Intrinsic third-order correlations in laser light near threshold*

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We present an experimental investigation of the third-order intensity correlations in the light from a single-mode intensity-stabilized He-Ne laser in the threshold region. Accurate results were obtained by utilizing a fast digital correlator operating in real time. The computation of the intrinsic third-order correlations requires also a knowledge of the second-order intensity correlation which was measured under the same operating conditions. The experimental data are in agreement with recent theoretical computations based on the usual Van der Pol model of the laser oscillator. A new definition of the intrinsic correlations, which seems more appropriate for the interpretation of the experimental results, is discussed.

I. INTRODUCTION

New interest in the behavior of a single-mode laser near threshold has been stimulated by recent papers[1,2] in which the analogy with phase transitions in thermal equilibrium and with instabilities in open systems has been discussed. From this point of view, a peculiar feature of the laser system is that it is possible to make evident experimentally[3] in a narrow region around threshold the non-Gaussian character of the field fluctuations due to the nonlinearity of the atom-field interaction. The fact that the laser field can no longer be treated as a Gaussian process implies, besides other effects which have already been thoroughly investigated,[3] that the behavior of third- and higher-order intensity correlations cannot be inferred from the knowledge of second-order intensity correlations.

Cantrell, Lax, and Smith[4] have recently computed the quantity

$$K_3 = \langle [I(t) - \langle I\rangle][I(t+\tau) - \langle I\rangle][I(t+\tau') - \langle I\rangle]\rangle / \langle I\rangle^3,$$

where $I(t)$ is the intensity of laser light, under the hypothesis that the laser field is a Markovian process. We present in this paper accurate measurements of third-order intensity correlations and describe the data-reduction procedure. Our results are in good agreement[5] with the calculations of Ref. 4.

A further point we discuss here is the definition of "intrinsic" correlation. Indeed, in Ref. 4, K_3 is called the intrinsic third-order correlation. That definition, however, does not appear satisfactory from a physical point of view, because one would expect an intrinsic third-order correlation to go to zero well below threshold, where the laser field is a Gaussian process and third-order correlations are predictable once second-order correlations are known. Since the quantity K_3 does not show this behavior, we propose here a different definition of intrinsic correlations.

II. EXPERIMENTAL SETUP AND DATA REDUCTION

A measurement of third-order correlations performed with a photon-counting technique yields directly the normalized third-order correlation function

$$g_3(\tau, \tau') = \langle I(t) I(t+\tau) I(t+\tau')\rangle / \langle I\rangle^3,$$

which is connected to K_3 by the relation

$$K_3(\tau, \tau') = g_3(\tau, \tau') + 2 - g_2(\tau) - g_2(\tau') - g_2(\tau - \tau'), \tag{1}$$

where $g_2(\tau) = \langle I(t) I(t+\tau)\rangle / \langle I\rangle^2$. Equation (1) shows that K_3 can be derived from the experimental g_3 after subtraction of other quantities which are also to be obtained experimentally. Since K_3 is much smaller than g_3 in the threshold region, the measurements must be very accurate.

In our experiment the laser source is an intensity-stabilized He-Ne laser working at 6328 Å on a single transverse and longitudinal mode. The laser light is suitably attenuated and detected by a photomultiplier tube. Single photoelectron pulses are amplified, standardized, and passed to a fast digital correlator built in our laboratory.[6] This instrument operates in real time up to a sampling frequency of 16 MHz over 107 delays and can perform autocorrelations and cross and triple correlations. In the triple-correlation mode the function $g_3(\tau, \tau')$ is obtained in a single measuring run for a fixed value of τ and for 107 values of τ'.

The signal at the input of the correlator contained also a contribution, uncorrelated with the laser signal, due to dark pulses from the photomultiplier tube and to fluorescent light from the laser discharge. It is particularly important to take into account uncorrelated contributions in a third-order correlation measurement, because their presence modifies also the time dependence of $g_3(\tau, \tau')$. Indeed, it is easy to show that the measured correlation $F_3(\tau, \tau')$ is connected to the normalized third-order laser correlation by the following expression:

$$\frac{F_3(\tau,\tau')}{\langle n\rangle_L^3} = g_3(\tau,\tau') + \frac{\langle n\rangle_N}{\langle n\rangle_L}[g_2(\tau) + g_2(\tau') + g_2(\tau-\tau')] + 3\frac{\langle n\rangle_N^2}{\langle n\rangle_L^2} + \frac{\langle n\rangle_N^3}{\langle n\rangle_L^3}, \tag{2}$$

where $\langle n\rangle_L$ and $\langle n\rangle_N$ are, respectively, the average number of photoelectrons due to laser light and of noise pulses. The second-order correlation $g_2(\tau)$ was measured for each operating point of the laser, together with the quantities $\langle n\rangle_L$ and $\langle n\rangle_N$. The third-order cumulant K_3 is then derived by using Eqs. (1) and (2).

