Excitation of the Ba and Ba^+ resonance lines by electron impact on Ba atoms*

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The relative optical excitation functions and polarizations of the Ba resonance line (5535 Å) and of the Ba^+ resonance lines (4554 and 4934 Å) have been measured, using crossed electron and barium-atom beams. The Ba $6\,^1P$ excitation function has been normalized to the Bethe theory in the high-energy limit, and the excitation functions of the ionic lines have been measured relative to that of the atomic resonance line. Using spontaneous-emission branching ratios for the $Ba^+(6\,^2P)$ levels, we have also obtained normalized cross sections for these ionizing excitation levels. From a few electron volts above threshold, the $Ba^+(6\,^2P)$ ionizing-excitation cross section has nearly the same energy dependence as has been reported for the total Ba^+ ionization cross section, but about 1/6 the magnitude. In particular, a broad maximum at ~ 22 eV, attributed to core-excited autoionizing levels, apparently contributes an equal fraction to both cross sections. This relationship holds also for the ionizing excitations versus total ionization of Sr. The cross sections for exciting the resonance levels of He, Mg, Ca, Sr, and Ba are also intercompared in reduced units and found to be strikingly similar at collision energies above ~ 3 times the threshold energies.

I. INTRODUCTION

The alkaline-earth elements are primarily two-electron heliumlike elements, but with visible resonance lines. Thus they provide an interesting and convenient testing ground for many of the questions raised by decades of $e^- + \mathrm{He}$ studies, e.g., threshold and high-energy polarization limits, resonance structures, and convergence to Born theory at high energy. The very core interactions that cause differences from He can also yield interesting features, such as core-excited resonances. The systematic versus specific behavior can also be seen by comparison of the Mg, Ca, Sr, and Ba cases. We have now measured the resonance-line excitation and ionizing excitation cross sections of these elements, although the ionizing excitation of Ca was omitted in the first measurement.[1-3] As this is the last of this series of measurements, we summarize and compare results here.

In this paper we present our measured optical excitation functions and polarizations for the Ba $6^1P \rightarrow 6^1S$ resonance line (5535 Å) and for Ba^+ $6^2P \rightarrow 6^2S$ resonance lines (4554 and 4934 Å) excited by electron impact on neutral barium atoms. Previous measurements of these Ba and Ba^+ excitation functions have been reported[4] with ±40% uncertainty in their absolute-cross section scale. Theoretical calculations of Ba $6\,^1P$ excitation cross section have been performed,[5-7] using either the Bethe approximation[5] or close coupling.[6-7] Comparisons are made to these results.

We have used crossed low-density beams of electrons and barium atoms, thereby minimizing space-charge and optical-depth problems. The measurements extend from threshold to 1497 eV, so that we are able to normalize the 5535-Å excitation function at high energy using the Bethe-approximation cross section and the known experimental optical oscillator strength for this resonance transition. This procedure allows us to obtain the absolute cross section for the Ba $6\,^1P$ level with about 5% uncertainty, limited primarily by knowledge of cascading. We have normalized the excitation functions of the ionic resonance lines to that of the atomic line by comparing the observed radiation intensities, thereby obtaining the absolute excitation cross sections for the Ba^+ $6\,^2P_{3/2}$ and $6\,^2P_{1/2}$ levels accurate to within about 11%.

II. MEASUREMENTS AND CORRECTIONS

The apparatus used in this experiment has been described in detail in previous papers reported from this laboratory.[1-3,8,9] Briefly, a beam of atoms from an oven intersects an electron beam at right angles and the resonance radiation in a cone along the third orthogonal axis is collected, filtered by an interference filter, and detected by a photomultiplier. A minicomputer is used to handle and acquire data. The measurements, after minor corrections described in Ref. 2, yield the polarization components parallel and perpendicular to the electron beam, $I_{\parallel}$ and $I_{\perp}$, of the radiation intensity propagating at right angles to the electron-beam axis. The polarization P is then $(I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp})$, and the relative line emission cross section R_T is proportional to $I_{\parallel} + 2I_{\perp}$. The barium-atom beam was optically thin (density $\lesssim 2\times 10^8$ atoms/cm^3) and the possible effect of radiation entrapment on polarization was found to be negligible when the beam density was varied. Using a retarding-potential analyzer, we have measured the electron-beam energy distribution; the energy spread was about 0.22 eV full width at half-maximum for energies below 13 eV and increased slightly at higher energies. We have adjusted the energy scales by reference to the excitation thresholds, an adjustment of less than 0.1 eV relative to the energy obtained with the retarding-potential analyzer.

The branching ratio for $6\,^1P \rightarrow 6\,^1S$ vs $6\,^1P \rightarrow 5\,^1D$ has been calculated[10] to be 600:1. The $6\,^1P \rightarrow 6\,^1S$ line emission cross section is thus effectively identical to the $6\,^1P$ level excitation cross section, including cascade contributions. The branching ratio B for $6\,^2P_{3/2} \rightarrow 6\,^2S$ versus $6\,^2P_{3/2} \rightarrow 5\,^2D$, and for $6\,^2P_{1/2} \rightarrow 6\,^2S$ versus $6\,^2P_{1/2} \rightarrow 5\,^2D$ of Ba^+ have been measured[11] to be 2.85 ± 0.3 and 2.77 ± 0.3, respectively. We have used these to obtain the level excitation cross sections, given by $(B+1)/B$, times the line emission cross sections. Finally, we have measured the ratios of the ionic line excitation functions to that of the atomic resonance line excitation function. Using filter transmission for each individual line, photomultiplier manufacturer's typical quantum efficiency curve (type "S-20," and "S" tubes have been used as a check), and the known branching ratios for each line, we normalize the excitation functions of the ionic levels to that of the atomic $6\,^1P$ level. The above normalization procedure had about $\pm 10\%$ uncertainty, primarily from uncertainty in the relative filter transmissions and the wavelength dependence of the photomultiplier's quantum efficiency.

