Atomic charge transfer in the presence of a laser field*

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(Received 19 January 1976)

Atomic charge-transfer cross sections in the presence of a laser field are calculated using proton–hydrogen-atom scattering as an example. Both strong and weak laser limits are obtained. An experiment is suggested to observe directly the modification of the differential cross section by the laser.

I. INTRODUCTION

The theory of atomic scattering processes is well understood in most cases and accurately applied in many. However, the introduction of lasers and their projected use in such processes as laser-induced fusion and laser-assisted isotope separation makes it desirable to be able to apply atomic scattering theory in the presence of a strong electromagnetic field. General theories of electron-atom[1] and atom-atom[2] scattering have been presented, as have some applications.[3] Here, we present the theory of low-energy symmetric charge transfer in atom-atom scattering as modified by the presence of an electromagnetic field.

The theory in the absence of the electromagnetic field has been understood now for many years, and we shall closely follow these old theories with the specific example of proton-hydrogen scattering. This example is somewhat special in that there is only one electron so that the "electron translation factor" can simply be inserted.[4] However, there are many ways of inserting these factors[5] so that there is an ambiguity in the method. An alternative way of inserting these factors[6] via the "switching function" is more general in that it allows for an arbitrary number of electrons in the problem. We shall neglect both forms of this refinement and ignore the translational factors here since their appearance has nothing to do with the electromagnetic fields. In addition, the impact-parameter method will be used. That is, we treat the internuclear coordinate classically and prescribe its time dependence to be unaccelerated motion. This limits the region of applicability of the theory to above about 100 eV.

We shall expand the total wave function in the molecular states of H_2^+ and further limit ourselves to a two-state approximation. This is a common approximation[5] in this problem in the absence of the field, and its corrections have been assessed. In the presence of the field it is, in addition, a limitation on the frequency and strength of the field since the laser photons are assumed to have insufficient energy and intensity to couple to higher molecular states. For the H_2^+ problem this is not too great a restriction on current lasers, but for heavier atoms (Ar_2^+) where levels are more dense it can invalidate the procedure. In all cases we make the dipole approximation for the field.

In Sec. II we treat the problem of a weak laser, and an experiment designed to observe the phenomena is suggested. In Sec. III a stronger laser is treated.

II. WEAK ELECTROMAGNETIC FIELDS

Our starting point is the Schrödinger equation in the impact-parameter approximation and the dipole approximation for the electromagnetic field:

$$\left(i\frac{\partial}{\partial t}-H\right)\Psi=0\ , \tag{2.1}$$

$$H=H_M+H_I+H_R+(e^2/2m)A^2\ ,$$

$$H_M=\frac{p^2}{2m}-\frac{e^2}{|r-R/2|}-\frac{e^2}{|r+R/2|}\ , \tag{2.2}$$

$$H_I=-(e/m)\vec{p}\cdot\vec{A}\ ,$$

where H_R is the Hamiltonian for the single-mode free radiation field of frequency ω,

$$H_r=\omega a^\dagger a \tag{2.3}$$

and

$$\vec{A}=\vec{\epsilon}(a+a^\dagger)\ . \tag{2.4}$$

The A^2 term can be diagonalized along with H_R to give a shift of the laser frequency due to the "loading" of the atoms. We therefore simply drop the A^2 term below.

The impact-parameter approximation is contained in the prescription

$$\vec{R}=\vec{b}+\vec{V}t\ ,\quad \vec{b}\cdot\vec{V}=0\ , \tag{2.5}$$

where b is the impact parameter and V the relative velocity of the protons. The initial condition on Ψ is at $t\rightarrow-\infty$

$$\Psi\rightarrow\psi_i=\phi(r-R/2)\,e^{-iE_0t}\ , \tag{2.6}$$

where ϕ is the $1s$ state of hydrogen and E_0 its energy, and we seek the amplitude for charge transfer to the ground state, which is the final state,

$$\psi_f = \phi(r+R/2)\, e^{-iE_0 t} . \tag{2.7}$$

For low collision velocity, $V \ll e^2/\hbar$, it is useful to expand in the molecular states defined by

$$[\,\overline{W}_n(R) - H_M(\vec{r},\vec{R})]\, u_n(\vec{r},\vec{R}) = 0 \tag{2.8}$$

and as discussed above, to keep only the lowest two molecular states with asymptotic properties

$$u_{0,1} \to (1/\sqrt{2})\,[\phi(r-R/2) \pm \phi(r+R/2)]\,, \quad W_{0,1}(R) \to E_0 . \tag{2.9}$$

