Range and stopping-power equations for heavy ions

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Starting with Bohr's semiclassical stopping-power equation, a set of stopping-power equations has been deduced for both partially and completely stripped heavy ions. Each of these equations is valid in a particular ion-energy region depending upon the ion-medium combination. A simple evaluation of $\int_E dE(dE/dx)_E^{-1}$ with the values of $(dE/dx)_E$ from the appropriate stopping-power equation valid in a particular ion-energy region gives the total range of a heavy ion. We have compared the available energy-loss data from other workers with the corresponding calculated values for a variety of heavy ions in different media (^{12}C, ^{14}N, ^{16}O, ^{19}F, and ^{20}Ne ions in Al; ^{9}Be ions in Al and Au; ^{10}B and ^{11}B ions in Al; ^{12}C ions in Ar and N_2; ^{40}Ar ions in Ar and N_2; ^{12}C, ^{14}N, and ^{16}O ions in Si; and ^{10}B, ^{11}B, ^{12}C, ^{14}N, ^{16}O, and ^{19}F in oxygen). In general the agreement is very good.

I. INTRODUCTION

Because of its wide applications in various fields, the energy loss of heavy ions in matter still constitutes a topic of substantial interest to many investigators.[1-10] The first theoretical formulation of the problem, by Bohr,[11] led to the following stopping-power equation:

$$\frac{dE}{dx}=\frac{4\pi z^2e^4n}{mV^2}Z\ln\left(\frac{1.123mV^3}{ze^2\bar{\omega}}\right), \tag{1}$$

where dE/dx is the energy lost per unit path length by a particle of velocity V and ionic charge ze, e and m are the electronic charge and mass; n is the number of atoms of the medium per unit volume; Z is the atomic number, and $\bar{\omega}$ is the geometric mean cyclic frequency of the orbital electrons of the medium.

The quantum-mechanical formulation by Bethe[12] yielded a stopping-power equation which is very similar to Eq. (1),

$$\frac{dE}{dx}=\frac{4\pi z^2e^4n}{mV^2}Z\left(\ln\frac{2mV^2}{I(1-\beta^2)}-\beta^2\right), \tag{2}$$

in which the additional parameter I represents the mean excitation potential of the medium and $\beta=V/c$, c being the speed of light. Bohr[13] has shown that the conditions for the validity of Eq. (1) and Eq. (2) are given, respectively, by $\chi\gg1$ and $\chi\ll1$, where $\chi=2zV_0/V$, $V_0=e^2/\hbar$. These two equations thus represent two extreme and limiting cases; over a broad energy region of the ion, as it slows down from its initial high energy at which Eq. (2) is valid, neither of these equations is usable and no effective means of computing the total range of an ion seems to be available. In 1948, Bohr[13] put forward a stopping-power equation which combines the classical features of Eq. (1) and the quantum mechanical features of Eq. (2),

$$\frac{dE}{dx}=\frac{2\pi z^2e^4n}{mV^2}\left\{\sum_s\ln(\eta_s^2[\chi]^{-2})+\sum_s\ln\left(\eta_s^2\left[\frac{\chi}{\eta_s}\right]^{-1}\right)\right\}, \tag{3}$$

in which $\eta_s=2V/U_s$ and $\chi=2zV_0/V$, U_s being the orbital velocity of the sth electron of the medium; the other symbols were defined earlier. The quantities within the square brackets, if less than unity, should be replaced by unity.

Mukherji and Srivastava[14] have shown recently that in the case of partially stripped heavy ions, such as fission fragments, for which neither $\chi\gg1$ nor $\chi\ll1$ holds, Eq. (3) may be used appropriately to obtain stopping powers which are in good agreement with the corresponding experimental data.

In the present work we have investigated the applicability of Eq. (3) in the cases of both partially and completely stripped heavy ions, particularly under conditions where Eq. (1) and Eq. (2) are not expected to be rigorously valid.

II. STOPPING-POWER EQUATIONS

In Eq. (3), $[\chi/\eta_s]^{-1}=1$ as long as $\chi/\eta_s\leq1$ or $U_s\leq2V\chi^{-1}$. Hence the summation terms may be written

$$\begin{aligned}\sum_s\ln(\eta_s^2[\chi]^{-2})+\sum_s\ln\left(\eta_s^2\left[\frac{\chi}{\eta_s}\right]^{-1}\right)&=\sum_{U_s=0}^{U_s'}\ln(\eta_s^2[\chi]^{-2})+\sum_{U_s=0}^{2V\chi^{-1}}\ln\eta_s^2+\sum_{U_s=2V\chi^{-1}}^{U_s''}\ln(\eta_s^3\chi^{-1})\\&=J_1+J_2+J_3.\end{aligned} \tag{4}$$

The lower limit of $U_s=0$ is an approximation by Bohr[13] and the upper limits U_s' and U_s'' correspond to those values of U_s at which the logarithmic terms in Eq. (4) become zero. These upper cutoff values signify that physically no negative energy loss is possible. For the sake of convenience, the three summation terms figuring in the right-hand side of Eq. (4) will be henceforth designated by the symbols J_1, J_2, and J_3, corresponding to the order in which they are written. In order to obtain a set of general stopping-power equations one has to evaluate the summation terms corresponding to different physical conditions determined essentially by the values of χ, z, Z, and V.

