Smith-Purcell radiation from small gratings

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A series of metallic gratings, having grating spacing of 2180 Å and known numbers of grooves ranging from 1000 to 4, were seen to produce Smith-Purcell radiation when exposed to a 50-keV electron beam. Bandwidths for the radiation from these gratings were measured and found to be dependent upon the number of grating grooves as well as the beam angle of incidence. This is interpreted as indicating a one-to-one correspondence between grooves in the grating and undulations in the radiated wave train produced by a single passing electron. With this assumption, a height for the zone of effective interaction for the electron is calculated to be ~1500 Å from the grating surface.

I. INTRODUCTION

Visible electromagnetic radiation produced by interaction of an electron beam with a metallic diffraction grating was first reported by Smith and Purcell.[1] The radiation resulted when high-energy (300 keV) electrons passed near and parallel to the surface of a grating in a direction approximately transverse to the grooves. The wavelength of the radiation was predicted and verified experimentally to be

$$\lambda = (d/n)(\beta^{-1} - \cos\theta) , \qquad (1)$$

where d is the grating groove spacing, β = (electron velocity)/c, θ is the angle between viewing direction and electron velocity, and $n = 1, 2, 3, \ldots$ is the order number.

Smith and Purcell proposed that the surface charge induced on the grating by the electron would radiate as it moved over the periodic surface. The induced charge could be thought of as equivalent to an image charge beneath the grating surface, and the radiation as being produced by oscillation of the electron-image dipole. Ishiguro and Tako further developed this theory, obtaining an expression for the radiation pattern.[2]

Attempts have been made to develop a classical theory based on solutions of Maxwell's equations with the grating surface as a boundary.[3-7] The complexity of the boundary-value problem has in every instance required the assumption of a periodic surface of infinite extent in two dimensions. At least one quantum-mechanical theory has been developed.[8]

Experimental investigations have been undertaken, and information concerning radiation patterns and angular spectra has been reported.[9-16] These investigations have been confined to large gratings with tens of thousands of grooves and electron-beam currents of the order of 1 mA. Renewed interest in the effect has been generated due to the recent proposal by Mizumo *et al.* of an inverse Smith-Purcell effect.[17]

The investigation described herein centered upon visual observations and spectrographic records of the first-order Smith-Purcell (SP) radiation from a series of gratings have decreasing numbers of grooves beginning with $N = 1000$. The groove number was decreased until the intensity of radiation diminished so that observation was impossible. The grating groove spacing of 2180 Å and electron energy of 50.0 keV were maintained for all observations.

II. EXPERIMENT

All gratings were produced in this laboratory by means of an interferometrically controlled ruling engine which used a diamond stylus having a ruling edge with an included angle of 90°, positioned to produce a symmetrical groove, i.e., no blaze. Examination of the gratings with a scanning electron microscope at magnifications up to 20kX allowed a check of grating quality. The ruled surface was of gold, vacuum sputtered over a thin layer of evaporated chromium on a glass substrate. Often several separate and parallel gratings were ruled on a single blank, permitting simultaneous observations of SP radiation from each. Intensity of the fundamental radiation was enhanced by deposition of a thin (~500–1000 Å) layer of vacuum-evaporated aluminum over the surface of the grating.[10] This aspect of the experiment was not investigated, but simply used as a means of increasing the intensity of SP radiation to maximize the signal-to-noise ratio.

The electron-beam apparatus was an RCA EMU-2D transmission electron microscope. This instrument provided a stable beam with a circular cross section adjustable from 1600 μm^2 to 1 mm^2 in area and a constant current of 0.46 μA, as measured at the point of the grating. The constant 50.00 kV accelerating voltage gave a value of 0.43

for β in Eq. (1). For these beam parameters, the average axial distance between electrons in the beam was 47 μm, or ~220 times the grating groove spacing. Thus, for gratings with $N \leq 200$, there would be only one electron at a time traversing the grating, so that radiation would be from single electrons.

Visual observations of SP radiation were made using a specially constructed $f/3.5$ microscope. The radiation was readily seen by a dark-adapted observer using this microscope to view through a glass window in the access port of the beam apparatus.

In addition to Smith-Purcell radiation from the grating, background radiation was observed at the point of impingement of the electron beam on the unruled surface. This light had the appearance of an indistinct, grey haze and was totally unpolarized. Its color did not change with viewing angle. In this way, the background radiation could be readily distinguished visually from the SP radiation.

Visual observations of SP radiation were made at various viewing angles θ from 97° to 125°, in a particular viewing plane (defined as a plane containing the electron velocity and k vector of the observed radiation) making an angle ϕ with respect to the plane of the grating. Observations were made in three different viewing planes: $\phi = 90°$, 45°, and 15°. In addition to the results summarized in Table I, SP radiation was observed from gratings having $N = 250$, 500, and 1000.