If we assume that the radiation field is initially a single mode of frequency ω and occupation number $N \gg 1$, then the initial state may be written as

$$(1/\sqrt{2})\,(u_0 + u_1)\,|N\rangle = (1/\sqrt{2})\,(|0,N\rangle + |1,N\rangle) . \tag{2.10}$$

We now assume that over the range of R of interest there is one and only one point at which the two molecular states are resonantly connected by a single-photon transition,

$$\overline{W}_1(R_x) - \overline{W}_0(R_x) = W(R_x) = \omega . \tag{2.11}$$

At that point the states $|0,N+1\rangle$, $|1,N-1\rangle$ are exactly degenerate with the two states in (2.10). Then all four states must be included in the expansion of Ψ:

$$\begin{aligned}\Psi = \Big\{ & B(N,0,t)\,|0,N\rangle \exp\Big[-i\int_{-\infty}^{t} dt\,(\overline{W}_0 - E_0)\Big] + B(N,1,t)\,|1,N\rangle \exp\Big[-i\int_{-\infty}^{t} dt\,(\overline{W}_1 - E_0)\Big] \\ & + B(N-1,1,t)\,|1,N-1\rangle \exp\Big[-i\int_{-\infty}^{t} dt\,(\overline{W}_1 - E_0) + i\omega t\Big] \\ & + B(N+1,0,t)\,|0,N+1\rangle \exp\Big[-i\int_{-\infty}^{t} dt\,(\overline{W}_0 - E_0) - i\omega t\Big]\Big\} e^{-iE_0 t - iN\omega t} .\end{aligned} \tag{2.12}$$

This form is then substituted back into (2.1) and projected onto each of the four states to yield the equations

$$\begin{aligned} i\dot{B}(N,0) &= \langle 0,N|H_I|1,N-1\rangle \exp\Big(i\omega t - i\int_{-\infty}^{t} dt\, W\Big) B(N-1,1)\,, \\ i\dot{B}(N-1,1) &= \langle 1,N-1|H_I|0,N\rangle \exp\Big(-i\omega t + i\int_{-\infty}^{t} dt\, W\Big) B(N,0)\,, \\ i\dot{B}(N,1) &= \langle 1,N|H_I|0,N+1\rangle \exp\Big(-i\omega t + i\int_{-\infty}^{t} dt\, W\Big) B(N+1,0)\,, \\ i\dot{B}(N+1,0) &= \langle 0,N+1|H_I|1,N\rangle \exp\Big(i\omega t - i\int_{-\infty}^{t} dt\, W\Big) B(N,1)\,. \end{aligned} \tag{2.13}$$

The four matrix elements are readily calculable in terms of the matrix elements of the photon creation and destruction operators. If we make the "laser approximation," $N^{1/2} \sim (N\pm 1)^{1/2}$, then

$$\begin{aligned}\langle 0,N|H_I|1,N-1\rangle &= \langle 0,N+1|H_I|1,N\rangle \\ &= \langle 1,N-1|H_I|0,N\rangle^* \\ &= \langle 1,N|H_I|0,N+1\rangle^* \\ &= \Lambda\end{aligned} \tag{2.14}$$

where

$$\Lambda = -(e/2m\omega)\vec{E}\cdot\vec{P}_{01}\,, \quad \vec{P}_{01} = \langle 0|\vec{P}|1\rangle = -m\,W(R)\vec{r}_{01}\cdot\hat{R}\hat{R}\,, \tag{2.15}$$

and $\vec{E}$ may be interpreted as the classical electromagnetic field intensity which approximately describes the single-mode laser. This is the same result as obtained from a semiclassical treatment of the electromagnetic field. With this approximation, the two pairs of coupled equations in (2.13) become identical. The initial conditions implied by (2.10) and (2.12) are

$$\begin{aligned} B(N,0,-\infty) &= B(N,1,-\infty) = 1/\sqrt{2}\,, \\ B(N-1,1,-\infty) &= B(N+1,0,-\infty) = 0\,, \end{aligned} \tag{2.16}$$

so that (2.13) implies

$$B(N,0) = B^*(N,1), \quad B(N-1,1) = B^*(N+1,0)\,, \tag{2.17}$$

and the conservation statements

$$|B(N+1,0)|^2 + |B(N,1)|^2 = |B(N,0)|^2 + |B(N-1,1)|^2 = \tfrac{1}{2} . \tag{2.18}$$