A. $\chi>1$

1. $V\geq\frac{1}{2}ZV_0\chi$. Since $\chi>1$, J_1 would include all values of U_s up to the velocity of the K-shell electron, which may be taken as ZV_0, if $V\geq\frac{1}{2}ZV_0\chi$. Writing $\eta_s^2=(2V/U_s)^2=2mV^2/I_s$, where the ionization potential I_s of the sth orbital electron of the medium is given by $I_s=\frac{1}{2}mU_s^2$, one obtains for J_1

$$J_1=\sum_{s=1}^{Z}\ln(\eta_s^2[\chi]^{-2})=Z\ln\left(\frac{2mV^2}{\bar{I}\chi^2}\right),\tag{5}$$

where the mean ionization potential $\bar{I}$ is defined by

$$Z\ln\bar{I}=\sum_{s=1}^{Z}\ln I_s.\tag{6}$$

As far as J_2 is concerned, since we have set the condition $V\geq\frac{1}{2}ZV_0\chi$, the upper limit $2V\chi^{-1}$ already corresponds to $U_s=ZV_0$. Hence

$$J_2=\sum_{U_s=0}^{ZV_0}\ln\left(\frac{2V}{U_s}\right)^2=\sum_{s=1}^{Z}\ln\left(\frac{2mV^2}{I_s}\right)=Z\ln\left(\frac{2mV^2}{\bar{I}}\right).\tag{7}$$

Furthermore, since the lower limit for U_s in J_3 corresponds to the maximum possible value for U_s, J_3 becomes redundant. Thus

$$J_1+J_2+J_3=2Z\ln(2mV^2/\bar{I}\chi).\tag{8}$$

2. $\frac{1}{2}ZV_0\chi>V\geq\frac{1}{2}ZV_0\chi^{1/3}$. Since $V<\frac{1}{2}ZV_0\chi$, the value for U_s' in J_1 is $2V\chi^{-1}$, J_1 being negative for $U_s>2V\chi^{-1}$. Adopting Bohr's[13] procedure, one can replace the summation by an integral,

$$J_1=\sum_{U_s=0}^{2V\chi^{-1}}\ln\left(\frac{2V\chi^{-1}}{U_s}\right)^2=\int_{U_s=0}^{U_s=2V\chi^{-1}}\ln\left(\frac{2V\chi^{-1}}{U_s}\right)^2dn(U_s),\tag{9}$$

where $n(U_s)$ is the number of orbital electrons with velocities less than a given velocity U_s. Mukherji and Srivastava[14] have shown that for a medium of atomic number Z, $n(U_s)$ is given by

$$n(U_s)=f(Z)U_s/V_0,\tag{10}$$

where $f(Z)=0.28Z^{2/3}$ for $Z\leq45.5$ and $f(Z)=Z^{1/3}$ for $Z\geq45.5$. Equation (10), however, suffers from the drawbacks that it does not yield the correct values for the velocities of the outermost or the innermost electron of an atom and that it does not satisfy the normalization condition

$$\int_{U_s=0}^{ZV_0}dn(U_s)=Z.\tag{11}$$

Mukherji[15] has attempted to avoid this difficulty by assuming that Eq. (10) is valid for all of the electrons of the medium except the two K-shell electrons. Since J_1 does not include the velocities of the K-shell electrons, one can apply Eq. (10) and write

$$J_1=\int_{U_s=0}^{2V\chi^{-1}}\ln\left(\frac{2V\chi^{-1}}{U_s}\right)^2dn(U_s)=\frac{f(Z)}{V_0}\int_0^{2V\chi^{-1}}\ln\left(\frac{2V\chi^{-1}}{U_s}\right)^2dU_s=\frac{4f(Z)\chi^{-1}V}{V_0}.\tag{12}$$

Similarly,

$$J_2=\int_{U_s=0}^{2V\chi^{-1}}\ln\left(\frac{2V}{U_s}\right)^2dn(U_s)=\frac{4f(Z)\chi^{-1}(1+\ln\chi)V}{V_0}.\tag{13}$$

Considering J_3, the maximum value for U_s would be the upper cutoff value $U_s''=2V\chi^{-1/3}$, and since we have set the condition $V\geq\frac{1}{2}ZV_0\chi^{1/3}$, one can write

$$J_3=\sum_{U_s=2V\chi^{-1}}^{U_s''}\ln(\eta_s^3\chi^{-1})=\int_{U_s=2V\chi^{-1}}^{ZV_0}\ln\left(\frac{2V}{U_s\chi^{1/3}}\right)^3dn(U_s).\tag{14}$$