Figure 1 shows a set of experimental curves obtained with a laser setting very close to threshold. Each curve presents a peak at the value $\tau=\tau'$. Indeed, it is immediately shown from the definition of the third-order correlation that $F_3(0,\tau') = F_3(\tau',\tau')$. The fractional statistical error at each point of $F_3(\tau,\tau')$ is about 0.3%. This result was achieved in a measuring time of about 20 min for each fixed value of τ'.

The comparison between experiment and theory is not straightforward. A first problem arises because all times in the theoretically computed correlation functions are expressed in dimensionless form in order to have results independent of the specific laser parameters. The time-scale factor is easily computed once the intensity correlation time τ_C of the laser at threshold is known.[3] In our case $\tau_C = 38$ μsec.

The second problem is the determination of the pump parameter a in the chosen operating conditions. The pump parameter is negative below and positive above threshold. Its definition in terms of the laser parameters can be found in Ref. 3. The laser threshold ($a=0$) is identified by the fact that the value of $g_2(0)$ is equal to 1.571 at threshold. Once the laser intensity at threshold is determined, the value of the pump parameter a for a different setting of the laser is computed from the ratio of the laser intensity at the working point and the laser intensity at threshold.[3]

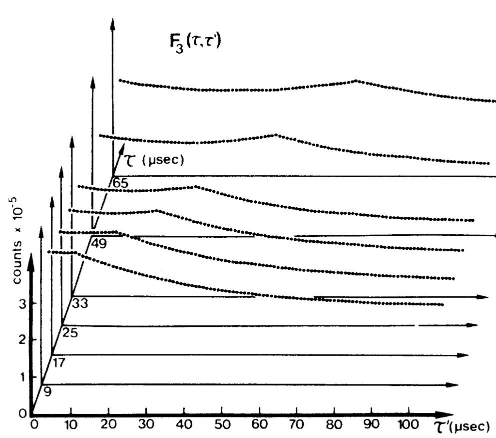

FIG. 1. Measured values of the third-order correlation function $F_3(\tau,\tau')$ for a laser operating at threshold. The standard deviations are smaller than the dot size on this scale. The measuring time was the same for all curves. Since the sampling frequency was 1 MHz, the number of microseconds on the time scale coincides with the channel number.

A further point to be considered is that our definition of K_3 is different from that given in Ref. 4. In this paper K_3 is expressed, consistent with the experimental procedure, as a function of the independent variables τ and τ', where τ is the delay between the first and the second sampling operation and τ' the delay between the first and the third. Since no time ordering is implied in our definition, τ' can be smaller than τ, as shown in Fig. 1. The theoretical definition instead implies time ordering and uses the two independent variables t and t', where t is the delay between the first and the second measurement and t' is the delay between second and third. Therefore once $g_3(\tau,\tau')$ is obtained in a single experimental run for a fixed value of τ and for 107 values of τ' the comparison between experiment and theory must be performed as follows: for $\tau' \leq \tau$ put $t=\tau'$ and $t'=\tau-\tau'$, for $\tau' \geq \tau$ put $t=\tau$ and $t'=\tau'-\tau$. This procedure is better understood by looking at Fig. 2: a measurement with $\tau = 17$ μsec and τ' ranging from 0 to 107 μsec gives $g_3(t,t')$ along the path ABC in Fig. 2. Furthermore, since $g_3(t,t')$ is a symmetric function of t and t', the path BAD is completely equivalent to ABC. The existence of crossing points can be exploited to check the internal consistence of the measurements and to estimate relative statistical errors. For instance, F_3 at point P ($t=17$, $t'=8$) is obtained from three independent experimental points: $\tau=17$, $\tau'=25$;

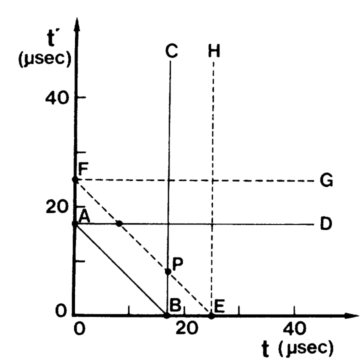

FIG. 2. Paths followed on the plane (t,t') of definition of $G_3(t,t')$ in a single experimental run with fixed τ and τ' ranging from 0 to 107. The paths shown refer to $\tau=17$ and 25 μsec. See further explanations in the text.

