Comments and Addenda

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Positron-impact $1s \rightarrow 2s$ excitation of atomic hydrogen in the eikonal approximation

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The eikonal approximation has been used to calculate differential and total cross sections for $1s \rightarrow 2s$ excitation of atomic hydrogen by 50-, 100-, and 200-eV incident positrons. As in the case of elastic scattering, the positron scattering cross sections are found to differ from the corresponding electron scattering cross sections. Comparison is made with the eikonal Monte Carlo results of Byron.

There has been much recent interest in applying Glauber and eikonal-type techniques to calculate cross sections for charged-particle–atom collisions in the intermediate energy range. In particular, the Glauber approximation has been utilized rather successfully to analyze a number of important atomic-collision problems.[1] However, the Glauber approximation (as well as the Born) predicts identical cross sections for both electrons and positrons scattering from an atomic target, for a given incident energy; one expects these cross sections to be different on physical grounds.[2]

The full eikonal approximation has recently been used to study electron and positron scattering from simple atomic targets.[3-7] Byron[3] employed the Monte Carlo method to evaluate the multidimensional integrals involved in applying the full eikonal method to scattering from hydrogen and helium. Even more recently, Gau and Macek[4] showed that the six-dimensional integral expression for the eikonal direct amplitude for electron-hydrogen collisions can be reduced to a double-integral form suitable for numerical evaluation. This reduced eikonal amplitude has been utilized to study elastic[5,6] and inelastic[5] electron-hydrogen scattering.

In a previous paper,[7] we noted that the reduced eikonal amplitude can also be used to calculate positron-hydrogen scattering cross sections, and we reported our results for the elastic scattering of 50-, 100-, and 200-eV positrons from hydrogen. It was found that the e^+-H and e^--H cross sections differ. In this addendum, we extend these calculations to excitation to the $2s$ state of hydrogen, and we report here our results for e^+-H ($1s \rightarrow 2s$) scattering for incident energies of 50, 100, and 200 eV. We present differential and total cross sections.

The reduced double-integral expression for the full eikonal amplitude for electron-hydrogen scattering is[8]

$$F_{fi}(\vec{q}) = \frac{-2^{4-i\eta}}{a_0}\pi\frac{\Gamma(1-i\eta)}{\Gamma(-i\eta)}C_{fi}D(\mu,\vec{\gamma})\mu\left(\frac{d}{d\mu^2}\right)^2\left[\int_0^\infty d\lambda\,\lambda^{-i\eta-1}\int_0^1 d\chi\,\chi^{-1}[\mathfrak{F}(1,0,0,0)-\mathfrak{F}(1,1,0,1)]\right]\Bigg|_{\vec{\gamma}=0}, \tag{1a}$$

where

$$\mathfrak{F}(m,p,r,s) = \lambda^s(1-\chi)^s\Lambda^{-p}(\Lambda^2+q'^2)^{i\eta-m}\times(\Lambda-iq'_z)^{-i\eta-r}, \tag{1b}$$

$$\Lambda = [\lambda^2(1-\chi)^2+\mu^2\chi+2i\lambda\chi(1-\chi)\gamma_z+\gamma^2\chi(1-\chi)]^{1/2}, \tag{1c}$$

$$\vec{q}' = \vec{q} - i\lambda(1-\chi)\hat{z} + \chi\vec{\gamma}. \tag{1d}$$

Here a_0 is the Bohr radius, $\vec{q}=\vec{k}-\vec{k}'$ is the momentum transfer to the target, $\eta = e^2/\hbar v = 1/k$, C_{fi} is a normalization constant, and $D(\mu,\vec{\gamma})$ is a differential operator which is introduced in the reduction of the six-integral amplitude to double-integral form.

TABLE I. Positron-hydrogen $1s \to 2s$ differential cross section vs positron scattering angle for 50-, 100-, and 200-eV incident positrons.