III. NORMALIZATION AND CASCADE

To obtain the absolute excitation cross section for the atomic resonance level, we normalize the measured 5535-Å relative line emission cross section at high energy to the Bethe direct cross section plus cascade contributions. According to the Bethe approximation, the high-energy behavior of the total Ba $6\,^1P$ cross section Q_T (including cascade contributions) can be expanded as[2,12]

$$Q_T E\ (\pi a_0^2\ \mathrm{eV}) = 1705 \sum_{\nu=6}^{\infty} \beta_\nu \frac{f_{0\nu}}{\Delta_{0\nu}} \log_{10} E + A + O(1/E), \quad (1)$$

where Q_T is in units of πa_0^2, E is the impact energy in eV, $\Delta_{0\nu}$ and $f_{0\nu}$ are, respectively, the excitation energy in eV and the optical oscillator strength from the ground state to the $\nu\,^1P$ state, $\beta_6 = 1$, β_ν for $\nu = 7, 8, \ldots$ is the fraction of atoms in state $\nu\,^1P$ which cascade into $6\,^1P$ (this occurs mainly via $5d^2\,^1D$ and $7\,^1S$), A is a constant, and $O(1/E)$ are higher-order terms that are neglected in the Bethe approximation. Using the known oscillator strengths[13,14] for some levels and the Coulomb approximation[15] to calculate those that are not given in Refs. 13 or 14, we obtain

$$B \equiv 1705 \sum_{\nu=6}^{\infty} \beta_\nu \frac{f_{0\nu}}{\Delta_{0\nu}} = (0.731 \pm 0.036) \times 1705 = 1246 \pm 61,$$

where about 97% of B is due to the direct $(\nu = 6)$ component ($f = 1.59$ from Ref. 13 has been used for the $6\,^1P$ state) and the remainder from the higher ^1P-state cascades. The uncertainties in B are mainly due to the uncertainties in the oscillator strengths and branching ratios. We now have to estimate A in Eq. (1) in order to obtain Q_T. The Bethe direct excitation cross section (Q_B) for $6\,^1P$ calculated in Ref. 5, using $f = 1.59$, is

$$Q_B E = 1211 \log_{10} E - 231\ ,$$

which implies that A in Eq. (1) will be equal to the sum of $-231 \pi a_0^2$ eV plus the cascade contributions $(Q'E)$. The cascade contributions $n\,^1D$ levels $(n = 6-9)$ are estimated by using the measured emission cross section in Ref. 4 for spectral lines terminating on the $6\,^1P$ level. They were extrapolated as E^{-1} above 30 eV, and indicated $Q'E \approx 56 \pm 22 \pi a_0^2$ eV. The data for cascade contributions from $n\,^1S$ levels are not available, so we have roughly extrapolated the cascade data for Mg, Ca, and Sr to estimate $Q'E \approx 12 \pm 6 \pi a_0^2$ eV for $n\,^1S$ cascade contributions. The contribution to Q_T at 1497 eV from these ^1S and ^1D terms is $\sim 2\%$, so that the uncertainty in these cascade terms does not significantly affect the final accuracy of the normalization. The only other contribution expected to be significant is the $5d^2\,^1D$ direct excitation followed by $5d^2\,^1D \rightarrow 6\,^1P$ decay. We estimate that contribution at no more than 2% of the $6\,^1P$ cross section and neglect it, except for an added uncertainty. The contribution of higher ^1P-state cascades to the A term are expected to be less than 1%, and therefore are ignored.

The sum Q_T of the Bethe $6\,^1P$ cross section plus the estimated total cascade terms thus has the form

$$Q_T E = -163 + 1246 \log_{10} E\ ,$$

at the high-energy limit. Our relative 5535 Å line emission cross section is normalized to this $Q_T E$ at high energy, as shown in Fig. 1. Note that our experimental data appear to converge uniformly to this expected theoretical energy dependence at high energy. We estimate that the uncertainty of the normalized cross section scale is about $\pm 5\%$, due to the 3% uncertainty in the optical oscillator

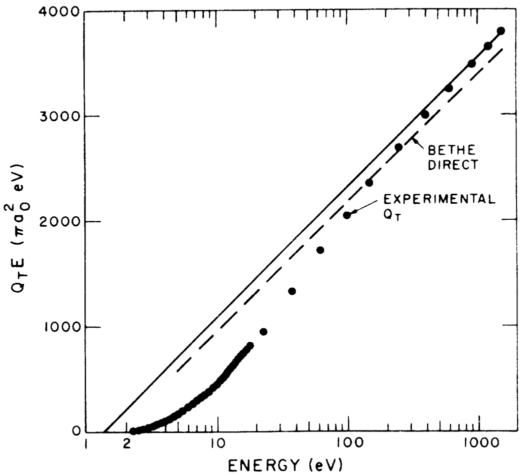

FIG. 1. Normalized total cross section Q_T for the Ba $6\,^1P$ level, including cascade contributions, times electron energy E, plotted vs $\log_{10}E$. This straight-line fit to the high-energy data is $Q_TE = -163 + 1246\log_{10}E$. The Bethe direct excitation cross section is from Ref. 5.

strength and therefore in the Bethe cross section, plus about 4% uncertainty in cascade contributions. As mentioned in Sec. II, the normalization of the Ba^+ resonance levels relative to the Ba resonance level was uncertain to ±10%; combining the uncertainties in quadrature yields ±11% as the uncertainty of the cross-section scale for the Ba^+ levels.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. Ba $6\,^1P \rightarrow 6\,^1S$ line

The present results are given in Table I and in Figs. 1–3. As noted in Sec. II, the corrections described in Ref. 2 have been taken into account. The uncertainty in the cross sections quoted in Table I includes uncertainties of the crossed beams's overlap, in correction factors, and the observed statistical uncertainty, but not the normalization uncertainty. The uncertainties in the polarization value are mainly from the counting statistics and the uncertainty in the instrumental polarization.