Since Eq. (10) is not applicable in the case of the K-shell electrons which happen to be included in J_3, one may separate the contributions of the outer $(Z-2)$ electrons from that of the two K-shell electrons and write

$$J_3=\frac{f(Z)}{V_0}\int_{U_s=2V\chi^{-1}}^{(Z-2)V_0/f(Z)}\ln\left(\frac{2V}{U_s\chi^{1/3}}\right)^3dU_s+2\ln(2V/ZV_0\chi^{1/3})^3.\tag{15}$$

The upper limit $U_s=(Z-2)V_0/f(Z)$ in the integral above represents the velocity of the $(Z-2)$th orbital electron and is taken from Eq. (10). An indirect justification of this artificial separation lies in the fairly good agreement between the experimental values of the mean excitation potential of the elements and the corresponding values calculated using the above procedure.[15] Evaluation of the integral leads to

$$J_3 = \frac{3f(Z)}{V_0}\left(\frac{(Z-2)V_0}{f(Z)} - 2V\chi^{-1} + 2V\chi^{-1}\ln\chi^{-2/3} - \frac{(Z-2)V_0}{f(Z)}\ln\frac{(Z-2)V_0\chi^{1/3}}{2Vf(Z)}\right) + \left(6\ln\frac{2V}{ZV_0} - 2\ln\chi\right). \tag{16}$$

From Eqs. (12), (13), and (16) we get

$$J_1 + J_2 + J_3 = 3(Z-2) + 2f(Z)V/V_0\chi + 3(Z-2)\ln[f(Z)/(Z-2)V_0] - Z\ln\chi - 6\ln(ZV_0) + 3Z\ln(2V). \tag{17}$$

3. $V<\frac{1}{2}ZV_0\chi^{1/3}$. In this case, both J_1 and J_2 would require appropriate cutoff values U_s' and U_s''. This has been considered in detail by Bohr,[13] and the general expression is given by[14]

$$J_1 + J_2 + J_3 = [2f(Z)/V_0](3\chi^{-1/3} + \chi^{-1})V. \tag{18}$$

B. $\chi<1$

1. $V\geq\frac{1}{2}ZV_0$. Equation (3) is expected[13] to yield Eq. (2), excluding the $\ln(1-\beta^2)$ and β^2 terms, if $\chi<1$ and the velocity of the particle is such that it is capable of ionizing even the innermost electron of the medium. The maximum velocity imparted by a heavy ion of velocity V to an electron is $2V$, and if the velocity of the K-shell electron is taken as ZV_0, then the second condition mentioned above is fulfilled if $V\geq\frac{1}{2}ZV_0$. For $\chi<1$, both J_1 and J_2 become identical and J_3 becomes redundant.[13] Thus

$$J_1 + J_2 + J_3 = 2\sum_{s=1}^{Z}\ln\left(\frac{2V}{U_s}\right)^2 = 2Z\ln\frac{2mV^2}{\bar{I}}. \tag{19}$$

2. $V<\frac{1}{2}ZV_0$. In this case, an appropriate cutoff value U_s' is needed for J_1 or J_2, since all of the electrons are not able to participate in the energy-loss process. Since the logarithmic term in J_1 or J_2 becomes zero if $U_s'=2V$, we have[13,14,16]

$$J_1 = J_2 = \int_{U_s=0}^{2V}\ln\left(\frac{2V}{U_s}\right)^2 dn(U_s) = \frac{4f(Z)V}{V_0}, \tag{20}$$

and

$$J_1 + J_2 + J_3 = 8f(Z)V/V_0. \tag{21}$$

After substitution of the values of $J_1+J_2+J_3$ from Eqs. (8), (17)–(19), and (21) in Eq. (3), conversion into proper units, and simplification, one obtains the following stopping-power equations valid under the stated conditions:

(i) $\chi>1$, $V\geq\frac{1}{2}ZV_0\chi$:

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{63.65z^2Z}{AV^2}\log_{10}\left(\frac{11.39V^2}{\bar{I}\chi}\right). \tag{22}$$

(ii) $\chi>1$, $\frac{1}{2}ZV_0\chi>V\geq\frac{1}{2}ZV_0\chi^{1/3}$:

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{13.79z^2}{AV^2}\left(3(Z-2) + 3(Z-2)\ln\frac{2f(Z)V}{(Z-2)V_0} + 6\ln\frac{2V}{ZV_0} + \frac{2f(Z)V}{V_0\chi} - Z\ln\chi\right). \tag{23}$$

(iii) $\chi>1$, $V<\frac{1}{2}ZV_0\chi^{1/3}$:

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{12.68f(Z)z^2}{AV}(3\chi^{-1/3} + \chi^{-1}). \tag{24}$$

(iv) $\chi<1$, $V\geq\frac{1}{2}ZV_0$:

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{63.65z^2Z}{AV^2}\log_{10}\left(\frac{11.39V^2}{\bar{I}}\right). \tag{25}$$

At relativistic energies of the ion, Eq. (25) should be identical with Eq. (2),

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{63.65z^2Z}{AV^2}\left(\log_{10}\frac{11.39V^2}{\bar{I}(1-\beta^2)} - \frac{\beta^2}{2.303}\right). \tag{26}$$

(v) $\chi<1$, $V<\frac{1}{2}ZV_0$:

$$\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = \frac{50.6f(Z)z^2}{AV}. \tag{27}$$

In all of the above equations both V and V_0 ($=2.184\times10^8$ cm/sec) are expressed in units of 10^8 cm/sec, $\bar{I}$ is in eV, A and Z are the mass number and the atomic number of the medium, and ze represents the ionic charge of the particle.

C. Values of z and $\bar{I}$

In order to calculate the stopping powers from the above equations, one must specify the values of z and $\bar{I}$. z in Eq. (3) stands for the ionic charge number (the actual ionic charge being ze) and is defined as the effective charge of the ion at a particular velocity V. Using Bohr's[13] idea that the effective charge is given by the number of orbital electrons of the ion whose velocities are less than the ion velocity V, it has been shown[14] that

$$z = f(Z_1)V/V_0, \tag{28}$$

where Z_1 is the atomic number of the ion and $f(Z_1)$ assumes the values $0.28Z_1^{2/3}$ at $Z_1\leq45.5$ and $Z_1^{1/3}$ at $Z_1\geq45.5$. Equation (28), however, is inapplicable if the ion is more than half stripped,[17] i.e., $z=\frac{1}{2}Z_1$. Thus z cannot be evaluated from Eq. (28) at velocities above $V=Z_1V_0/2f(Z_1)$. To calculate z Northcliffe[18] used the relationship

$$\gamma^2 = (z/Z_1)^2 = 1 - 1.85e^{-2V/V_k}, \tag{29}$$

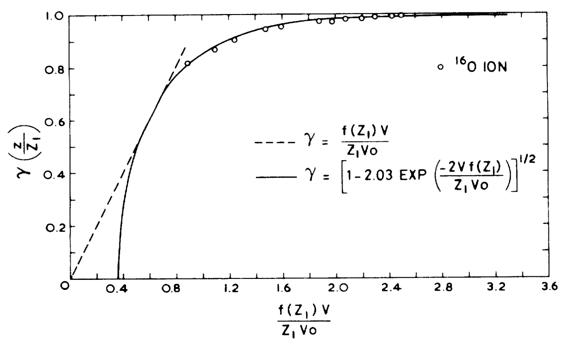

FIG 1. Variation of the effective charge parameter γ $(=z/Z_1)$ with ion velocity, expressed in the form $f(Z_1)V/Z_1V_0$. Open circles are the experimental data for ^{16}O from Northcliffe Ref. 18.

where γ stands for the fractional effective charge z/Z_1 and V_k is the velocity of the K-shell electron of the ion. From the point of view of practical usefulness, Eq. (29) should have a smooth continuity with Eq. (28) at the lower velocities. Equation (29), however, fails in this respect, since at $V = Z_1V_0/2f(Z_1)$, at which Eq. (28) predicts $z = \frac{1}{2}Z_1$, for ^{16}O it yields a negative value for z. Following Northcliffe's[18] suggestion that z should be calculated from an expression of the type

$$z = Z_1[1 - c\exp(-2V/Z_1^{2/3}V_0)]^{1/2}, \qquad (30)$$

where c is a constant and $Z_1^{2/3}V_0$ represents the Thomas-Fermi velocity for the ion, we have used the following relationship:

$$z = Z_1[1 - c\exp(-2f(Z_1)V/Z_1V_0)]^{1/2}, \qquad (31)$$

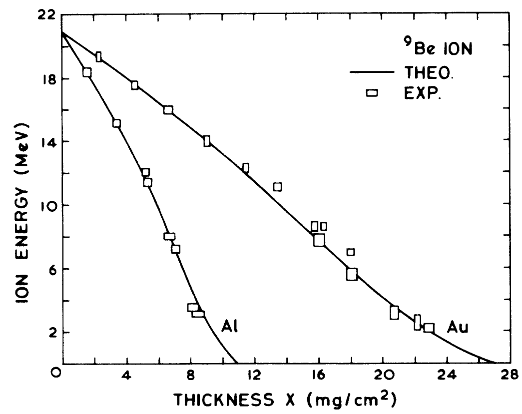

FIG. 2. Energy-loss curves for ^{9}Be in Al and Au along with the experimental data from Hower and Fairhall, Ref. 19.