An $f/3.5$ prism spectrograph was constructed expressly for this experiment. An entrance aperture allowed acceptance of radiation over a range of viewing angles, $\Delta\theta = 1.3°$. All spectrographic observations were made at $\phi = 90°$ and $\theta = 102°$. The fundamental radiation wavelength was thus 5800 Å with an inherent linewidth due to $\Delta\theta$ of 46 Å.

The observations were recorded on Kodak Tri-X Panchromatic film developed in D-19 for $6\frac{1}{2}$ min at 68 °F. The films were scanned with a Photometrics EDP scanning microdensitometer, model 1000 B, in the contour mode. Density measurements taken from equal density interval contours were converted to intensity profiles, from which were obtained measurements of the radiation band full width at half-height, $\Delta\lambda$.

Spectrographic observations were made on a set of several gratings having different N, and ruled on the same blank. The gratings were parallel and spaced a distance of 0.3 mm apart. In this way, spectrograms from all of the gratings could be encompassed in one frame of the film, eliminating effects of film and development upon intensity measurements. Parameters involving the electron-beam apparatus would also be held constant since it was possible to expose all four gratings to the beam simultaneously. Observations were made at different values of i, the angle of incidence of the beam on the grating surface.

TABLE I. Angle between plane of polarization of Smith-Purcell radiation and viewing plane for various viewing planes ($\theta = 125°$).

Number of grooves, N	Polarization angle (±5°) [a] $\phi = 90°$	$\phi = 45°$	$\phi = 15°$
100	0	45	60
32	0	45	60
16	0	45	... [b]
8	0	45	... [b]
4	... [b]	45	... [b]

[a] ±10° for $\phi = 15°$.

[b] Radiation too weak to determine a polarization angle.

III. RESULTS

A. Minimum number of grooves

Bachheimer has suggested[5] that a minimum number of grooves might be required for production of the SP radiation. At $i = 1°$ the number $N_0 \cong \beta\lambda / 2\pi d(1-\beta^2)^{1/2} \sin i$ is approximately ten grooves.

To check the prediction, a single blank was ruled with seven parallel gratings having $N = 100$, 32, 16, 8, 4, 2, and 1 grooves, respectively. Visual observations were made in the $\phi = 45°$ plane at three different values of θ from 97° to 125° when the gratings were exposed to the beam incident at $i = 1°$. SP radiation was observed from all gratings except $N = 2$ and $N = 1$.

B. Polarization of radiation

Previously reported experimental investigations of the SP effect were limited to observations in a viewing plane normal to the grating surface ($\phi = 90°$). In this viewing plane the SP radiation was reported to be plane polarized in a direction parallel to the electron beam. Observations in other viewing planes were not reported.

In this experiment, SP radiation was observed in other viewing planes. Intensity of the radiation was found to vary with ϕ, with radiation in the $\phi = 45°$ plane being the most intense of the three planes used. Radiation in the $\phi = 15°$ plane was markedly weaker than the other two, to the extent that radiation was undetectable from gratings with $N \leq 32$. The SP radiation was found to be plane polarized for all values of ϕ but with polarization angle dependent upon ϕ, as shown in Table I.

C. Dependence of bandwidth upon N

Bachheimer has presented extensive measurements of the SP radiation bandwidth $\Delta\lambda$ as a function of the beam incidence angle,[16] and for N large (> 1000) has shown good agreement with the value predicted by his theory:

$$\Delta\lambda = \frac{2d(1-\beta^2)^{1/2}}{n\beta} \sin i \ . \tag{2}$$

Measurements were made of bandwidth of SP radiation from a set of four parallel gratings on one blank having $N = 100$, 32, 16, and 8 in an attempt to discover any dependence of $\Delta\lambda$ on N. The measurements were made for $\phi = 0°$ and $\theta = 102°$ at various values of i. Measurements of $\Delta\lambda$ were obtained for all gratings except $N = 8$. The radiation in that case was not sufficiently intense to allow measurement of $\Delta\lambda$, although the radiation was detected photographically. The results are shown in Fig. 1. Solid curves for each of the values of N are drawn through the data points.

The figure also displays a dashed line showing the value of $\Delta\lambda$ predicted by Eq. (2) for $d = 2180$ Å, $n = 1$, and $\beta = 0.43$. The results of Bachheimer for a grating with N large are also shown. Although his experimental parameters ($d = 8000$ Å, $n = 2$, $\beta = 0.6$) differed from those of this experiment, Eq. (2) predicts almost the same functional dependence of $\Delta\lambda$ on i. Hence, a comparison of results should be valid.