The data were obtained at energy intervals of less than 0.1 eV below 18 eV, and at 11 energy values between 23 and 1497 eV. Table I contains all of the data above 23 eV, and representative averaged values below that. The original data below 12 eV can be seen in Fig. 3. Because of the smooth behavior of the high-energy Q_T data shown in Fig. 1, we have represented our data as a continuous curve in subsequent plots.

The threshold behavior of our cross section is obscured by the energy spread of the electron

TABLE I. Experimental results for electron-impact excitation of the Ba 5535-Å resonance line including cascade contributions.

Energy [a] (eV)	Polarization (%)	Q_T [b] (πa_0^2)
2.30(1) [c]	78.4(22) [c]	2.11(7) [c]
2.40(1)	75.4(17)	3.52(9)
2.50(1)	70.9(13)	5.03(11)
2.70(1)	61.9(10)	8.68(16)
3.00(1)	51.1(7)	13.23(20)
3.50(1)	37.3(6)	20.05(30)
4.00(1)	31.9(5)	28.48(34)
5.00(1)	26.5(3)	36.74(39)
6.00(2)	22.5(2)	41.06(43)
7.00(2)	18.9(2)	43.96(46)
7.50(2)	16.5(2)	44.39(47)
8.00(2)	16.7(2)	43.92(46)
8.35(2)	16.7(2)	42.98(46)
9.00(3)	16.5(2)	44.57(47)
10.00(3)	16.2(2)	45.45(49)
11.00(3)	15.2(1)	46.78(48)
12.60(3)	13.3(1)	56.92(48)
15.34(5)	10.6(1)	46.71(32)
22.81(6)	4.3(1)	42.05(24)
37.6(2)	−3.2(1)	35.34(20)
62.4(2)	−10.9(1)	27.50(16)
98.3(2)	−16.9(1)	20.87(12)
148.9(2)	−22.0(1)	15.85(9)
247.9(2)	−27.0(1)	10.86(6)
399.5(3)	−30.9(2)	7.512(48)
600.4(3)	−33.9(2)	5.415(35)
897.6(6)	−36.6(2)	3.880(26)
1198.3(8)	−38.3(2)	3.054(22)
1497.0(10)	−39.8(2)	2.534(17)

[a] Mean energy of the incident electrons, corrected by reference to the $6\,^1P$ excitation threshold (2.238 eV). The electron energy resolution was ≈0.22 eV full width at half-maximum for energies below 13 eV. Besides the uncertainty given for each point, there is an additional uncertainty of ±0.05 eV in the energy scale discussed in Sec. IV.

[b] Q_T is the normalized level excitation cross section including cascade contributions. The uncertainties given in columns do not include the uncertainty of ±5% to our normalized-cross-section scale.

[c] Number in parentheses gives the uncertainty in the last places of the previous number. In columns 2 and 3, the quoted uncertainties include the observed statistical uncertainties (roughly 2σ) and estimated systematic uncertainties.

beam. To find the threshold energy we have assumed the cross section has the form $Q_T \propto (E-\Delta)^{1/2}$ or $Q_T \propto (E-\Delta)$ in the first 0.2 eV above threshold and is convoluted with the electron energy distribution. Both forms yield satisfactory fits to our data, while they reach zero cross section at different points about 0.05 eV apart along the energy axis. We have adopted the average of these two zero-cross-section points as the known excitation

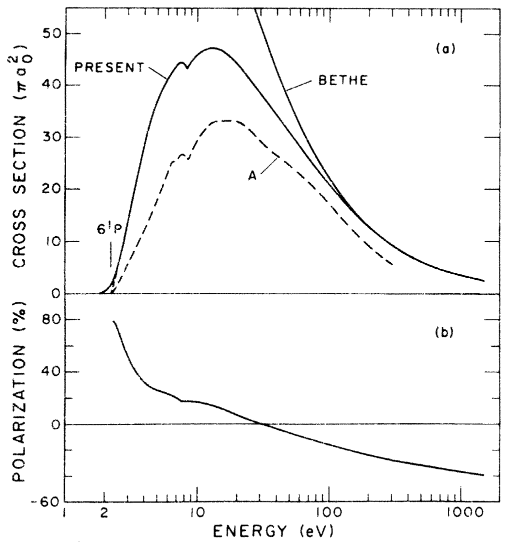

FIG. 2. Normalized total excitation cross section for the Ba $6\,^1P$ level and polarization of the 5535-Å resonance line ($6\,^1P \rightarrow 6\,^1S$). The present cross section is compared with previous measurements by Aleksakhin *et al.* (Ref. 4) (A) and Bethe theory (Ref. 5). All experimental data include cascade, while the Bethe theory represents only the direct excitation cross section.

energy ($\Delta = 2.238$ eV) of the $6\,^1P$ level and used it to correct our energy scale. From this convolution procedure we assume that our energy scale is uncertain to ± 0.05 eV, in addition to the uncertainties given for individual energy points in Table I.

The normalized excitation function and polarization are shown in Figs. 2 and 3. In Fig. 2(a) the present results are compared with earlier measurements by Aleksakhin *et al.*[4] Both results include cascade contributions. Aleksakhin *et al.* used a crossed-beam apparatus and spectrometer to carry out their absolute measurements for the excitation function and quoted $\pm 40\%$ for their experimental uncertainty. Both results show similar structure near 8 eV, which may be caused by cascade contributions. However, their excitation function was somewhat different in shape, as well as in magnitude, from ours; we suspect that their measurements might be, to a certain degree, affected by Ba resonance radiation entrapment, since they used a much larger atom-beam density than that used in the present work ($\gtrsim 500$ times larger).