where c is a constant. For an ion with $Z_1 \geq 45.5$, $Z_1V_0/f(Z_1)$ gives the Thomas-Fermi velocity $Z_1^{2/3}V_0$, while for ions with $Z_1 \leq 45.5$, $Z_1V_0/f(Z_1)$ stands for something analogous to the Thomas-Fermi velocity in the case of the heavier ions. Imposition of the constraint that both Eqs. (28) and (31) should match at the velocity $V = Z_1V_0/2f(Z_1)$, at which Eq. (28) predicts $z = \frac{1}{2}Z_1$, leads to the following expression:

$$z = Z_1[1 - 2.03\exp(-2Vf(Z_1)/Z_1V_0)]^{1/2}. \qquad (32)$$

Figure 1 shows a plot of γ $(=z/Z_1)$ vs $f(Z_1)V/Z_1V_0$ using Eqs. (28) and (32), and it can be seen that there is indeed a region of overlap followed by

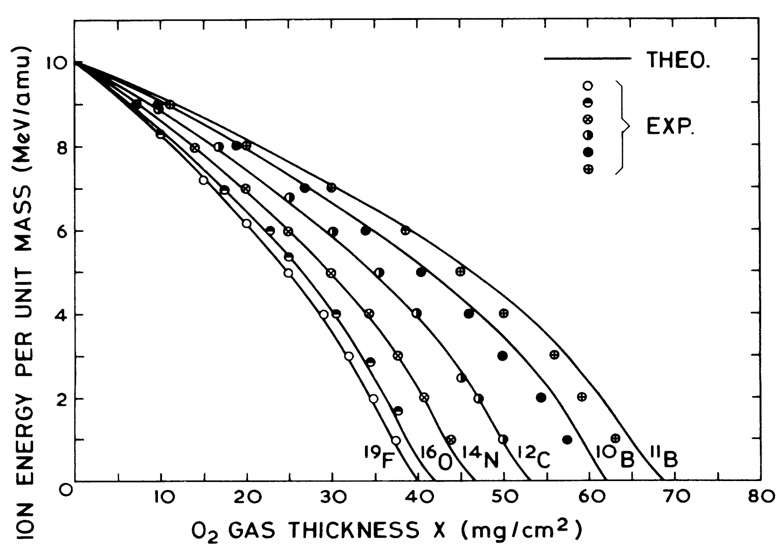

FIG. 3. Energy-loss curves for ^{10}B, ^{11}B, ^{12}C, ^{14}N, ^{16}O, and ^{19}F in oxygen. Experimental data are from Roll and Steigert, Ref. 20.

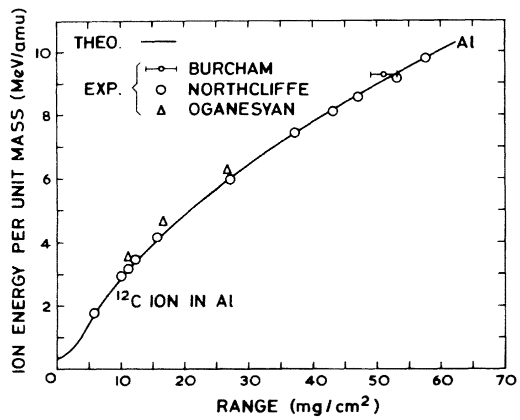

FIG. 4. Range-energy curve for the ^{12}C ion in Al. Experimental data are from Northcliffe (Ref. 18), Burcham (Ref. 21), and Oganesyan (Ref. 22).

a smooth transition. Furthermore, Fig. 1 also shows a comparison between the values of γ predicted by Eq. (32) and those obtained experimentally by Northcliffe[18] in the case of ^{16}O ions. The excellent agreement between the two allows one to use Eq. (32) with some degree of confidence.

Thus the values of z in Eqs. (22)–(27) would be given by Eq. (28) if $V \leq Z_1 V_0/2f(Z_1)$ and by Eq. (32) if $V \geq Z_1 V_0/2f(Z_1)$.

The values of $\bar{I}$ have been obtained from the following equation proposed by Mukherji[15]:

$$Z \ln \bar{I} = (Z-2)\ln\{13.6[(Z-2)/2.717f(Z)]^2\} + 2\ln(13.6Z^2), \qquad (33)$$

where $\bar{I}$ is in eV and $f(Z)$ has been defined earlier.

