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High-momentum-transfer atomic collisions *

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Recent results due to Kelsey for certain high-momentum-transfer collisions are generalized to all such processes. A simple model for scattering in this region is presented which provides considerable insight into these processes. The model predicts that all processes in the regions where first Born terms fail are proportional to the nuclear Rutherford-like scattering, and gives a simple formula for calculating the cross section in terms of electronic form factors.

Recently there has been some interest[1-4] in collisions at high but nonrelativistic energies and large momentum transfers. The interest has focused on inelastic electron scattering off simple atoms, where there is reason to believe that the first Born term fails in this region.[5] Kelsey has shown[2] that for these processes the second Born term indeed dominates the first. It was found that the cross section at fixed energy does not drop rapidly with increasingly large momentum transfer but remains proportional to a Rutherford-like scattering off the nucleus. The purpose of this comment is to point out that this feature is a general property of all atomic collisions at high momentum transfer, and to develop a simple model which circumvents the difficulties of computing higher-order effects. Such a model facilitates the evaluation and comparison of data and aids in identifying atomic phenomena specific to this region. To the author's knowledge, no such model has been discussed in the literature.

The present model is based on the following semiclassical ideas. It is well known that the elastic collisions of atoms or ions at high energies are accurately described by the screened Coulomb potential of the nuclei, away from the forward direction. The distance of closest approach in an elastic collision may be much smaller than the atomic dimension, which shows that the electrons play no dynamical role, merely passively providing a screen for the internuclear interaction. The scattering is due entirely to the forces felt by the nuclei as they reach the distance of closest approach.

In order to generalize this to inelastic processes at high momentum transfer, it is argued that during such a collision an electron in an atom typically changes its momentum by an amount of the order of the normal momentum of a bound electron. This holds also for ionization reactions where the probability of producing a high-energy electron is small compared to a lower-energy electron. The point is that an electron, because of its negligible mass, may change its energy by a large amount and yet change its momentum by a small amount, usually of order $1/a_0$. At large momentum transfers this is a negligible reaction on the nuclei. Therefore one expects the motion of the nuclei to be almost identical for both elastic and inelastic processes; the dynamical role of the electrons is irrelevant in determining the angle of scatter. As far as an atomic electron is concerned it responds to the changing fields during the collision but provides a negligible reaction on the nuclei providing the fields. Thus one may regard the nuclei as undergoing a Rutherford-like scattering, which at high momentum transfers is independent of the screening lengths, and at the same time providing fields that perturb the atomic electrons. Thus these fields may be obtained using the motion of the nuclei to provide a specified source for the perturbing fields. This picture leads to a factorized scattering amplitude

$$f = F_N A_e\,, \tag{1}$$

where F_N describes the nuclear amplitude and A_e is computed using perturbation theory to describe the electronic transition produced by the nuclear fields. The basic approximation here is the neglect of the reaction of the electronic process on the nuclei, thus allowing the semiclassical approach of treating the electronic transition producing fields using a specified c-number source. The validity of the model depends on a very close nuclear collision, and therefore requires high energy and large momentum transfer. The basic idea is similar to the straight-line approximation for low-energy distant collisions between a charged particle and an atom or ion. The physical basis is, however, completely different.

The cross section may be obtained directly from Eq. (1):

$$\frac{d\sigma}{d\Omega} = \left| \frac{4\alpha M_R Z_1 Z_2}{q^2} + f_N \right|^2 P(q^2)\,, \tag{2}$$

where M_R is the reduced mass and f_N is the nuclear strong-interaction amplitude, which becomes important at large q^2. In first Born approximation P is simply a product of atomic form factors, and depends on q^2. At high momentum transfer the Born approximation to P drops rapidly in q^2 for inelastic processes, remaining finite and equal to unity only for truly elastic processes. We shall show that the present model for P gives unity for elastic processes and a quantity of order $(\alpha c/v)^2$ for inelastic processes. The model gives P in terms of electronic form factors; we now turn to its evaluation.

The current of an atom scattering from a momentum $\vec{p}$ to $\vec{p}'$, is well known from the theory of bremsstrahlung[6,7]:

$$j_\mu^1(x) = e\int d^4k\,(2\pi)^{-4} j_\mu^1(k) e^{-ik\cdot x} \tag{3}$$

where

$$j_\mu^1(k) = F_1(k^2)\left(\frac{2p-k}{2p\cdot k - k^2 + i\epsilon} - \frac{2p'-k}{2p'\cdot k + k^2 - i\epsilon}\right)_\mu . \tag{4}$$

The index μ runs from 0 to 3, and all vectors in these expressions are four-vectors. The current is proportional to a form factor, which is the (screened) nuclear charge, and has the limits

$$\lim_{k^2\to-\infty} F_1(k^2) = Z_1 \tag{5}$$

and

$$F_1(0) = Z_1 - N , \tag{6}$$

where N is the number of electrons. In this semiclassical approach the nuclear charge is the dominant contributor to electronic transitions. The current produces an electromagnetic field of a Lienard-Wiechert[6] type,

$$A_\mu^1(k) = j_\mu^1(k)/(k^2 - i\epsilon) , \tag{7}$$

which produces transitions in the other atom. These may be computed by simple perturbation theory. The wave function for atom 2 may be written as follows

$$\psi^{(2)}(x_j, t) = \sum_n C_n^{(2)}(t, v)\psi_n^{(2)}(x_j) \times \exp\left(i(E_n + \tfrac{1}{2}m_e v^2)t - \sum_j i m_e \vec{v}\cdot\vec{x}_j\right), \tag{8}$$