In Fig. 2(a) the Bethe cross section for the direct excitation of $6\,^1P$ state calculated by Kim and Bagus[5] also has been compared with our normalized cross section (including cascade contributions). The only Ba excitation cross section calculation other than the Bethe approximation was made by Fabrikant.[6,7] The calculations in Ref. 6 are for

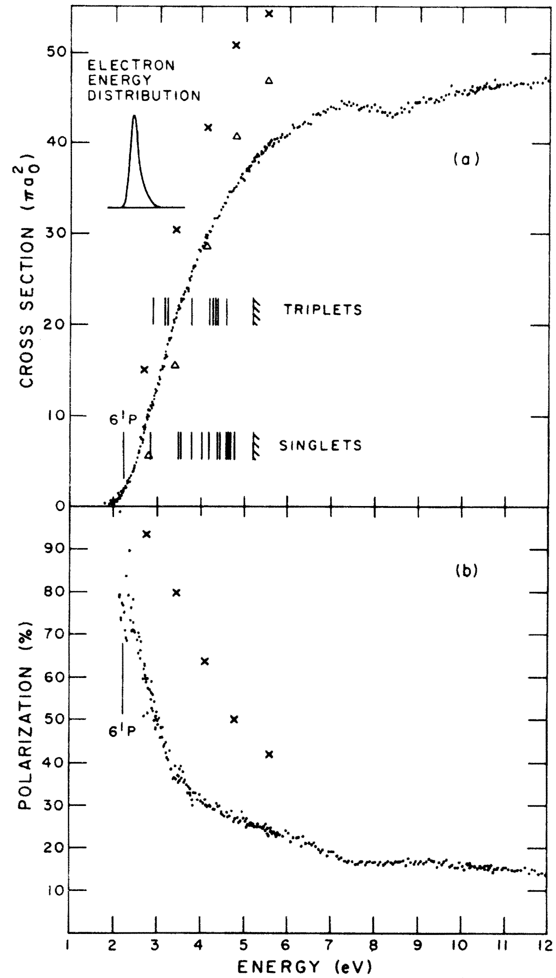

FIG. 3. Detailed low-energy experimental data (dots) for the $6\,^1P$ level, compared with the results of the $6\,^1S$, $6\,^1P$, $6\,^3P$ close coupling (×, Ref. 6, Tables 7 and 19) and of the $6\,^1S, 6\,^1P, 5\,^1D$ close coupling (△, Ref. 6, Table 10). Experimental data include cascade contributions, while the theoretical results are for direct $6\,^1P$ excitation only. Excitation thresholds for some of the higher singlet and triplet levels, which may have cascade contributions or affect the direct cross section, are marked by bars.

$6\,^1S$, $6\,^1P$, $6\,^3P$ close coupling, and $6\,^1S$, $6\,^1P$, $5\,^1D$ close coupling. Both results are shown in Fig. 3(a) for comparison. Note that the inclusion of $5\,^1D$ in the calculation yields a much lower $6\,^1P$ cross section as Damburg and Fabrikant have pointed out in Ref. 7. The polarization function from Ref. 6 is also compared in Fig. 3(b). This polarization function is for zero nuclear spin, whereas our measured polarization function was expected to be affected by resonance of the nonzero nuclear spin isotope contained in the natural barium; this will be discussed in more detail below.

Cascade contribution to the $6\,^1P$ level begins at 2.86 eV and may cause the structure discernible in the data of Fig. 3. Unfortunately, there is not sufficient information about cascade available for further discussion.

In the simple case of $^1S_0 \rightarrow {}^1P_1$ excitation followed by $^1P_1 \rightarrow {}^1S_0$ decay with zero nuclear spin the threshold polarization predicted by Percival and Seaton[16] is +100% and the high-energy limit is −100%. This predicted threshold behavior has often been obscured by resonances in He excitations,[17] but is clearly seen in Mg, Ca, and Sr excitation. In the present experiment, the natural barium used contains 18% of ^{135}Ba and ^{137}Ba, both of which have a nuclear spin of $\frac{3}{2}$. The hyperfine separation[18] of the ^{135}Ba and ^{137}Ba is much larger than the natural level width, and therefore the 5535-Å line radiation from ^{135}Ba and ^{137}Ba should be largely depolarized. The expected threshold polarization thus corresponds to $(I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp})$, with $I_{\parallel} \cong 0.82 + 0.18/3$ and $I_{\perp} \cong 0.18/3$, or about 87%. Our threshold polarization data in Fig. 3(b) are consistent with the expected threshold limit, considering the electron energy spread and the large statistical experimental uncertainty for the small signals near threshold. At high energies, our polarization data are consistent with the expected logarithmic convergence to about −77%.[1,2]

B. $Ba^+\ 6\,^2P \rightarrow 6\,^2S$ lines

Our results for the ionic resonance doublets are shown in Figs. 4 and 5. The data were obtained at energy intervals of about 0.1 eV below 20 eV, at energy intervals of about 0.15 eV between 20 and 30 eV, and at ten energy values between 38 and 1497 eV. To produce the results shown in Figs. 4 and 5, we have plotted smooth curves through the Q_T data. The uncertainty of the relative excitation function is about ± 2–3% for all energies.

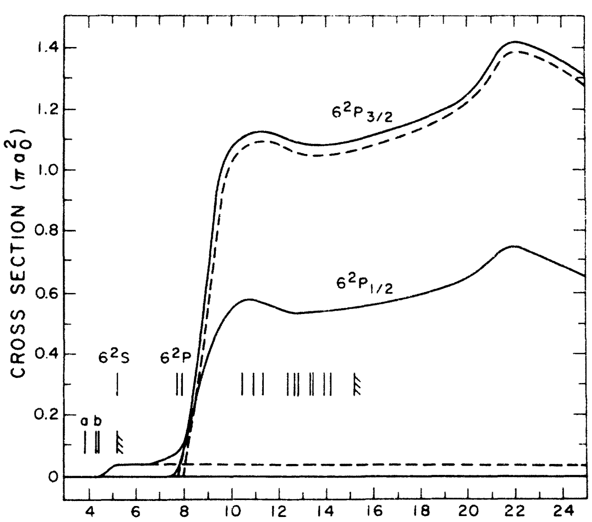

FIG. 4. Normalized total cross section for ionizing excitation of the Ba+ $6\,^2P_{3/2}$ and $6\,^2P_{1/2}$ levels (including cascades) from the ground state of Ba. Spectral leakages from nearby Ba lines into the $6\,^2P_{3/2} \rightarrow 6\,^2S$ line cause the signals below 8 eV; this has been extrapolated above 8 eV (dashed line) and subtracted from the raw data (solid curve), yielding the corrected cross section (dashed curve). No leakage correction was necessary for the $6\,^2P_{1/2}$ level. Thresholds for some cascade-producing terms are indicated by bars.

