Differential spin exchange in the elastic scattering of low-energy electrons by rubidium*

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The atomic-beam recoil technique has been used to obtain the ratio of spin-flip to full differential cross sections for the elastic scattering of electrons by rubidium at 0.80, 1.10, and 1.40 eV. Data are presented for a range of angles between 30° and 180° at each of these energies. These measurements were partly motivated by the expectation that relativistic effects, particularly the spin-orbit interaction, will be significant in electron scattering by the heavier alkali metals, even at low energies. In this context, we show how our measurements are related to the relativistic scattering amplitudes recently discussed by Burke and Mitchell for one-electron atoms. These experiments were performed at intermediate magnetic fields, where the nuclear and valence electron magnetic moments are not fully decoupled. The necessary corrections are discussed and taken into account.

I. INTRODUCTION

An ever increasing body of results for low-energy electron scattering by the alkali-metal atoms has been accumulating in the last few years, both in experiment and theory. This is not surprising given the characteristics of the alkali elements, which are simple to handle experimentally on the one hand, and relatively easy to treat by the theorist on the other.

Most of the experimental work has been performed with sodium and potassium and includes the measurement of total,[1-3] differential,[2,4-6] direct differential,[6] and exchange differential[2,7,8] elastic cross sections, as well as some differential n^2S-n^2P cross sections,[9,10] both with and without spin analysis. The unifying feature of these experiments is the generally good agreement which obtains between measurements and the results of few-state close-coupling calculations,[11-13] although some discrepancies exist.

Recently Moores,[14] in connection with a three-state close-coupling computation, has summarized the status of comparison of theory and experiment for potassium, as well as between several close-coupling calculations. He obtains good agreement with the elastic differential cross section at fixed angle observed by Eyb and Hoffmann[5] near the first excitation threshold, and also with the absolute elastic differential measurements of Collins, Bederson, and Goldstein,[2] and Slevin, Visconti, and Rubin.[1] In most, although not all, cases good agreement is also obtained with other measurements, including total $4s$-$4p$ excitation, absolute inelastic angular distributions, and absolute total cross sections.

With the state of agreement between theory and experiment for sodium and potassium in such relatively good shape, one is now led to explore both the heavier alkalis and lithium, in order to question possible omissions from the simple close-coupling formulation which would not necessarily reveal themselves in the study of sodium and potassium. Recent work performed in our laboratory on lithium will be presented in a separate paper. Of particular interest to us here is the question of the contribution of relativistic effects, particularly the spin-orbit interaction, which is not usually included in the electron-atom Hamiltonian. Even for the lighter alkalis some experimental evidence indicates that spin-orbit interaction effects, while small, are not necessarily negligible. For example, Wilmers[15] has observed the spin-orbit interaction in sodium at 5 eV by detecting the small polarization produced by elastic scattering of unpolarized electrons as a function of scattering angle.

Spin effects arise, even for light atoms where the spin-orbit interaction is negligible, because of electron exchange. In this case, nonrelativistic close-coupling calculations have been quite successful in predicting spin-dependent cross sections, as shown by the reasonable agreement obtained with the results of spin-analyzed experiments.[2,6-10] For heavier atoms, the spin-orbit interaction becomes more important, and it also gives rise to spin-polarization effects. Initially, the tendency in both theory and experiment was to concentrate on electron scattering from spin-zero targets, where electron exchange does not appear as an explicit cause of spin polarization. But lately, increasing attention is being directed towards processes where electron exchange and relativistic effects are both important.[16-18] Spin-analyzed electron scattering by the heavy alkali atoms presents the best opportunity to investigate those processes. Electron exchange will still play a dominant role in such collisions, but relativistic effects should be observable. The recoil technique, as described in Sec. II, is well suited

to the study of such processes, because the spin state of the atoms, both before and after the collision, can be easily determined.

As for the choice of target, both cesium and rubidium seem likely candidates to show the expected relativistic effects. Cesium may look like a better choice because of its higher Z, but rubidium possesses a smaller hyperfine-structure splitting, enabling us to decouple nuclear and electronic spins more easily when analyzing the atomic beam spin state after the collision. The present work will permit a test of the adequacy of nonrelativistic close-coupling calculations for the heavier alkalis when they become available, several such calculations being currently in progress.