Both (2.17) and (2.18) are true for all times. Thus it is necessary to solve only the pair

$$i\dot{B}(N,0)=\Lambda e^{-i\varphi(t)}B(N-1,1)\ ,$$
$$i\dot{B}(N-1,1)=\Lambda^{*}e^{i\varphi(t)}B(N,0)\ , \tag{2.19}$$

where

$$\varphi(t)=\int_{-\infty}^{t}dt'\,W(R)-\omega t \tag{2.20}$$

Here and below the time dependence of the B's is suppressed. The final-state amplitude may be extracted from (2.12) with the result for the exchange amplitude

$$A_x(b)=\frac{1}{\sqrt{2}}\Big\{|N\rangle\Big[B(N,0)\exp\Big(-i\int_{-\infty}^{\infty}dt(\overline{W}_0-E_0)\Big)-B(N,1)\exp\Big(-i\int_{-\infty}^{\infty}dt\,(W_1-E_0)\Big)\Big]$$
$$+|N+1\rangle B(N+1,0)\exp\Big(-i\omega t-i\int_{-\infty}^{\infty}dt\,(\overline{W}_0-E_0)\Big)+|N-1\rangle B(N+1,1)\exp\Big(i\omega t-i\int_{-\infty}^{\infty}dt(\overline{W}_1-E_0)\Big)\Big\}\ , \tag{2.21}$$

where the B's are evaluated at $t=+\infty$. This amplitude evidently consists of subamplitudes for exchange with change of laser occupation number by 0, ±1. They add incoherently and since their individual observation is difficult we calculate only the total probability for exchange (with no observation of photon number). Using the conservation conditions, (2.18) and (2.17), we obtain

$$P_x(b)=\frac{1}{2}\Big[1-2\mathrm{Re}B^2(N,0)\exp\Big(i\int_{-\infty}^{\infty}dt\ W(R)\Big)\Big]\ . \tag{2.22}$$

Now returning to (2.19) we note that

$$\dot{\phi}=W(R)-\omega\gg\Lambda \tag{2.23}$$

except in the immediate vicinity of the resonant point $R=R_x$. This condition is our definition of weak coupling. Under these circumstances the Eqs. (2.19) are identical with those obtained in the conventional atom-atom scattering problem with the condition of one real curve crossing (see Fig. 1). The approximate S matrix for this case is well known. Thorson[7] has given a systematic presentation from which our result may be obtained. One simply has to modify the usual assumption that the coupling Λ^2 is an even function of t. It is not necessarily so for our case. With that modification we obtain

$$B^{*}(N,0)=(1/\sqrt{2})[1-z^2(+)]^{1/2}[1-z^2(-)]^{1/2}$$
$$+(1/\sqrt{2})z(+)z(-)e^{-i[\Gamma_2(+)+\Gamma_2(-)]} \tag{2.24}$$

where

$$z(\pm)=(1-e^{-\pi T(\pm)})^{1/2} \tag{2.25}$$

and

$$\Gamma_2(\pm)=\arg\Gamma(\tfrac{1}{2}iT(\pm))+\tfrac{1}{2}T(\pm)[1-\ln\tfrac{1}{2}T(\pm)]$$
$$+\tfrac{1}{4}\pi+\int_0^{\tau_x}dt[W(R)-\omega] \tag{2.26}$$

where

$$\tau_x=(1/V)(R_x^2-b^2)^{1/2}\ , \tag{2.27}$$

$$T(\pm)=\Big(\frac{2}{V}\Lambda(\pm)^2\Big|\frac{dW}{dR}\Big|^{-1}\Big)_{R=R_x}\ , \tag{2.28}$$

$$\Lambda(\pm)=\frac{e}{2}\frac{\vec{r}_{01}\cdot\vec{R}}{R^2}[\vec{b}\cdot\vec{E}\pm\hat{V}\cdot\vec{E}(R^2-b^2)^{1/2}]_{R=R_x}\ . \tag{2.29}$$