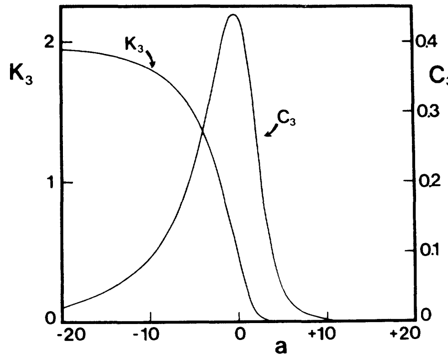

FIG. 3. Intrinsic third-order intensity correlation C_3 and third-order intensity cumulant K_3 at zero delay vs the pump parameter a in the laser threshold region.

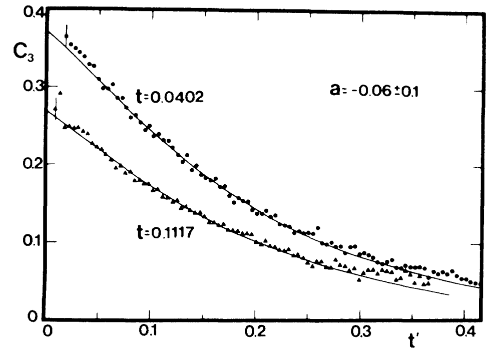

FIG. 4. Intrinsic third-order intensity correlation $C_3(t, t')$ vs t' for two distinct values of t. The laser is operating at threshold. The error bars are almost the same for all the points of the same curve. For the sake of clarity of presentation they are given only for the first point. The delays t and t' are expressed in dimensionless form, as explained in the text. Solid lines represent theoretical results.

$\tau = 25$, $\tau' = 17$; $\tau = 25$, $\tau' = 8$. The three obtained values, in arbitrary units but with the same rate and measuring time, at $a = -2$, are, respectively, $F_3(17, 25) = 476.13$, $F_3(25, 17) = 474.73$, and $F_3(25, 8) = 475.21$.

III. INTRINSIC CORRELATIONS

We define as the intrinsic nth-order correlation that part of the total nth-order correlation which is not predictable from the knowledge of all correlations up to order $n-1$. For the sake of simplicity, let us discuss explicitly the special case of all delay times equal to zero. The generalization of the results to time-dependent correlations is straightforward. In this case we need only $P(\mathfrak{a})$, the probability density of the complex field amplitude $\mathfrak{a}$. The cumulants k_{mn} of $P(\mathfrak{a})$ are defined through the following series expansion[7]:

$$k(\lambda) = \ln Q(\lambda) = \sum_{m=n=0}^{\infty} k_{mn} \frac{(i\lambda)^m (i\lambda^*)^n}{n!\,m!}, \tag{3}$$

where $Q(\lambda) = \langle e^{-i(\lambda^* \mathfrak{a} + \lambda \mathfrak{a}^*)} \rangle$ is the moment-generating function. It can be shown that the cumulants are a measure of intrinsic correlations. If, for instance, the optical field is a Gaussian process we know that correlations at any order are predictable from the simplest correlation $k_{11}(\tau) = \langle \mathfrak{a}^*(t)\mathfrak{a}(t+\tau)\rangle$. Correspondingly, we find that all cumulants except k_{11} are equal to zero.

The situation is different if we start from the intensity probability density $p(I)$, where $I = |\mathfrak{a}|^2$. If we consider again a Gaussian field, $p(I)$ is exponential and its cumulants at any order are different from zero. We conclude therefore that intensity cumulants are not a good measure of intrinsic correlations as we have defined them at the beginning of this section. The normalized third-order cumulant for a Gaussian field, expressed in terms of the second-order cumulant $K_2(\tau) = g_2(\tau) - 1$, is

$$K_3(\tau, \tau') = 2[K_2(\tau)K_2(\tau')K_2(\tau - \tau')]^{1/2}. \tag{4}$$

Equation (4) suggests the following expression for the intrinsic normalized third-order intensity correlation C_3 of an optical field:

$$C_3(\tau, \tau') = -\{K_3(\tau, \tau') - 2[K_2(\tau)K_2(\tau')K_2(\tau' - \tau)]^{1/2}\}. \tag{5}$$

The minus sign at the right-hand side of Eq. (5) is introduced only to make C_3 non-negative.

We show in Fig. 3 the quantity $C_3(0, 0)$, computed from theory,[8] versus the pump parameter a in the laser threshold region. Also, $K_3(0, 0)$ is shown for comparison. As expected, $C_3(0, 0)$ is significantly different from zero only in a narrow region around threshold ($a = 0$) where the field fluctuations are markedly non-Gaussian.

We have already shown in Ref. 5 that the experimental values of K_3, as computed from the raw data through Eqs. (1) and (2), are in good agreement with the calculations of Ref. 4. Of course, that agreement is not modified by using C_3 instead of K_3. As an example, we present some of our results in Fig. 4, together with the theoretical curves computed from Eq. (25) of Ref. 4 and our Eq. (5).

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