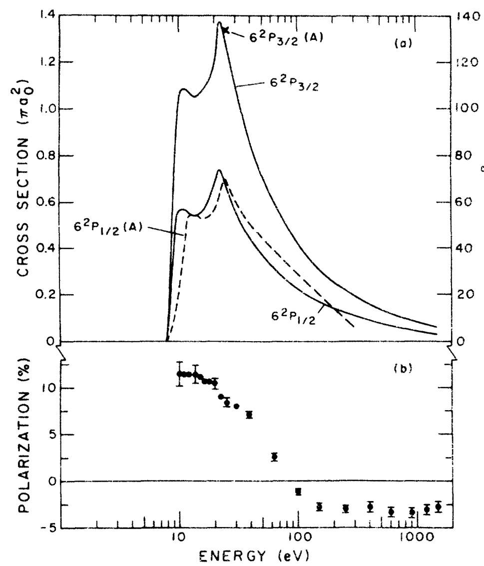

FIG. 5. Normalized total cross section for the Ba+ $6\,^2P_{3/2}$ and $6\,^2P_{1/2}$ levels and polarization for the 4554-Å ($6\,^2P_{3/2} \rightarrow 6\,^2S_{1/2}$) line. The present cross sections (solid curves) are compared with measurements of Aleksakhin *et al.* (A). These level excitation cross sections are obtained by multiplying the 4554- and 4934-Å line excitation cross sections of both experiments by 1.35 and 1.36, respectively, due to branching ratios discussed in Sec. II. All of the experimental data include cascades. The present result for $6\,^2P_{3/2}$ level has been corrected for spectral leakage of atomic Ba lines as indicated in Fig. 4.

The atomic Ba spectrum contains a number of lines too close to the Ba+ 4554-Å resonance line to be completely rejected by our filter. Figure 4 shows how we corrected the 4554-Å excitation function for assumed leakage of these unwanted lines. It appears that the Ba lines that can account for the leakage to the 4554-Å ionic resonance line arise from the triplet terms, i.e., $6s7p(^3P) \rightarrow 6s5d(^3D^e)$ marked "*a*" and $6p^2(^3P^e) \rightarrow 6s6p(^3P^o)$ marked "*b*" in Fig. 4. These triplet-term excitation functions are spin forbidden and therefore are expected to decrease faster than the ionizing excitation function at high energy.[4,19] We have removed the effects of these leakages from the data in the Ba+ $6\,^2P_{3/2}$ threshold region by extrapolating from the observations below 8 eV, as shown in Fig. 4. We regarded this leakage as inconsequential at higher energies. For lack of sufficient information about these leakages, their effect on our observed ionic line polarization has been ignored. No atomic-line leakage through the 4934-Å line was observed, as shown in Fig. 4.

It is also possible that both ionic resonance lines have been affected by the leakages from the Ba^+ $7\,^2S \rightarrow 6\,^2P$ doublets (4525 and 4900 Å). In our experiment the contributions from the unwanted ionic lines could not be identified and subtracted out, as we did for the unwanted atomic lines mentioned above. However, the contributions of the $7\,^2S \rightarrow 6\,^2P$ lines to our observed ionic resonance lines were estimated by comparing the measured excitation functions[4] and the relative filter transmission curves for lines involved; it then accounts for only 0.2% or less.

The present excitation functions are compared with previous measurements by Aleksakhin *et al.*[4] in Fig. 5. For this comparison, we have multiplied their excitation cross sections by the $(1+B)/B$ factors in Sec. II (1.35 and 1.36 for the 4554- and 4934-Å lines, respectively) to yield level excitation cross sections. The measured ratio of the $6\,^2P_{3/2}$ to $6\,^2P_{1/2}$ level excitation cross section is 1.91(5) in the present experiment and essentially independent of collision energy, compared with 1.92 from the data reported in Ref. 4 [times the $(1+B)/B$ factors] at one energy where the cross sections are maximum. *LS* coupling without variations of radial wave functions predicts[16] a ratio of 2:1. This is in contrast to the Sr^+ case, where we measured a ratio of 1.56:1 for the equivalent $5\,^2P_{3/2}$ to $5\,^2P_{1/2}$ level ionizing excitations.

V. COMPARISON TO SIMILAR ELEMENTS

A. Atomic resonance lines

In Fig. 6 we compare the electron-impact excitation cross sections of the resonance levels of helium[20] and the alkaline-earth elements, as measured in this laboratory.[1-3] These are similar excitation processes, since the He transition is a $1s^2\,^1S_0 \rightarrow 1s2p\,^1P_1$ transition, with $f = 0.276$, and the alkaline earths are $ns^2\,^1S_0 \rightarrow nsnp\,^1P_1$ transitions with $f = 1.5-2.0$. According to the Born and Bethe approximations, the quantity $Q_T\Delta^2/f$ will become a universal function of E/Δ for dipole transitions in the high-energy limit, so we have used these units in Fig. 6. In addition to the Bethe[5] cross sections shown in Fig. 6, Robb[21] has calculated Born cross sections for Ca and Mg, which are 5–15% larger at ~3 times the threshold energies and essentially the same at high energy. The Born and Bethe cross sections for He are the same at $E/\Delta \geq 1.5$. As one can see in Fig. 6, the measured cross sections in these reduced units (including cascading, except for He estimated as typically