III. RANGES

As a heavy ion initially at $\chi < 1$ starts slowing down, the energy-loss rate would be given by the appropriate equation chosen from Eqs. (22)–(27), depending upon the instantaneous ion velocity V. The particular sequence in which the stopping-power equations must be used depends upon the ion-medium combination under consideration. The total range R corresponding to an initial ion energy E_0 is given by

$$R(\mathrm{mg/cm^2}) = \sum_{E_0}^{E_1} \frac{\delta E}{(dE/dx)_1} + \sum_{E_1}^{E_2} \frac{\delta E}{(dE/dx)_2} + \cdots + \sum_{E_{n-1}}^{E_n} \frac{\delta E}{(dE/dx)_n}, \qquad (34)$$

where δE is a small but finite amount of energy loss (e.g., ~0.01 MeV). In the energy region E_0–E_1 a particular stopping-power equation is valid and $(dE/dx)_1$ represents the values of the stopping powers to be obtained from that equation at intervals of δE until the ion energy comes down to E_1, after which the computation would continue with another stopping-power equation which is valid in the energy region E_1–E_2, and so on. The final energy E_n corresponds to a velocity $V_0 = e^2/\hbar$, since at $V < V_0$ large-angle scatterings[14] due to nuclear collisions lead to insignificant additions to the total penetration depth in the beam direction. The values of the ranges and energy losses presented in Sec. IV have been obtained by the aid of a computer (IBM 7044) into which the relevant information in the form of the stopping-power

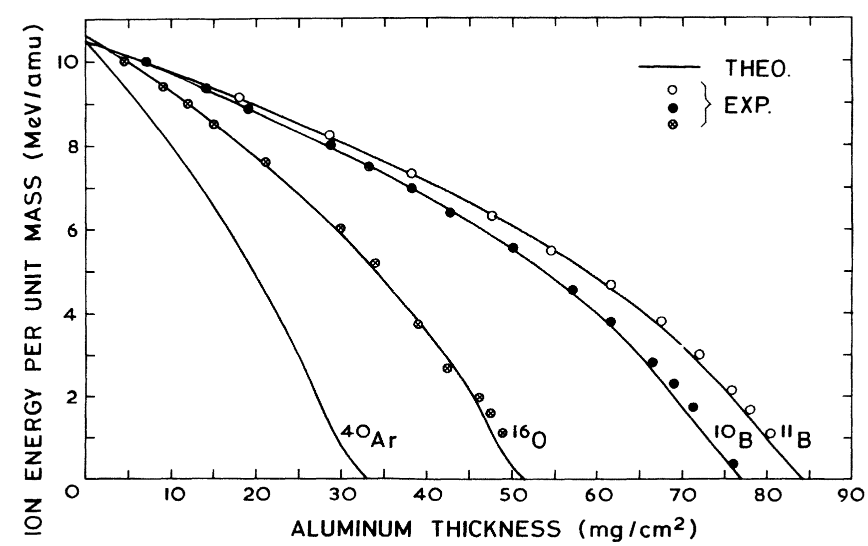

FIG. 5. Energy-loss curves for ^{10}B, ^{11}B, ^{16}O, and ^{40}Ar in Al, along with the experimental values from Northcliffe, Ref. 18.

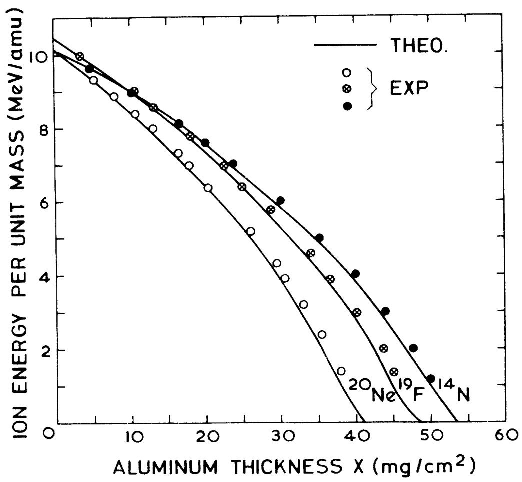

FIG. 6. Energy-loss curves for ^{14}N, ^{19}F, and ^{20}Ne ions in Al. Experimental data are from Northcliffe, Ref. 18.

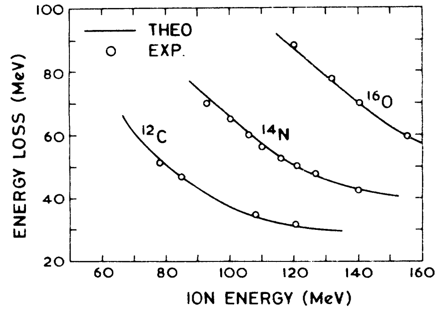

FIG. 8. Energy-loss vs incident energy of ^{12}C, ^{14}N, and ^{16}O ions in 94.6 μm of silicon. Experimental data are from Kelley *et al.*, Ref. 8.

equations [i.e., Eqs. (22)–(27)] and their respective regions of validity, as well as Eqs. (28) and (32) for the calculation of the effective charges and Eq. (33) for the calculation of mean ionization potentials, have been fed; calculations were performed with $\delta E = 10$ KeV in Eq. (34).

