) γ_k vanishes. Physically this result may be understood as follows: Consider the problem of one electron in an electromagnetic field described by $\vec{A}(t)$ and moving in a potential V (the plasmon field). We have

$$H = \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_0^2 + V,$$

where $\frac{1}{2}mv_{\parallel}^2$ and $\frac{1}{2}mv_0^2$ are the longitudinal and transverse energies of the electron, respectively. Then $\omega \rightarrow \omega_c$ ($v_0 \rightarrow \infty$), the transverse energy is much larger than V , and the electron-plasmon interaction is "frozen."

Finally, Eq. (10) tells us that for ω near ω_c , but not necessarily at resonance, the plasmon popula-

tion may in principle grow (amplification) with time for values of k such that $k_{\perp}v_0 > \omega_k$. One may say that the effect of the radiation field is to give a drift velocity $v_0 = eE_0/m|\omega - \omega_0|$ to the plasma electrons in the direction perpendicular to the magnetic field. For \vec{k} parallel to the z axis we have only damping, whereas in the perpendicular direction we have only amplification. In a general direction, however, we may have amplification provided $k_{\perp}v_0 > \omega_k$. Since the plasmons are well-defined excitations only for k 's up to $k_D = \omega_p/v_T$, it follows that one has amplification of plasma waves only for $v_0 \geq v_T$ or $E_0 \geq E_c$, where

$$E_c = (m|\omega - \omega_c|/e)(2k_B T/m)^{1/2}.$$

For a plasma with $T = 10^3$ °K in a magnetic field of the order of 10^4 G, the critical power turns out to be of the order of 10 kW/cm² at a frequency of about 10^{11} sec⁻¹.

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