For $b>R_x$ the crossing is not effective for transitions, and the transition probability becomes

$$P_x=\sin^2\Big(\int_0^{\infty}dt\,W(R)\Big)\ , \tag{2.30}$$

which is the usual result in the absence of the field. For some optical lasers, $E\simeq 2\times10^4$ V/cm. This yields an extremely small value of $T(\pm)$, ~10^{-8} at colliding energy of about 500 ev. Thus the modification induced by the laser is small,

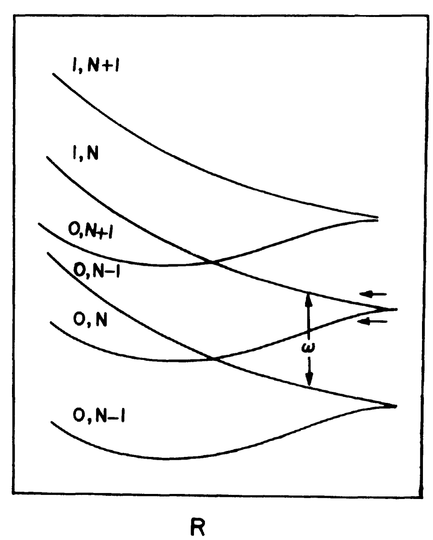

FIG. 1. Energy levels for the case of one crossing. Arrows indicate initially occupied states.

$$P_x(b) = \sin^2\int_0^\infty dt\, W(R) + \frac{\pi}{2}\left\{[T(+)+T(-)]\cos\left(2\int_0^\infty dt\, W(R)\right) + 2[T(+)T(-)]^{1/2}\sin\left(2\int_{\tau_x}^\infty dt\, W(R) + 2\omega\tau_x\right)\right\}. \tag{2.31}$$

The term in curly brackets is small and proportional to the laser intensity and is the quantity of interest here. It could be observed by illuminating the charge-exchange region with a chopped laser and measuring the charge-transfer current at a fixed angle (fixed b) in synchronization with the modulated laser beam. This would eliminate the larger cross section which is independent of the laser. It would be interesting to make the measurement (1) as a function of intensity, (2) as a function of polarization, (3) as a function of frequency, and (4) as a function of collision energy.

(1) The signal should be proportional to the laser intensity.

(2) It should depend on the laser direction with respect to the direction of the incident beam through the factors $T(+)+T(-)$ and $[T(+)T(-)]^{1/2}$. (We assume that the scattering angle is small, so that the incident and final directions are essentially the same.) These parameters have the dependence

$$T(+)+T(-) \simeq b^2 E_\perp^2 + E_\parallel^2(R_x^2 - b^2)\,,$$

$$[T(+)\,T(-)]^{1/2} \simeq b^2 E^2 - R_x^2 E_\parallel^2\,,$$

where $E_\parallel$ and $E_\perp$ are the components of the field respectively parallel and perpendicular to the atomic beam.

(3) The frequency dependence is complicated since ω determines R_x (2.11). The frequency also appears explicitly as a factor ω^{-2} in both T's and in the sine in (2.31).

(4) The signal should have an overall factor of V^{-1} and an oscillating behavior at two different frequencies since the arguments of the sinusoidal functions are different,

$$2\int_0^\infty dt\, W(R) = \frac{2}{V}\int_b^\infty \frac{dR\, R}{(R^2-b^2)^{1/2}}\, W(R)\,,$$
$$2\int_{\tau_x}^\infty dt\, W(R) + 2\omega\tau_x = \frac{2}{V}\left(\int_{R_x}^\infty \frac{dR\,R}{(R^2-b^2)^{1/2}}\, W(R) + W(R_x^2 - b^2)^{1/2}\right). \tag{2.32}$$