IV. COMPARISON WITH EXPERIMENTAL DATA

The ranges and stopping powers corresponding to different initial ion energies have been obtained for the following ions in the specified media: ^{9}Be ion in Al and Au; ^{10}B, ^{11}B, ^{12}C, ^{14}N, ^{16}O, and ^{19}F in O_2; ^{10}B, ^{11}B, ^{12}C, ^{14}N, ^{16}O, ^{19}F, ^{20}Ne, and ^{40}Ar in Al; ^{12}C, ^{14}N, and ^{16}O in Si; ^{12}C in N_2 and Ar; ^{40}Ar in N_2 and Ar; and ^{12}C and ^{16}O in Cu. For comparison with experimental data presented in the form of ion energy E against the thickness X of the medium, we have plotted the ion energy E against $R(E_0) - R(E)$, where $R(E_0)$ is the computed total range corresponding to the initial experimental ion energy E_0 and $R(E)$ is the computed range corresponding to a particular energy E of the ion. Figure 2 shows a comparison between the theoretical energy-loss curves for ^{9}Be in Al and Au and the experimental ones of Hower and Fairhall.[19] Figure 3 shows a similar comparison in the case of ^{10}B, ^{11}B, ^{12}C, ^{14}N, ^{16}O, and ^{19}F ions in oxygen gas, using the experimental data of Roll and Steigert.[20] Figure 4 snows a plot of the range R against the ion energy E in Al in the case of ^{12}C compared with the experimental values of Northcliffe,[18] Burcham,[21] and Oganesyan.[22] In Fig. 5 are shown the theoretical energy-loss curves for ^{10}B, ^{11}B, ^{16}O, and ^{40}Ar in Al and their comparison with the corresponding experimental values in the cases of ^{10}B, ^{11}B, and ^{16}O from Northcliffe.[18] Figure 6 shows Northcliffe's[18] experimental energy-loss values for ^{14}N, ^{19}F, and ^{20}Ne ions compared with the corresponding theoretical curves. Figure 7 shows the theoretical range energy curves for ^{12}C, ^{14}N, and ^{16}O ions in Si. In Fig. 8 we have plotted the theoretical values of the energy loss ΔE suffered by ^{12}C, ^{14}N, and ^{16}O ions in passing through 94.6-μm-thick Si at different incident energies E_i and compared them with the corresponding experimental values from Kelley *et al.*[8] The value of ΔE at a given E_i has been obtained from Fig. 7 as follows: A

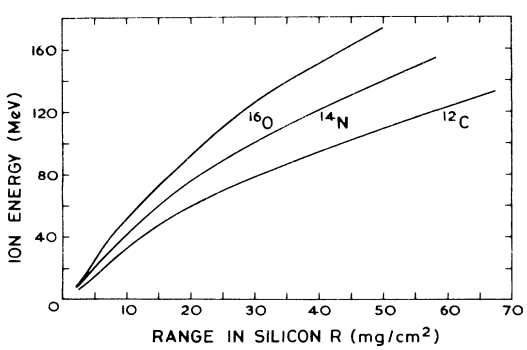

FIG. 7. Theoretical range vs energy curves for ^{12}C, ^{14}N, and ^{16}O ions in silicon.

TABLE I. Calculated and experimental values of the stopping power of ^{20}Ne ions in aluminum.

Energy (MeV)	$(dE/dx)_{calc}$ [a] (MeV cm^2/mg)	$(dE/dx)_{expt}$ [b] (MeV cm^2/mg)
18.4	9.46	9.4 ± 0.6
16.9	9.05	10.6 ± 1.8

[a] This work.
[b] Shane and Seaman, Reference 9.

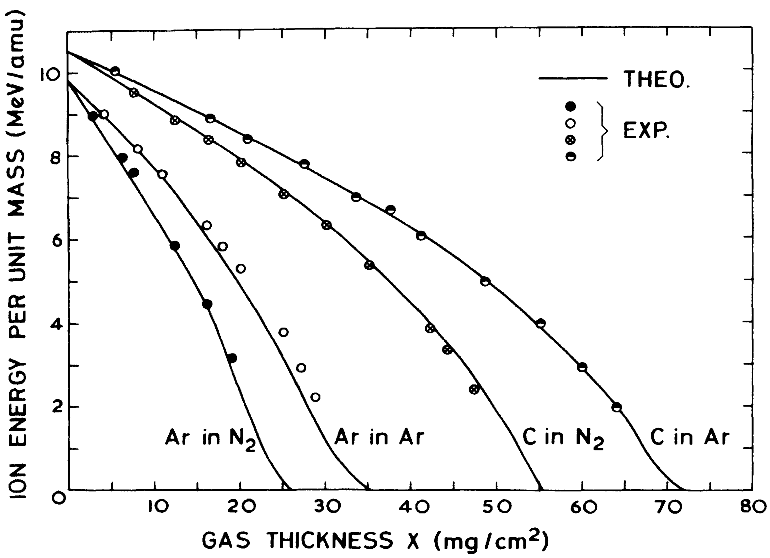

FIG. 9. Energy-loss curves for ^{12}C in Ar and N_2 and ^{40}Ar in Ar and N_2. Experimental data are from Martin and Northcliffe, Ref. 23.