II. METHOD AND APPARATUS

Rubin *et al.*[19] developed the "recoil technique" method for the study of low-energy electron-atom collisions. The main difference between this and other experimental techniques is that observation is made on the scattered atoms, rather than on the electrons. The atomic recoil angles are large enough to allow not only the determination of total scattering cross sections by measuring the attenuation of the atom beam when crossfired by electrons, but also the study of differential scattering, by differentially collecting atoms scattered away from the beam axis. Scattering experiments can also be performed with state selection before and state analysis after the interaction.

A sketch of the experimental arrangement is shown in Fig. 1. The apparatus consists of an alkali oven source, a Stern-Gerlach velocity selector and polarizer magnet, a scattering region, an E-H gradient-balance magnet,[20] and a surface ionization detector. In normal operation, the alkali oven is offset from the beam axis. The Stern-Gerlach magnet permits one of the "effective moment" states to pass through the apparatus, and at the same time velocity-selects that state with a resolution $\Delta V/V$ of approximately 0.08.

The characteristics of the electron gun and scattering region are described by Collins *et al.*[21] The electron energy resolution is about 250 meV, with the gun usually operated at an interaction current of 100 μA. The electron energy is corrected for contact-potential differences.

The E-H gradient-balance magnet[20] operates by mutual cancellation of the electric and magnetic forces acting on the effective magnetic and induced electric dipole moments for a particular spin state of the atom. It is a characteristic of this magnet that atoms satisfying the balance condition are transmitted with no significant alteration of their trajectories. This characteristic is very desirable, because it simplifies the transformation from atomic to electron scattering angles. In normal operation, the E-H gradient-balance magnet (spin analyzer) is adjusted so as to pass only those atoms that have changed their spin state as a result of the scattering process.

The polarizer-analyzer combination employed in this experiment is about 97% efficient, i.e., only about 3% of the unwanted spin state arrives at the detector with both magnets operating.

The surface ionization detector is a platinum-tungsten wire (92% Pt, 8% W), 0.025 cm in diameter. At the operating temperature, the ionization efficiency of the hot wire for rubidium atoms is nearly 100%. The alkali ions pass through a simple magnet mass spectrometer, and are detected by a channeltron electron multiplier operated in the current mode.

The spin-analyzer magnet and detector can rotate about the center of the interaction region, in the plane determined by the atomic and electron beams. The rotation is driven by a motor, and the detector position is accurately determined by a dial gauge.

The electron-gun current was chopped at a frequency of 15 Hz, and a phase-sensitive lock-in detector was employed to observe the scattering signals. The lock-in output was usually digitized and integrated by using a voltage-to-frequency converter and a scaler. Alternatively, it could be displayed as the ordinate on an X-Y recorder whose abscissa was the output of a linear potentiometer which gives a voltage proportional to the detector displacement.

We will discuss the theory of the recoil method only very briefly. A more complete analysis is contained in the article by Rubin *et al.*[9] Let (θ,φ) be the electron scattering angles, and (ψ,χ) the atomic recoil angles, ψ in the plane determined by both incident beams and χ in the normal plane containing the incident atomic beam. It is then easy to show that for elastic scattering and in the approximation in which $\alpha = mv/MV \ll 1$ where

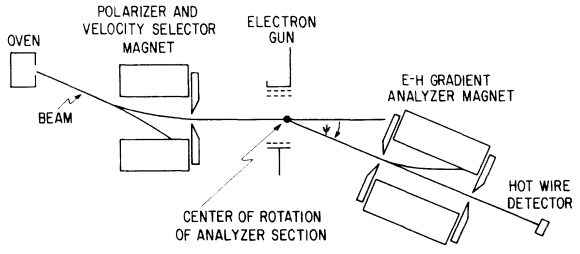

FIG. 1. Experimental arrangement. The analyzer and detector rotate about the scattering center.

mv and MV are the electron and atom momenta, respectively,

$$\psi \cong \alpha(1-\cos\theta) , \tag{1}$$

$$\chi \cong \alpha \sin\theta \sin\varphi , \tag{2}$$

to first order in α. If L is the distance between detector and scattering region and x is the detector displacement from the beam axis, then $x = L\tan\psi \cong L\psi$.

Equation (1) gives a one-to-one relationship between the polar electron scattering angle θ and atomic "recoil" angle ψ independently of the azimuthal electron scattering angle φ. However, different φ's correspond to different χ's, as given by Eq. (2). Assuming no beam obstructions, a detector of height greater than $2\alpha L$ will collect all atoms scattered into ψ, thus effectively integrating over φ for any value of θ. But if the effective height of the detector is smaller than $2\alpha L$, as it actually is in our case because of the presence of the analyzer magnet, then only a fraction of the atoms scattered into ψ will be collected, and, furthermore, that fraction will depend on θ, as shown by Eq. (2). In determining absolute differential cross sections, this necessitates the introduction of an "azimuthal form factor" (see Ref. 9). In the present experiment, where only ratios are determined, this correction is not necessary.