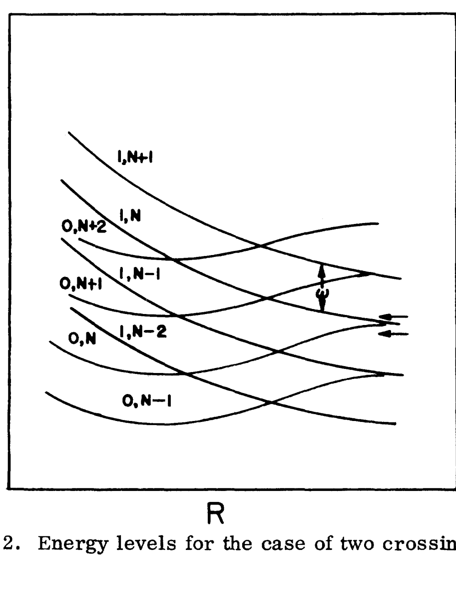

FIG. 2. Energy levels for the case of two crossings.

The effect of the laser on the cross section [the last term in (2.31)] is new and therefore of interest in itself, but in addition it can be used as a probe of the "quasimolecular" properties of the collision. For example, the oscillation (as a function of V) in (2.32) is different from that of the leading term thereby giving a different probe of $W(R)$. In addition, measurement of $T(\pm)$ is a probe of dW/dR at variable R_x determined by the choice of ω.

In the above we imposed the restriction upon ω that there be only one solution to (2.11). For smaller ω there can be more than one solution. Figure 2 is an illustration of the case where there are two such solutions. That is, not only are the states $|1,N\rangle$ and $|0,N+1\rangle$ degenerate for a particular R, but $|1,N\rangle$ can also be degenerate with $|0,N+2\rangle$ for some smaller value of R. It would therefore seem to be necessary to couple in additional states in (2.12). This is not the case since the coupling between $|1,N\rangle$ and $|0,N+2\rangle$ vanishes. Thus, in a single scattering (in along one curve and out along another) with the restriction of energy-conserving coupling, there is no population of these additional states.

III. MORE INTENSE FIELDS

The probability of the laser-induced transition becomes larger as the field gets more intense. When the probability is no longer small, it is more useful to diagonalize the two-state-plus-field problem[2] before dealing with the collision. Since we still restrict the expansion by keeping only two molecular states, this means that the laser cannot be too intense. The assumption that the coupling to other molecular states is negligible is expressed by

$$|\Lambda|/|\Delta W(R) - \omega| \ll 1\,,$$

where $\Delta W(R)$ is the separation between either one of the states in question and any higher state. Provided that there is no resonance of this kind in regions of R explored by the collision, this is satisfied for intense lasers.

Since we are dealing with not very intense fields, diagonalization will be performed in the rotating-wave approximation.[8] In that case the two eigenstates, for fixed R, can be written

$$\phi_\pm(N) = \frac{1}{\sqrt{2}}\left(1 - \frac{\Delta}{\epsilon_\pm}\right)^{1/2} |0,N\rangle + \frac{\Lambda}{\sqrt{3}\,\epsilon_\pm}\left(1 - \frac{\Delta}{\epsilon_\pm}\right)^{-1/2} |1,N-1\rangle \tag{3.1}$$

with eigenvalues

$$E_\pm(N) = \tfrac{1}{2}(\overline{W}_0 + \overline{W}_1) + (N - \tfrac{1}{2})\omega + \epsilon_\pm\,, \tag{3.2}$$

where

$$\epsilon_\pm = \pm(\Delta^2 + \Lambda^2)^{1/2} = \pm\epsilon \tag{3.3}$$

and the energy defect is

$$\Delta(R) = \tfrac{1}{2}[W(R) - \omega]. \tag{3.4}$$

We have used the fact that the coupling Λ in (2.14) is real. For large R, Λ vanishes exponentially and $\Delta = -\frac{1}{2}\omega$, a negative number, so that

$$\begin{aligned} &\phi_+(N) \to |0,N\rangle, \quad \phi_-(N) \to (-\operatorname{sgn}\Lambda)\,|1,N-1\rangle \\ &E_+(N) \to E_0 + N\omega, \quad E_-(N) \to E_1 + (N-1)\omega. \end{aligned} \tag{3.5}$$

The energy-level curves are shown in Fig. 3.