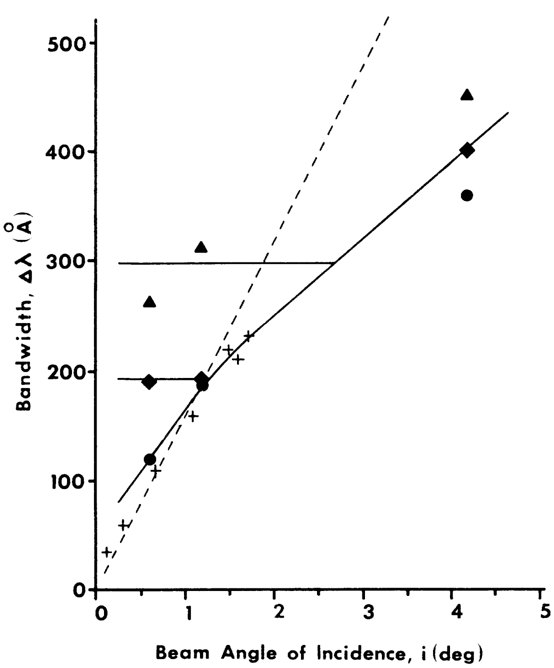

FIG. 1. Plot of radiation bandwidth vs angle of incidence of electron beam on the grating for various gratings. ▲, results for $N = 16$. ♦, results for $N = 32$. ●, results for $N = 100$. +, results from Ref. 16 for N large but indeterminate.

IV. CONCLUSIONS

Over the range of angles in which comparison is possible, the results of Bachheimer agree closely with our own results for $N = 100$. For smaller N however, significant differences are seen, with the curves leveling off as i decreases. Extrapolation of the curves to $i = 0°$ indicates that the bandwidths approach different minimum levels, $\Delta\lambda_0$, characteristic of N, and that $\Delta\lambda_0$ increases as N decreases.

This result may be readily explained by the following argument. At $i = 0°$ the electron velocity is parallel to the grating surface so that the electron which emits SP radiation does so while passing over all N grooves of the grating.

We then assume that a one-to-one correspondence exists between grooves in the grating and undulations in the train of electromagnetic waves produced by interaction of a single passing electron with the surface. The wave train produced would have length $L = N\lambda$, where λ is given by Eq. (1). A standard textbook exercise[18] shows that "chopping" of such a wave train to length L results in broadening of the wavelength band to yield a width at one-half maximum intensity of

$$\Delta\lambda_0 = 2\lambda\left(\frac{1}{1 - 1.392/\pi N} - 1\right). \tag{3}$$

The bandwidth increases with a decrease in N, the number of grating grooves.

Table II shows values of $\Delta\lambda_0$ predicted by (3) together with results obtained by extrapolation of the curves of Fig. 1 for the three values of N employed. The agreement seems to indicate the correctness of our assumption regarding the length of the radiated wave train.

The theory of Bachheimer, which yields Eq. (2), assumes that an electron radiates in crossing the grating only if it is near the surface, i.e., at a height above the grating less than some distance h_0. He refers to this region as the "zone of effectiveness." As the angle i increases, the electron crosses fewer grooves after entering the zone of effectiveness, and hence generates a wave train

TABLE II. Bandwidths of radiation due to electron incident upon grating at $i = 0°$ for various numbers of grooves.

Number of grooves, N	Bandwidth (Å) Predicted by Eq. (3) [a]	Extrapolated from Fig. 1
100	98	70
32	209	195
16	376	300

[a] Includes 46 Å broadening due to $\Delta\theta$.

of lesser length. This results in the bandwidth increase which is verified experimentally.

For gratings with N small, this same model predicts that the bandwidth should decrease with decreasing angle of incidence until an angle i_0 is reached for which the electron radiates in passing over all grooves. A further decrease in i should not affect the bandwidth. The value of i_0 would, of course, depend upon N. This behavior is suggested by the shapes of the curves for $N=32$ and $N=16$ in Fig. 1.

The data obtained in this experiment may be used to estimate the height, h_0, of the zone of effectiveness. From the curves of Fig. 1 we obtain a value of i_0 for a particular N. Assuming a straight-line path for the electron, we then have $h_0 = Nd\tan i_0$. The values which we calculate for h_0 are 1650 and 1450 Å for $N=16$ and $N=32$, respectively.

V. SUMMARY

The results of this experiment may be summarized as follows: (1) Smith-Purcell radiation was observed from gratings having as few as four grooves; (2) the intensity of radiation in a viewing plane oblique to the grating surface was found to be greater than in a plane normal to the grating; (3) the polarization of the radiation was found to vary with ϕ; and (4) the width of the band of wavelengths generated was found to be a function of the number of grating grooves, for i small.

The latter result may be interpreted as indicating a one-to-one correspondence between undulations in the radiated wave train and grooves in the grating. Assuming the correctness of this interpretation, we have calculated the height of the zone of effectiveness for the electrons.

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