The relationship between detector displacement x and electron polar angle θ is given by

$$\theta = \arccos(1 - x/\rho) , \tag{3}$$

where ρ, the radius of the scattering sphere in configuration space, is given by

$$\rho = \alpha L = (2mE)^{1/2} L/MV , \tag{4}$$

and thus requires the knowledge of the mean atom beam speed V. This quantity was determined as described by Collins *et al.*[2] This method takes advantage of the fact that the detector displacement corresponding to inelastic scattering with energy loss E_0 is given (to first order in α) by

$$x \simeq \alpha L[\,1 - (1 - E_0/E)^{1/2}\cos\theta\,] . \tag{5}$$

Because of the finite detector height, the differential scattering signal for $E > E_0$ exhibits a well-defined peak at the displacement x corresponding to forward inelastic scattering. The displacement of this peak from the beam axis can be used to determine the beam velocity.

In normal operating conditions, with the oven at about 300 °C, the mean beam velocity is about 500 m/sec. The full range of detector displacement, 2ρ, corresponding to θ varying from 0° to 180°, is typically about 1 cm.

Rubin *et al.*[9] have shown that the scattering signal $S(x)$ at the detector position x is

$$S(x) = \text{const} \times \int dE\, E^{-1/2} N(E)\gamma(\theta, E)\sigma(\theta, E) , \tag{6}$$

neglecting the spread in atom velocities, and assuming a "pencil" atom beam of negligible height and width. The constant includes various geometric and beam factors. $N(E)$ is the electron energy distribution, which can be determined from retarding potential measurements in the electron gun. $\gamma(\theta, E)$ is the azimuthal form factor.

If the spin analyzer is turned off, then all spin states can reach the detector. The signal $S_{\text{off}}(x)$ will thus be proportional to the full differential cross section $\sigma(\theta, E)$. If we assume that $\sigma(\theta)$ is a slowly varying function of energy, Eq. (6) can be written

$$\sigma(\theta) = S_{\text{off}}(x) \Big/ \text{const} \times \int dE\, E^{-1/2}\gamma(\theta, E)N(E) . \tag{7}$$

If the spin analyzer is now turned on, and adjusted to transmit the atoms which changed their spin state in a collision (here we are neglecting any mention of hyperfine structure; the subject will be discussed in Sec. III), then an equation similar to (7) can be written for the spin-flip cross section $\sigma_{\text{SF}}(\theta)$,

$$\frac{\sigma_{\text{SF}}(\theta)}{2} = S_{\text{on}}(x) \Big/ \text{const} \times \int dE\, E^{-1/2}\gamma(\theta, E)N(E) , \tag{8}$$

where $S_{\text{on}}(x)$ is the scattering signal at x with the analyzer operative. We are here again assuming that $\sigma_{\text{SF}}(\theta)$ is a slowly varying function of energy. The factor $\frac{1}{2}$ appears because the electron beam is unpolarized, and thus only half of the electrons participate in this measurement.

The simplest quantity to measure in our experiments is obviously the ratio $R(\theta)$ of the two signals,

$$R(\theta) = S_{\text{on}}(x)/S_{\text{off}}(x) = \tfrac{1}{2}\sigma_{\text{SF}}(\theta)/\sigma(\theta) , \tag{9}$$

since in this determination most of the apparatus parameters cancel out. The measurement of $R(\theta)$ is straightforward, and does not merit further discussion here except that we point out that Eq. (9) has to be modified to allow for the subtraction of background signal. The measured values of $R(\theta)$ were corrected for the analyzer transmission and the beam depolarization according to the prescription described by Collins *et al.*[2]

$R(\theta)$ was measured at $E = 0.80$, 1.10, and 1.40 eV. The measurements at each energy and detector

position were repeated a certain number of times (typically ten). The results were averaged, and standard deviations and standard errors were computed.