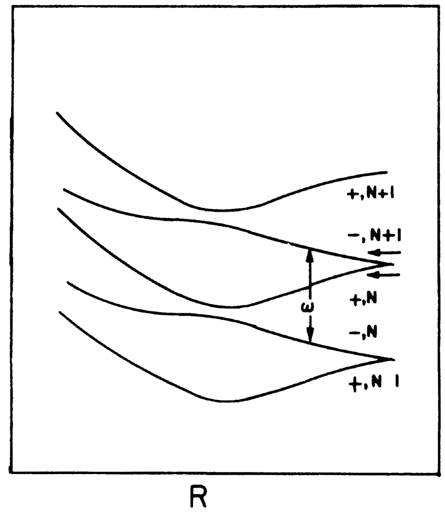

FIG. 3. Energy levels for the case of one avoided crossing.

For the case where there is only one solution to (2.11), there is only one avoided crossing, and again using the concept that coupling of states will occur only at real crossings or avoided crossings, we need only use four states in our expansion of Ψ,

$$\begin{aligned}\Psi = {} & A_+(N,t)\phi_+(N)\exp\left(-i\int_{\infty}^{t}\Delta E_+(N,t)\,dt - iE_+(N,-\infty)t\right) + A_-(N,t)\phi_-(N)\exp\left(-i\int_{-\infty}^{t}\Delta E_-(N,t)\,dt - iE_-(N,-\infty)t\right) \\ & + A_-(N+1,t)\phi_-(N+1)\exp\left(-i\int_{-\infty}^{t}\Delta E_-(N+1,t)\,dt - iE_-(N+1,-\infty)t\right) \\ & + A_+(N+1,t)\phi_+(N+1)\exp\left(-i\int_{-\infty}^{t}\Delta E_+(N+1,t)\,dt - iE_+(N+1,-\infty)t\right),\end{aligned} \tag{3.6}$$

where

$$\Delta E_\pm(N,t) = E_\pm(N,t) - E_\pm(N,-\infty). \tag{3.7}$$

The initial conditions implied by (3.5) and (2.10) are

$$\begin{aligned} &A_+(N+1) = A_-(N) = 0, \\ &A_+(N) = 1/\sqrt{2}\,, \quad A_-(N+1) = -(1/\sqrt{2})\operatorname{sgn}\Lambda. \end{aligned} \tag{3.8}$$

Substitution of (3.6) into (2.1) with the approximation that $\phi_\pm(N)$ are eigenfunctions of H for fixed R yields

$$i\dot{A}_\pm(N) = -i\left\langle \phi_\pm(N), \frac{\partial}{\partial t}\phi_\mp(N)\right\rangle e^{\pm i\chi(t)}A_\mp(N), \tag{3.9}$$

where only the couplings of states at the avoided crossing have been retained. A similar equation with N replaced by $N+1$ is obtained for the other pair.

The phase is defined by

$$\chi(t) = 2\int_{-\infty}^{t} dt'\,(\epsilon - \omega/2) + \omega t \tag{3.10}$$

and the coupling matrix element is given by

$$\left\langle \phi_-(N), \frac{\partial}{\partial t}\phi_+(N)\right\rangle = -\left\langle \phi_+(N), \frac{\partial}{\partial t}\phi_-(N)\right\rangle^* = \frac{\Lambda}{|\Lambda|}\,\frac{\Delta\dot{\Lambda} - \Lambda\dot{\Delta}}{\epsilon}. \tag{3.11}$$

These equations are identical in form with those obtained for the atom-atom scattering problem in the case of an avoided crossing. The minimum energy separation occurs near R_x [Eq. (2.11)] and is approximately $|\Lambda|$. Equations (3.9) and (3.10) can be combined to show that

$$A_+(N) = -\left.\frac{\Lambda}{|\Lambda|}\right|_{t=-\infty} A_-^*(N+1),$$
$$A_-(N) = -\left.\frac{\Lambda}{|\Lambda|}\right|_{t=-\infty} A_+^*(N+1). \qquad (3.12)$$