Positron scattering angle (deg)	$d\sigma/d\Omega$ (a_0^2/sr) 50 eV	100 eV	200 eV
0	2.67	1.82	2.84
2	2.48	1.87	2.24
4	2.07	1.43	1.41
6	1.57	0.909	0.864
8	1.15	0.594	0.526
10	0.842	0.411	0.320
20	0.204	7.84×10^{-2}	3.25×10^{-2}
30	6.57×10^{-2}	1.95×10^{-2}	6.66×10^{-3}
40	3.21×10^{-2}	7.23×10^{-3}	2.25×10^{-3}
60	9.48×10^{-3}	1.64×10^{-3}	4.22×10^{-4}
90	2.39×10^{-3}	3.45×10^{-4}	8.94×10^{-5}
120	4.85×10^{-4}	1.49×10^{-4}	3.81×10^{-5}

To apply Eq. (1a) to $1s \to 2s$ excitation of hydrogen by positron impact, we let $\eta \to -\eta$,[9] and use $C_{fi} = 1/4\pi\sqrt{2}$, $D(\mu, \vec{\gamma}) = (2 + d/d\mu)$, and then set $\mu = \frac{3}{2}$ and $\vec{\gamma} = 0$.

We have numerically evaluated Eq. (1a) above for $1s \to 2s$ excitation, for incident positron energies of 50, 100, and 200 eV, and scattering angles from 0 to 120°. As in our previous work, we have done the λ integration by parts to obtain numerical convergence. The differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \left| F_{1s\to 2s}(\vec{q}) \right|^2 . \qquad (2)$$

TABLE II. Total positron-hydrogen and electron-hydrogen $1s \to 2s$ scattering cross sections for 50-, 100-, and 200-eV incident positrons and electrons.

Incident energy (eV)	Total scattering cross section (πa_0^2) Positrons (interpolated from Byron's work)	Positrons (this work)	Electrons
50	0.097	0.122	0.084
100	0.053	0.054	0.048
200	...	0.041	0.037

In Table I our numerically computed differential cross sections are presented, and in Table II we compare our total cross sections (obtained from the differential cross sections by the use of Simpson's-rule numerical integration[10]) with the corresponding electron scattering cross sections.[11] We find again, as in the case of elastic scattering, that the positron and electron cross sections differ at lower energies, and merge at higher energies. Comparing our positron cross sections with the eikonal Monte Carlo results of Byron[3] (also given in Table II), it is seen that the two calculations agree very well at 100 eV; however, at 50 eV our result is somewhat higher than the interpolated value of Byron's calculation.

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[1]See E. Gerjuoy and B. K. Thomas, Rep. Prog. Phys. 37, 1345 (1974), for a detailed review of the applications of Glauber theory to atomic collisions; see also, e.g., K. C. Mathur, Phys. Rev. A 9, 1220 (1974); J. Z. Terebey, J. Phys. B 7, 460 (1974); F. T. Chan and S. T. Chen, Phys. Rev. A 9, 2393 (1974); S. Kumar and M. K. Srivastava, Phys. Rev. A 12, 801 (1975); and T. Ishihara and J. C. Y. Chen, J. Phys. B 8, L417 (1975).

[2]E. Gerjuoy and B. K. Thomas, in Ref. 1 above.

[3]F. W. Byron, Jr., Phys. Rev. A 4, 1907 (1971).

[4]J. N. Gau and J. Macek, Phys. Rev. A 10, 522 (1974).

[5]J. N. Gau and J. Macek, Phys. Rev. A 12, 1760 (1975).

[6]G. Foster and W. Williamson, Jr., Phys. Rev. A 13, 936 (1976).

[7]G. Foster and W. Williamson, Jr., Phys. Rev. A 13, 1265 (1976).

[8]Reference 4, p. 525, Eq. (17a).

[9]G. Foster and W. Williamson, Jr., in Ref. 7.

[10]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, NBS Appl. Math. Ser. 55 (U. S. GPO, Washington, D.C., 1964). We have interpolated between our values of $d\sigma/d\Omega$ given in Table I so that Simpson's rule could be applied using 1° intervals from 0 to 10°, 2° intervals from 10 to 40°, and 5° intervals from 40 to 120°.

[11]As a check to our calculations, we have recalculated the corresponding cross sections for electron impact; our results are in reasonable agreement with both the Gau and Macek results of Ref. 5 and Byron's Monte Carlo results of Ref. 3.