III. RESULTS

Had our experiments been performed on "ideal alkali atoms,"[22] the relationship between the measured quantity $R(\theta)$ and the scattering amplitudes, neglecting relativistic effects, would be almost trivial. Such an atom possesses a single valence electron in an s state, no nuclear spin, no core angular momentum, and no spin-orbit interaction during the collision. Below the first inelastic threshold, the scattering process is completely represented by two independent scattering amplitudes. For a particular choice of such amplitudes, namely, f and g, the direct and exchange scattering amplitudes, we can write the relevant cross sections as

$$\sigma(\theta) = \tfrac{3}{4}|f-g|^2 + \tfrac{1}{4}|f+g|^2 , \tag{10}$$

$$\sigma_{SF}(\theta) = \sigma_{ex}(\theta) = |g|^2 , \tag{11}$$

and then

$$R(\theta) = \frac{1}{2}\frac{\sigma_{SF}(\theta)}{\sigma(\theta)} = \frac{\frac{1}{2}|g|^2}{\frac{3}{4}|f-g|^2 + \frac{1}{4}|f+g|^2} . \tag{12}$$

The problem becomes more complicated if the spin-orbit interaction (or other relativistic effects) becomes important. In that case, Burke and Mitchell[17] have shown that for elastic electron scattering by hydrogenlike atoms, coupling the total spin $\vec{S}$ to the total orbital angular momentum $\vec{L}$ to form $\vec{J}$, the total angular momentum of the system, leads to six rather than two independent scattering amplitudes. They give the collision matrix M in terms of two different sets of such amplitudes.

The differential scattering cross sections $\sigma(\alpha, \beta, \theta)$ are

$$\sigma(\alpha,\beta,\theta) = \mathrm{Tr}[M\rho(\alpha)M^{\dagger}\rho(\beta)]/\mathrm{Tr}\rho(\alpha) , \tag{13}$$

where α, β are the initial and final spin states of the electron-atom system, $\rho(\alpha)$ is the density matrix for the incident beams, and $\rho(\beta)$ is the projection operator onto (or density matrix for) the desired final states. This definition incorporates the usual averaging over initial states and summing over final states.

In our spin-analyzed experiment, unpolarized electrons are scattered by polarized atoms. After the collision, we detect only those atoms which have changed their spin state, without any reference to the spin state of the scattered electrons. The axis of quantization is parallel to the momentum of the incident electrons. Hence if $|\pm,\pm\rangle$ represents the state of the system when the projections of the atomic and electron spin on the quantization axis are $\pm\frac{1}{2}$, then

$$\rho(\alpha) = |+,+\rangle\langle+,+| + |+,-\rangle\langle+,-| , \tag{14}$$

$$\rho(\beta) = |-,+\rangle\langle-,+| + |-,-\rangle\langle-,-| . \tag{15}$$

The cross section for the process under study is

$$\sigma(\alpha,\beta,\theta) = |A_2|^2 + |A_4|^2 + \sin^2\tfrac{1}{2}\theta|A_5|^2 + \cos^2\tfrac{1}{2}\theta|A_6|^2 , \tag{16}$$

in terms of the set $\{A_i\}$ of scattering amplitudes, as defined by Burke and Mitchell, neglecting all hyperfine-structure effects. In terms of the same amplitudes, the full differential cross section is[17]

$$\sigma(\theta) \equiv I_0 = |A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2 + |A_5|^2 + |A_6|^2 , \tag{17}$$

and the quantity that we measure in our experiment, $R(\theta)$, is

$$R(\theta) = \sigma(\alpha,\beta,\theta)/\sigma(\theta) . \tag{18}$$

In the limit in which the relativistic interactions are negligible, $A_1 = f - \frac{1}{2}g$, $A_2 = A_3 = 0$, and $A_4 = A_5 = A_6 = -\frac{1}{2}g$. In this case, $R(\theta)$ is given again by (12).

Real alkali atoms do possess nuclear spin, which causes hyperfine splitting of the ground state and complicates the analysis of electron-alkali atom scattering experiments. One way to avoid these complications is to perform the experiments in a magnetic field high enough to decouple electron and nuclear spins completely. The electron-spin projection, $M_s = \pm\frac{1}{2}$, then becomes a good quantum number, as it is for the "ideal alkali atom," and Eq. (12) becomes valid again provided that the spin-orbit interaction can still be neglected during the collision. In this way, experiments similar to the one we are describing were performed with potassium[2] and sodium.[8]