Again the exchange amplitude can be extracted from (3.6) as was done in obtaining (2.21). Again there are three incoherent amplitudes for the three different photon states. The result for the total charge-exchange probability is

$$P_x(b) = \frac{1}{2}\left[1 - 2\,\mathrm{Re}A_+^2(N)\exp\left(-i\int_{-\infty}^{\infty} dt\,(2\epsilon - \omega)\right)\right]. \qquad (3.13)$$

An interesting but somewhat unrealistic result emerges when $\Lambda|_{R=R_x}$ is large enough so that the curves are well separated at the avoided crossing. In that case the coupling in Eq. (3.9) can be neglected with the result that $A_+(N) = 1/\sqrt{2}$ so that

$$P_x(b) = \sin^2\int_{-\infty}^{\infty} dt\,(2\epsilon - \omega)$$
$$= \sin^2\frac{1}{V}\int_b^{\infty}\frac{dR\,R}{(R^2-b^2)^{1/2}}\{[\Delta^2(R)+\Lambda_-^2]^{1/2} + [\Delta^2(R)+\Lambda_+^2]^{1/2} - \omega\} \qquad (3.14)$$

where

$$\Lambda_\pm = (e/2\omega R)[\vec{E}\cdot\vec{b} \pm E_\parallel (R^2-b^2)^{1/2}]W(R)\vec{r}_{01}\cdot\hat{R}\,. \qquad (3.15)$$

The result is unrealistic in that it requires that $T(\pm) > 1$; so we shall not pursue it further here.

For the weak coupling limit of Eqs. (3.1)–(3.13), the avoided-crossing curves approach each other more closely and the slopes of the energies (3.2) become discontinuous so that we return to the real crossing or weak coupling limit. Therefore, there seems to be no reason to pursue Eqs. (3.9) and (3.13) further since both their weak and strong coupling limits are given.

For the case where ω is small enough that (2.11) has an additional root at $E_+(N) = E_-(N+1)$ or

$$\omega = \tfrac{1}{2}W(R) + 2\Lambda^2(R)/W(R)\,, \qquad (3.16)$$

there is a real crossing as shown in Fig. 4. In this case the coupling matrix element does not vanish but is

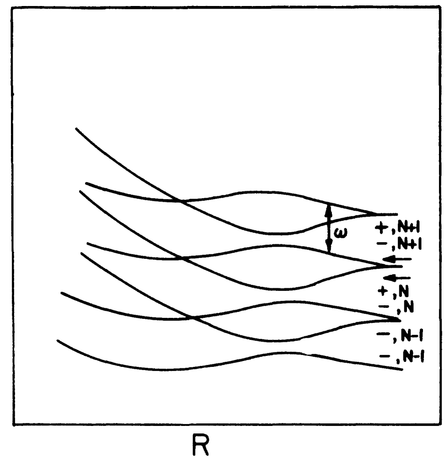

FIG. 4. Energy levels for the case of one avoided crossing and one real crossing.

$$\left\langle\phi_+(N), \frac{\partial}{\partial t}\phi_-(N+1)\right\rangle = -\frac{\Lambda}{2|\Lambda|}\frac{\epsilon-\Delta}{\epsilon}\langle 0|\frac{\partial}{\partial t}|1\rangle\,. \qquad (3.17)$$

Then the four additional states $\phi_\pm(N+2)$ and $\phi_\pm(N-1)$ are coupled at this crossing. These populate exchange states with photon states $|N\pm 2\rangle$ so that two-photon emission and absorption is obtained. The reason for their appearance here and not in Sec. II is that the rotating-wave approximation diagonalization of the (no motion) problem has already incorporated multiphoton transitions in the zero-order states [Eq. (3.1)].

For still smaller ω there are additional real crossings, but one can show that the coupling matrix elements vanish at these points in this approximation.

To summarize, we have obtained the small change in the differential cross section for symmetric charge transfer caused by the presence of an electromagnetic field in the interaction region. The change in the cross section has an interesting dependence on the field parameters, and an experiment has been suggested to investigate this.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge several stimulating discussions with Professor Joel Gersten. Thanks are also due to the Aspen Center for Physics for hospitality during the period when the ideas described here were developed.

*Research supported by U. S. Office of Naval Research Contract No. N 00014-76-C-0014.

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