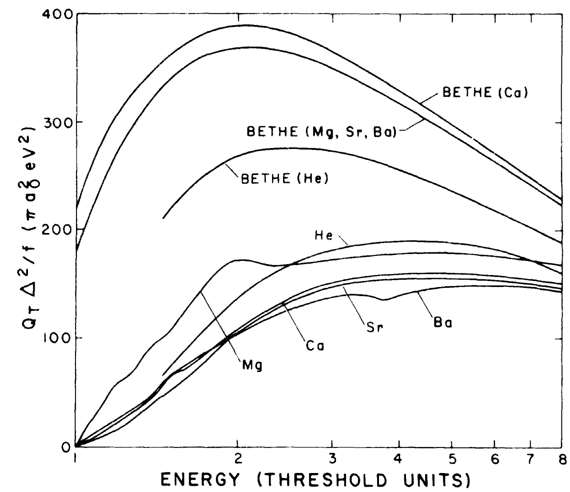

FIG. 6. Comparison of excitation cross sections for the atomic resonance levels of helium (Ref. 20) and of the alkaline-earth elements, Mg (Ref. 1), Ca (Ref. 2), Sr (Ref. 3), and Ba (present work). For each case we have plotted $Q_T\Delta^2/f$ vs E/Δ, where Q_T is the cross section including cascades (except for He), Δ the excitation threshold energy, and f the resonance-line optical oscillator strength. The Bethe direct excitation cross sections for Mg, Ca, and Ba are from the high-energy analytic formula by Kim and Bagus (Ref. 5) and are not considered to be reliable at low energy. The Bethe direct excitation for the He resonance level is given by Ref. 20. Apart from minor differences due to cascades all of the curves merge at high energy.

5–15%) converge within ±15% of a common result by three threshold energies. On the other hand, the Born and Bethe cross section are typically ~100% too high in this energy range and 15% convergence to the actual cross sections occurs only at about 20 threshold energies for Mg, Ca, Sr, and Ba (see Fig. 2 and Refs. 1–3) and 10 thresholds for He (Ref. 20). If the alkali-metal resonance-line excitations of Refs. 8 and 9 (also with cascades) are included in Fig. 6, they average about 10% above the average of the cross sections shown. Thus it appears that a fairly universal result for allowed dipole transitions is emerging from these measurements. As it is not specific to any species, a relatively simple universal theory should explain it. While Glauber, close-coupling, and other approximations do give reasonable agreement with measurements for some cases, these presumably depend strongly on the species and atomic wave functions. Thus we are not aware of any electron collision theory which has proposed or implied such a universal result. The idea of simple universal approximations for intermediate and low energies has been investigated in Refs. 22–24. We hope these comparisons will help stimulate a search for a simple universal

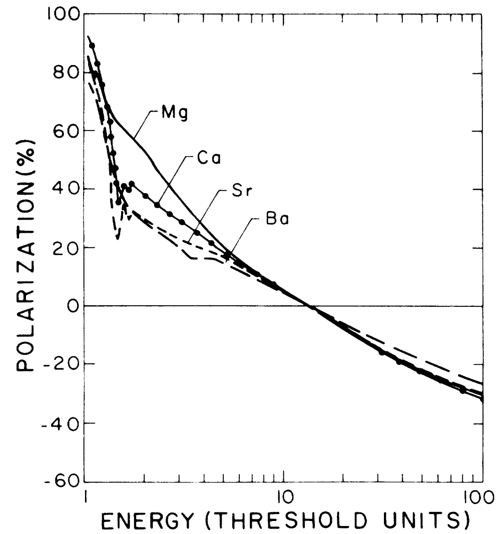

FIG. 7. Comparison of the measured polarization of the Mg, Ca, Sr, and Ba resonance lines.

improvement on the Born and Bethe approximations for this intermediate-energy range.

For use in comparisons with other measurements, theories, and applications we will note a simple analytic expression which fits this average result as accurately as it can be defined at all energies:

$$(\Delta^2/f)Q_{\text{total observed}}/(\pi a_0^2\ \text{eV}^2)$$

$$\cong (1-X^{-1/2})[(280+1705\log_{10}X)/X], \quad (2)$$

where $X=E/\Delta$. For the Mg, Ca, Sr, and Ba cases the expression in brackets is very close to the

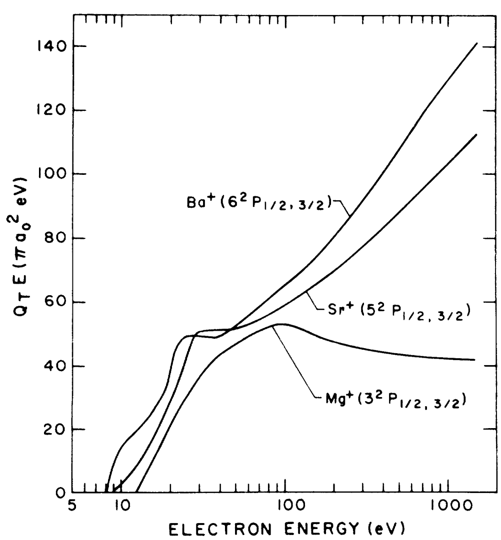

FIG. 8. Comparison of the high-energy behavior of the ionic resonance level ($^2P_{1/2,3/2}$) excitation cross sections for Mg^+, Sr^+, and Ba^+.