As a rule of thumb, in order to be able to use the high-field approximation, one needs magnetic fields which are well above the value that will produce the highest zero effective moment state.[23] Rubidium is a mixture of two isotopes, ^{85}Rb (72%) and ^{87}Rb (28%). ^{85}Rb has a nuclear spin $I = \frac{5}{2}$, and the highest zero effective moment state appears at $H = 722$ G. The corresponding figures for ^{87}Rb are $I = \frac{3}{2}$, $H = 1221$ G. In our experiments, the magnetic field in the interaction region was 850 G, which is certainly not large enough to decouple the nuclear and electron spins. This means that Eq. (12) is no longer valid. In these conditions the experimentally determined $R(\theta)$ is not equal to the ratio of the spin flip to full differential cross sections. In the case in which two ampli-

tudes are adequate to describe the collisions under study (i.e., whenever the "nonrelativistic" analysis can be used), we can correct our data for hyperfine-structure effects by using the formalism developed by Glassgold and Walker,[24] who studied the general problem of spin-exchange collisions in uniform magnetic fields. We are not aware of any such treatment that could be used with the six-amplitude description, and so we will not attempt to correct our data for hyperfine-structure effects within the context of a "relativistic" analysis.

Glassgold and Walker[25] consider all magnetic interactions to be negligible during the collision, so that the only effect of an external magnetic field is to change the character of the asymptotic states. They show that the unitary transformation, which diagonalizes the Breit-Rabi Hamiltonian for a single paramagnetic atom, relates scattering in the absence of field to scattering in the presence of an arbitrary field H. They then give a general expression for $\sigma(\nu, M; \nu', M')$, the scattering cross section when the atom initially in the state (ν, M) ends in the state (ν', M'). The quantum numbers (ν, M) characterize the hyperfine state of the atom in the presence of an arbitrary magnetic field. M is the eigenvalue of $J_z + I_z$, where $\vec{J}$ and $\vec{I}$ are the atomic and nuclear momenta, respectively, and ν depends upon the sign of the radical in the Breit-Rabi formula. For $H = 0$, $F = I + \frac{1}{2}\nu$ if $|M| \neq I + \frac{1}{2}$, and $F = I + \frac{1}{2}$ if $|M| = I + \frac{1}{2}$. For $H \to \infty$, $\nu = \pm 2M_J$ (see Fig. 2).

The cross section $\sigma(\uparrow, \downarrow)$ for atoms prepared in spin-up states by the Stern-Gerlach magnet, and elastically scattered by unpolarized electrons into the spin-down states transmitted by the analyzer, can be calculated using Glassgold and Walker's expression for $\sigma(\nu, M; \nu', M')$ by summing over final states and averaging over initial states:

$$\sigma(\uparrow, \downarrow) = \sum_{M=-I+\frac{1}{2}}^{I+\frac{1}{2}} \sum_{M'=-I-\frac{1}{2}}^{I-\frac{1}{2}} A_M \sigma(1, M; -1, M')$$

$$= \tfrac{1}{4}|g|^2 \sum_{M=-I+\frac{1}{2}}^{I+\frac{1}{2}} \sum_{M'=-I-\frac{1}{2}}^{I-\frac{1}{2}} A_M \Gamma(1, M; -1, M') \ , \tag{19}$$

where the coefficients $\Gamma(\nu, M; \nu', M')$ are given by Glassgold and Walker. The selection rule $\Delta M = 0, \pm 1$ is satisfied. A_M is the statistical weight of the state $(1, M)$ in the incident atomic beam after state selection by the Stern-Gerlach magnet.

In our experiments, the magnetic field in both polarizer and analyzer magnets was 1240 G. For ^{85}Rb, this field is high enough for the polarizer to transmit all the $\nu = 1$ states ($M = -2$ to 3) and for the analyzer to transmit all the $\nu = -1$ states ($M = -3$ to 2) (see Fig. 2). But for ^{87}Rb, both $M = -1$ states ($\nu = 1$ and $\nu = -1$) have zero effective magnetic moment at 1221 G. In consequence, the polarizer transmits only the $\nu = 1$, $M = 0, 1, 2$ states, and the analyzer the $\nu = -1$, $M = -2, 0, 1$ states. Assuming that for each of the two isotopes all states in the beam transmitted by the Stern-Gerlach magnetic are equally populated, then $A_M = \frac{1}{6}$ for all $\nu = 1$, ^{85}Rb states; $A_{-1} = 0$, $A_M = \frac{1}{3}$ for $M \neq -1$ for the $\nu = 1$, ^{87}Rb states. Using Eq. (19), and adding the contributions of both isotopes weighted by their natural abundances, we obtain $\sigma(\uparrow, \downarrow) = 0.457|g|^2$, or

$$\sigma_{SF}(\theta)/\sigma(\theta) = 2.188R(\theta) \ . \tag{20}$$

Figure 3 shows our experimental results, corrected using Eq. (20), for $E = 0.80$, 1.10, and 1.40 eV. The vertical bars give the standard (statisti-