where $\vec{v}$ is the velocity of the atom. If very large momentum transfers are excluded, namely

$$q a_0 \ll M_2/m_e , \tag{9}$$

then the velocity dependence of the coefficients C_n may be neglected, and the amplitude for the electrons to be in a final state s is

$$C_s^2 = \delta_{si} + ie\int_{-\infty}^{\infty} dt\, e^{i(E_s - E_i)t}(\psi_s^{(2)} A_0^{(1)} j_0^{(2)} \psi_i^{(2)}) , \tag{10}$$

where the space components of the electromagnetic potential have been dropped. The electronic transition probability P is then given by

$$P_{st} = |C_s^{(2)} C_t^{(1)}|^2 . \tag{11}$$

It is emphasized that only the electronic contribution to $j_0^{(2)}$ is to be included in Eq. (10). The nuclear term is to be excluded since it represents a higher-order approximation to the nuclear scattering amplitude, which in this analysis is a correction to $d\sigma_0/d\Omega$ and not a contribution to P. [This is already implicit in Eq. (8) where only electronic coordinates appear.] To evaluate the coefficients C_s, write

$$(\psi_s^{(2)}, j_0^{(2)}\psi_i^{(2)}) = \int F_{si}^{(2)}(k) e^{i\vec{k}\cdot\vec{x}} d^3k/(2\pi)^3 \tag{12}$$

to obtain from (10)

$$C_s^{(2)} = \delta_{si} + ie\alpha\int \frac{d^3k}{(2\pi)^3} F_{si}^{(2)}(k)\frac{j_0^{(1)}(k_0, \vec{k})}{k_0^2 - k^2 + i\epsilon} , \tag{13}$$

where

$$k_0 = E_s - E_i . \tag{14}$$

The dependence on k_0 is very weak, and neglecting it gives a simple perspicuous result for C_s:

$$C_s^{(2)} = \delta_{si} + \frac{\alpha c}{2\pi}\left(\frac{1}{v_1} + \frac{1}{v_1'}\right)\int_0^\infty \frac{dk}{k} F^1(k) F_{si}^{(2)}(k)\frac{d\Omega_k}{4\pi} . \tag{15}$$

Together with Eqs. (11) and (2) this completes the model.

In the case of the elastic scattering of two ions or an ion and a charged particle, the integral in (15) diverges. However, this occurs only for the elastic case, in which the integral is formally of order α compared to the constant term. This is the same phenomenon as occurs in second Born approximation and is actually related to the charge renormalization of quantum electrodynamics,[7] which possesses an infrared divergence of this type. Consequently, it is permissible to absorb that divergence into the charge renormalization constants; the end result is simply to strike out the integral in Eq. (15) and replace it by a finite quantity of order α, which can now be neglected.

Then we obtain

$$C_i^{(2)} = 1 \tag{16}$$

and for $s \neq i$

$$C_s^{(2)} = -\frac{\alpha c}{2\pi}\left(\frac{1}{v_1}+\frac{1}{v_1'}\right)\int_0^\infty \frac{dk}{k}\, F^1(k)\, F_{si}^{(2)}(k)\, \frac{d\Omega_k}{4\pi}\,. \tag{17}$$

The model is clearly very simple to use and allows the immediate comparison of several excitation processes, since the final electronic state appears only in F_{si}. Rather than attempting to tabulate the cross sections for a large number of processes, it is preferable to have the model in its present analytic form so that any process of interest may be immediately calculated by the reader as required. The F_{si} form factors may be obtained from data at lower energies or from simple models. The F form factors describing the screened nuclear charge may be taken to be the ground-state form factor, or a screened potential where the screening length is some weighted average of the initial and final-state screening lengths. This is expected to be sufficiently precise if these screening lengths are not very different. If ionization occurs, then this approximation may not be sufficient. In that case, Eq. (17) may be replaced by the following:

$$C_s^{(2)} = -\frac{\alpha c}{2\pi}\int_0^\infty \frac{dk}{k}\left(\frac{F^1}{v_1}+\frac{F^{1\prime}}{v_1'}\right) F_{si}^{(2)}(k)\, \frac{d\Omega_k}{4\pi} \tag{18}$$

where the F' refers to the final state of the atom or ion.

The application of the results to electron scattering is weaker in that the arguments based on the inertia of the atomic electrons are weaker. The assumption of neglecting their reaction on the impinging electron is nonetheless expected to be quite good. The reason is that atomic electrons typically change the projectile momentum by something of the order of $1/a_0$, which is small compared to the momentum transfer between the projectile and the nucleus. The results of applying this model to electron scattering off simple atoms give results very close to those of Kelsey,[2] and provide insight into his results.

In conclusion, a general, simple model of large momentum transfer processes has been presented. The model predicts that *all* such processes are proportional to the nuclear Rutherford-like cross section, where the coefficient is independent of angle [(cf. (18)] and of order $(\alpha c/v)^2$. Any deviation from such behavior would indicate new and interesting phenomena in this region.

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[1]S. Geltman and M. B. Hidalgo, J. Phys. B 4 1299 (1971).
[2]E. J. Kelsey, Phys. Rev. A 14, 56 (1976).
[3]B. R. Junker, Phys. Rev. A 11, 1552 (1974).
[4]E. J. Kelsey and J. Macek, Phys. Rev. A 12 1127 (1975).
[5]C. Quigg and C. J. Joachim, Rev. Mod. Phys. 46 279 (1974).
[6]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), Chap. 14.
[7]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Chap. 19.