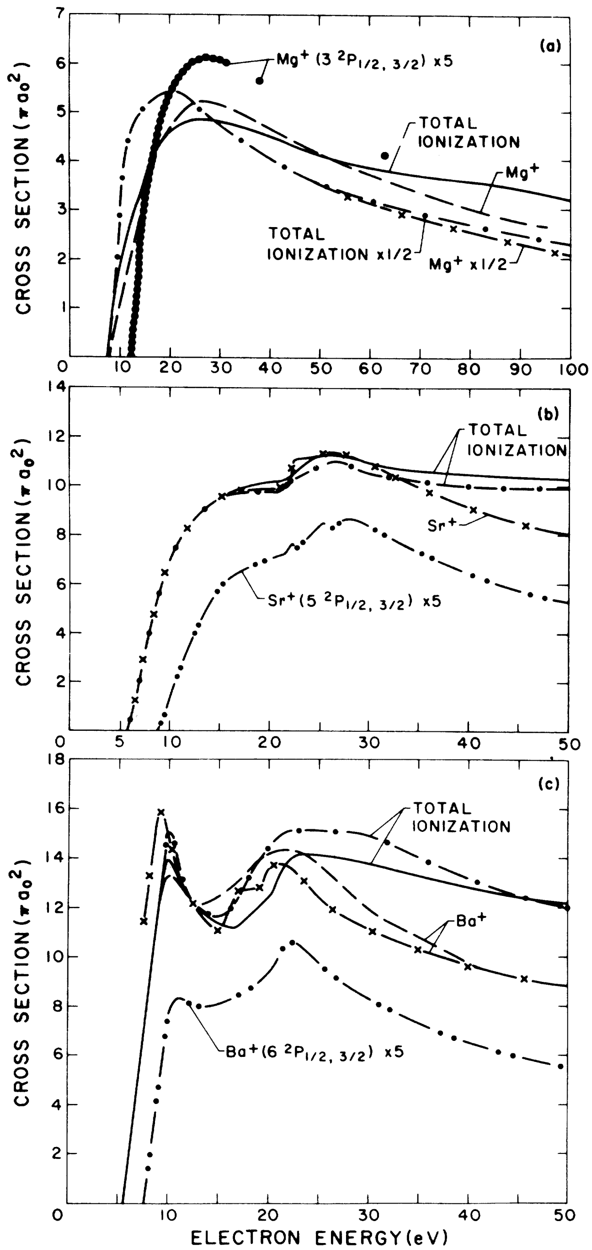

FIG. 9. Comparison of the Mg, Sr, and Ba ionization functions. (a) ···: Leep and Gallagher (Ref. 1); ——: Vainshtein *et al.* (Ref. 26), absolute measurement; —·—: Okuno *et al.* (Ref. 27), absolute measurement; — —: Kaneko (Ref. 29), normalized here to Ref. 26 at 20 eV; —×—: Okudaira *et al.* (Ref. 31), normalized to Ref. 27. (b) —··—: Chen *et al.* (Ref. 3); ——: Vainshtein *et al.* (Ref. 26), absolute measurement; —·—: Okuno (Ref. 28), normalized here to Ref. 26 at 15 eV; —×—: Okudaira (Ref. 32), normalized here to Ref. 26 at 15 eV. (c) —·-—: this work; ——: Vainshtein *et al.* (Ref. 26), absolute measurement; —·—: Okuno (Ref. 28), normalized here to Ref. 26 at 12.4 eV; — —: Ziesel and Abouaf (Ref. 30), normalized here to Ref. 26 at 12.4 eV; —×—: Okudaira (Ref. 32), normalized here to Ref. 26 at 12.4 eV.

Bethe cross section. For He the number 280 is replaced by a different constant in the Bethe cross section. In fact all of the cross sections in Fig. 6 can be quite accurately fitted using Eq. (2) with different constants in place of the number 280.

We have plotted the polarizations of the Mg, Ca, Sr, and Ba resonance lines versus threshold energies in Fig. 7 for comparison. For the alkaline-earth elements, the resonance-line emission is due to the $^1S_0 \rightarrow {}^1P_1$ excitation followed by $^1P_1 \rightarrow S_0$ decay. In the case of zero nuclear spin, Percival and Seaton[16] predicted +100% for threshold polarization and −100% for the high-energy limit. Considering the possible influence by the nonzero-nuclear-spin isotopes contained in our experimental samples, we thus expected the threshold polarizations, observed at 90° to the electron beam, to be +100%, +100%, +95%, and +87% for Mg, Ca, Sr, and Ba, respectively, and −100%, −100%, −91%, and −77%, respectively, for the high-energy limits. From Fig. 7, one can see that our polarization data are consistent with the expected threshold limits and a logarithmic convergence to these high-energy limits. (The convergence is logarithmic because the $\Delta m_l = \pm 1$ excitations decrease as E^{-1} and the $\Delta m_l = 0$ as $E^{-1}\log_{10}E$, as can be obtained for example from Eq. (6.13) of Ref. 16). All of the polarization functions appear to cross zero polarization at about 12 times the threshold energy. When the nuclear spin differences are allowed for, the polarizations are very similar above ~4 times the threshold energy, further indicating the universal nature of the intermediate-energy cross sections.

B. Ionic resonance lines

The $Q_T E$ vs $\log_{10}E$ plots of the ionizing excitations for Mg^+, Sr^+, and Ba^+ are compared in Fig. 8. Note that the Mg^+ ionizing excitation cross section appears to behave as E^{-1} in the high-energy limit, as expected for a dipole-forbidden transition from the ground state of Mg, while the ionizing excitation functions for Sr^+ and Ba^+ behave as $E^{-1}\log_{10}E$. One possible explanation[25] for this dipole-allowed transition behavior for Sr^+ and Ba^+ is that their ground-state wave functions are predominantly $ns^2\,{}^1S$ but also contain a substantial admixture of the $np^2\,{}^1S$ configuration. Thus the ionization of one of the np electrons in a dipole-allowed transition leaves an excited $np\,{}^2P$ ion.