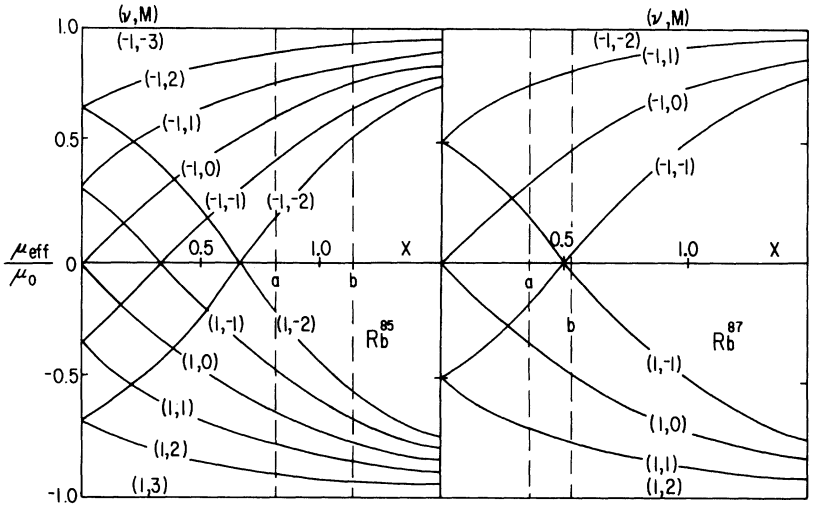

FIG. 2. Effective magnetic moment diagrams for ^{85}Rb and ^{87}Rb. $x = g_J \mu_0 H/\Delta W$, where ΔW is the ground-state hyperfine splitting. The dashed lines indicate the operating points for (a) the interaction region (850 G) and (b) the polarizer and analyzer magnets (1240 G).

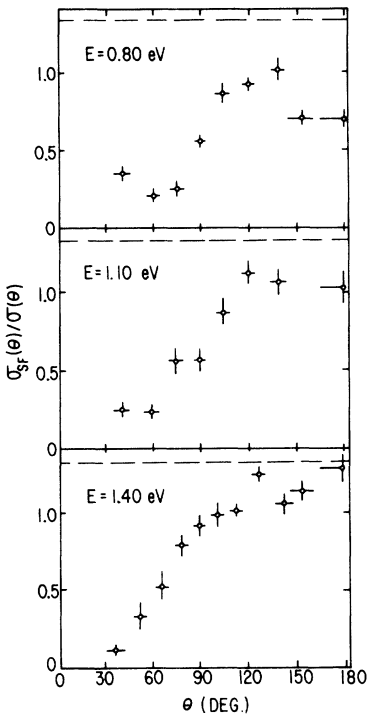

FIG. 3. Ratio $\sigma_{SF}(\theta)/\sigma(\theta)$ as a function of electron scattering angle at $E=0.80$, 1.10, and 1.40 eV. Vertical bars give standard errors; horizontal bars indicate overall apparatus angular resolution.

cal) error in the cross-section ratio. The horizontal bars are indicative of the overall angular resolution, and take into account geometrical factors as well as the uncertainty in atomic beam velocities and electron energies.

The assumption of equal populations among the hyperfine states, which results in the relation given in Eq. (20), introduces a systematic error into the determination of $\sigma_{SF}(\theta)/\sigma(\theta)$. Reasonable alternative assumptions concerning these population distributions lead to results for this ratio which differ at most by about 1%. Thus this uncertainty does not significantly affect the overall error estimate.

Assuming that the two-scattering-amplitudes description is valid, then the largest value that the cross-section ratio can take is $\frac{4}{3}$, obtained when $f=\frac{1}{2}g$. This is shown by the dashed lines in Fig. 3. Our experimental results were always below that theoretical limit. In general, they are small at small angles and larger at large angles. This is to be expected, because on the average atoms scattered through large angles correspond to small-impact-parameter collisions, where exchange contributions should become important.

We are not aware of any other experiment that could be compared with the present results. At the present time there are no close-coupling computations for rubidium, although calculations are now in progress by several groups, and results are expected within the next several months.

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