thickness of 94.6 μm of Si corresponds to 22.05 mg/cm^2 of Si. From Fig. 7, the range R_i corresponding to the incident energy E_i is first read out and then the emergent energy E_e corresponding to the range $R_i - 22.05$ is obtained from the same curve. The difference between E_i and E_e represents the energy loss ΔE at the incident energy E_i. Finally, in Fig. 9 are shown a comparison between the theoretical energy-loss curves for ^{12}C in N_2 and Ar and ^{40}Ar in N_2 and Ar and the corresponding experimental values from Martin and Northcliffe.[23]

In Table I are listed the experimental stopping powers from Shane and Seaman[9] in the case of ^{20}Ne ions in Al, along with the corresponding calculated values from Eq. (24). Table II lists the theoretical and experimental ranges of ^{12}C and ^{16}O ions in Cu and the theoretical and experimental ranges of ^{11}B, ^{12}C, ^{16}O, ^{20}Ne, and ^{40}Ar in Al.[24]

TABLE II. Calculated (R_{calc}) and experimental (R_{expt}) ranges of various ions in aluminum and copper.

Ion	Medium	Energy (MeV/amu)	R_{expt} (mg/cm^2)	R_{calc} [a] (mg/cm^2)
^{11}B	Al	10.4 ± 0.2	82 [b]	82.5 ± 1
^{12}C	Al	10.4 ± 0.2	62.6 [b]	63.5 ± 1
^{16}O	Al	10.4 ± 0.2	49.4 [b]	50 ± 1.5
^{20}Ne	Al	10.4 ± 0.2	42.4 [b]	42.5 ± 1
^{40}Ar	Al	10.4 ± 0.2	34 [b]	33 ± 1
^{12}C	Cu	4.33	18.9 [c]	20.0
		5.5	27.1 [c]	28.5
		5.79	32.0 [c]	30.9
^{16}O	Cu	4.7	18.8 [c]	19.0
		6.03	27.1 [c]	25.0

[a] This work.
[b] Reference 24.
[c] Reference 22.

V. DISCUSSION

An examination of Figs. 2–6, 8, and 9 shows that except in the cases of ^{10}B and ^{11}B in O_2 and ^{40}Ar in Ar, the agreement between the theoretically calculated energy loss and the corresponding values from literature is generally very good. Since the experimental values of the energy loss of ^{10}B and ^{11}B ions in Al are in excellent agreement with the corresponding theoretical values in Al, as Fig. 5 shows, and the same holds in the case of the other ions (i.e., ^{12}C, ^{14}N, ^{16}O, and ^{19}F) in O_2, one cannot ascribe the difference between the theoretical and experimental values in the cases of ^{10}B and ^{11}B in O_2 (Fig. 3) to any systematic error in the formulation of either the effective charge of the ion or the mean ionization potential of the medium. Since the theoretical energy loss of ^{12}C in Ar and of ^{40}Ar in N_2 are in excellent agreement with the corresponding experimental values shown in Fig.9, the large deviation in the case of ^{40}Ar in Ar is not easy to explain. The energy-loss curves for several ions in Ni provided by Roll and Steigert[20] are in considerable disagreement with our calculated values and are not shown here. Table II shows, however, that the experimental ranges of ^{12}C and ^{16}O in Cu, which is close to Ni, is in fair agreement with the corresponding calculated values, although neither the detailed experimental procedure used by Oganesyan[22] nor the method of assigning the experimental errors are clearly stated. Furthermore, the excellent agreement of the calculated ranges in Al with the corresponding experimental values of Brustad[24] is particularly satisfying, since the experimental ranges were obtained by a direct recording of the ion current, instead of by extrapolation from the energy-loss curve. Finally, the excellent agreement between the calculated and experimental[8] values of the energy loss of ^{12}C, ^{14}N, and ^{16}O in silicon, shown in Fig. 8, possibly indicates that z^3 correction[5-8,10] may not be necessary for energy-loss calculations.

The only available tables of computed values of ranges and stopping-powers for heavy ions are those of Northcliffe and Schilling.[25] Although the approach is empirical, these tables are very useful for fairly-high-energy heavy ions but are not sufficiently accurate at lower energies at which the ions are incompletely stripped, and the total range would reflect this uncertainty to some extent. The present work provides a reasonable logical basis for the computation of the stopping powers and ranges of heavy ions which is free from empiricisms and adjustable parameters. Further work on the computation of ranges and stopping powers of heavy ions in complex media, such as nuclear emulsions, and solid dielectric media, such as mica and plastics, is in progress.

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