In Fig. 9, we compare our Mg^+, Sr^+, and Ba^+ ionizing excitation cross sections with measured ionization cross sections.[26-32] The data by Vainshtein *et al.*[26] and Okuno and co-workers[27,28] are total-ionization cross sections, while others are single-ionization cross sections. Only the data in Refs. 26 and 27 were given in absolute values. The data of Okudaira *et al.*[31] for Mg^+ were normalized by the authors to the absolute total-ionization cross section of Mg in Ref. 27. To simplify comparisons we have normalized the other relative measurements to the absolute total-ionization cross sections in Ref. 26 at the energy where only the single-ionization can occur.

Every ionization curve for Sr and Ba shows two peaks in the low-energy region. The higher-energy peak is considered to be due to a number of autoionizing levels arising from an excited inner-shell p electron.[33] In particular, the $p^5ds^2\,{}^1P$ autoionizing level may make a major contribution. Hansen[33] has pointed out that the atoms of the p^5ds^2 levels can autoionize to either a singly or doubly ionized final state, while Hotop and Mahr[34] have measured the 584.3-Å (21.2 eV) photoelectron spectrum of Ba and clearly detect substantial population in a number of excited Ba^+ levels. Furthermore, they have found the ratio of $6\,{}^2P_{3/2}:6\,{}^2P_{1/2}$ to be 1.95(15), as we find for electron collisions and as expected from statistical weights. A similar inner-shell excitation autoionizing feature has also been observed in the ionization of Sr^+ and Ba^+ by Peart and co-workers[35,36] at electron energies consistent with that of the second peak in Fig. 9. No experimental evidence for autoionization resonances has been observed for Mg.

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[1]D. Leep and A. Gallagher, Phys. Rev. A 13, 148 (1976).
[2]V. J. Ehlers and A. Gallagher, Phys. Rev. A 7, 1573 (1973), and 9, 1026(E) (1974).
[3]S. T. Chen, D. Leep, and A. Gallagher, Phys. Rev. A 13, 947 (1976).
[4]I. S. Aleksakhin, I. P. Zapesochnii, I. I. Garga, and V. P. Starodub, Opt. Spektrosk. 38, 228 (1975) [Opt. Spectrosc. 38, 126 (1975)].
[5]Y.-K. Kim and P. S. Bagus, Phys. Rev. A 8, 1739 (1973).
[6]I. I. Fabrikant, in *Atomnye Protsessy* (Atomic Processes) (Izd. Zinatne, Riga, 1975), p. 80.
[7]R. J. Damburg and I. I. Fabrikant, in *Electronic and Atomic Collisions, Abstracts of the Papers of the Ninth International Conference on the Physics of Electronic and Atomic Collisions*, edited by J. S. Risley and R. Geballe (Univ. Washington Press, Seattle, 1975), Vol. 2, p. 1117.
[8]E. A. Enemark and A. Gallagher, Phys. Rev. A 6, 192

(1972).
[9]D. Leep and A. Gallagher, Phys. Rev. A 10, 1082 (1974).
[10]P. McCavert and E. Trefftz, J. Phys. B 7, 1270 (1974).
[11]A. Gallagher, Phys. Rev. 157, 24 (1967).
[12]M. Inokuti, Rev. Mod. Phys. 43, 297 (1971).
[13]B. M. Miles and W. L. Wiese, At. Data 1, 1 (1969).
[14]H. Friedrich and E. Trefftz, J. Quant. Spectrosc. Radiat. Transfer 9, 335 (1969).
[15]D. R. Bates and A. Damgaard, Philos. Trans. R. Soc. A 242, 101 (1949).
[16]I. C. Percival and M. J. Seaton, Philos. Trans. R. Soc. A 251, 113 (1958).
[17]B. L. Moiseiwitsch and S. J. Smith, Rev. Mod. Phys. 40, 238 (1968).
[18]A. Lauri, Phys. Rev. 136, A376 (1964).
[19]K. C. Kulander and J. S. Dahler, Phys. Rev. A 6, 1436 (1972).
[20]F. G. Donaldson, M. A. Hender, and J. W. McConkey, J. Phys. B 5, 1192 (1972).
[21]W. B. Robb, J. Phys. B 7, 1006 (1974).
[22]A. E. S. Green and S. K. Dutta, J. Geophys. Res. 72, 3933 (1967).
[23]L. Vainshtein, L. Presnyakov, and I. Sobelman, Zh. Eksp. Teor. Fiz. 45, 2015 (1963) [Sov. Phys.-JETP 18, 1383 (1964)].
[24]L. Presnyakov, I. Sobelman, and L. Vainshtein, in *Proceedings of the Third International Conference on the Physics of Electronic and Atomic Collisions, London, 1963*, edited by M. R. C. McDowell (North-Holland, Amsterdam, 1964), p. 243.
[25]George Victor (private communication).
[26]L. A. Vainshtein, V. I. Ochkur, V. I. Rakhovskii, and A. M. Stepanov, Zh. Eksp. Teor. Fiz. 61, 511 (1971) [Sov. Phys.-JETP 34, 271 (1972)].
[27]Y. Okuno, K. Okuno, Y. Kaneko, and I. Kanomata, J. Phys. Soc. Jpn. 29, 164 (1970).
[28]Y. Okuno, J. Phys. Soc. Jpn. 31, 1189 (1971).
[29]Y. Kaneko, J. Phys. Soc. Jpn. 16, 2288 (1961).
[30]J. P. Ziesel and R. Abouaf, J. Chim. Phys. 64, 702 (1967).
[31]S. Okudaira, Y. Kaneko, and I. Kanomata, J. Phys. Soc. Jpn. 28, 1536 (1970).
[32]S. Okudaira, J. Phys. Soc. Jpn. 29, 409 (1970).
[33]J. E. Hansen, J. Phys. 8, L403 (1975); 7, 1902 (1974); 8, 2759 (1975).
[34]H. Hotop and D. Mahr. J. Phys. B 8, L301 (1975).
[35]B. Peart and K. T. Dolder, J. Phys. B 8, 56 (1975).
[36]B. Peart, J. G. Stevenson, and K. T. Dolder, J. Phys. B 6, 146 (1973).