Parity nonconservation in Tl and Bi atoms*

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In connection with experiments which have been carried out and are planned, we have calculated the violation of parity conservation due to neutral weak currents for transitions to the first excited states of ^{205}Tl and ^{209}Bi. A Green's-function technique is used to evaluate the parity-nonconserving transition matrix. The dependence of the results on various input data (e.g., the Weinberg angle) is discussed.

I. INTRODUCTION

Although the existence of weak neutral currents has been established from muon-neutrino–induced reactions,[1] it is not yet known whether these currents produce an interaction between charged leptons ($\mu^{\pm}, e^{\pm}$) and hadrons. Furthermore, the space-time properties of the neutral currents remain to be determined[2]; for instance, are these neutral currents purely vector,[2] purely axial, or are they mixed V-A as required by theories such as that of Weinberg?[3] Suggested methods for determining the parity-nonconserving (PN) nature of the neutral currents include tests in muonic and electronic atoms.[4]

Atomic tests have several advantages. Weak-current effects are coherent and can thus be enhanced. Measured effects often involve an interference of the weak force with the well-known electromagnetic force. Furthermore, atomic wave functions are reasonably well known. Hence, an experimental observation can yield the sign as well as the magnitude and perhaps the space-time character (e.g., V-A) of the interaction. In addition, by varying the atom it is possible to establish the isospin dependence of the weak force due to neutral currents. These quantities are crucial ingredients of any theory.

Early papers by Bouchiat and Bouchiat[5] and by others[6] demonstrated the feasibility of parity-nonconservation searches in heavy electronic atoms. Such tests have been undertaken in various laboratories. More careful theoretical evaluations are therefore required. While the current work was in progress, a method for doing so was outlined by Bouchiat and Bouchiat[5] and a calculation for atomic ^{209}Bi was carried out by Brimicombe, Loving, and Sandars.[7]

Our method differs somewhat from both of these, although the basic tool remains a Green's-function technique. We present numerical results for two atoms which are being examined and actively considered at the University of Washington by Fortson and co-workers.[8] Our method is adaptable to other cases.

II. THEORY

The main assumptions we shall make are that the weak semileptonic e-nucleon interaction arises from both V and A currents and that the e-e PN force can be neglected.[5] (The parity-conservation violation due to the weak forces between electrons is small, but not primarily due to the Coulomb repulsion.[5]) The PN part is then proportional to

$$H_{\mathrm{PN}} = \frac{G}{\sqrt{2}}\left(\bar{N}\gamma_\mu\gamma^5\frac{f_0+f_1\tau_3}{2}N\bar{e}\gamma^\mu e + \bar{N}\gamma_\mu\frac{g_0+g_1\tau_3}{2}N\bar{e}\gamma^\mu\gamma^5 e\right), \tag{1}$$

where $G = 89.6$ eV fm^3 is the usual weak-coupling constant and f_0, g_0 and f_1, g_1 are the isoscalar and isovector contributions, respectively. In the Weinberg theory

$$f_0 = 0, \quad g_0 = -2\sin^2\theta_W, \qquad f_1 = (1-4\sin^2\theta_W)g_A, \quad g_1 = 1-2\sin^2\theta_W, \tag{2}$$

where g_A is the axial-vector renormalization constant $\simeq 1.25$, and θ_W is the Weinberg angle with $\sin^2\theta_W \simeq \frac{1}{3}$.

For the case of heavy atoms, which we are considering, the contribution of the first term in Eq. (1) is negligible because it is proportional to the nuclear spin, which is of order unity, whereas the second term gives rise to a coherent nuclear contribution roughly proportional to A, the atomic number. For the same reason the dominant part of the second term comes from the time component, so that for heavy atoms and nonrelativistic nucleons Eq. (1) can be approximated by

$$H_{\mathrm{PN}} = \frac{G}{\sqrt{2}}\frac{Q}{2}\int \rho_A(\vec{\mathbf{r}})\,\psi_e^\dagger\gamma^5\psi_e\,d^3r, \tag{3}$$

where

$$Q = \langle A \mid g_0 + g_1\tau_3 \mid A\rangle, \tag{4a}$$

$\rho_A(\vec{r})$ is the nuclear density and ψ_e is the relativistic electron wave function. In the Weinberg model

$$Q \to Q_W = Z(1 - 4\sin^2\theta_W) - N\,, \tag{4b}$$

where Z and N are the nuclear charge and neutron number. In a nonrelativistic approximation for the electron, we can neglect the nuclear size and obtain

$$H_{\rm PN} \to V_{\rm PN} = \frac{G}{\sqrt{2}}\,\frac{Q}{2}\left(\frac{\vec{\sigma}\cdot\vec{p}}{2mc}\,\delta^3(\vec{r}) + \delta^3(\vec{r})\,\frac{\vec{\sigma}\cdot\vec{p}}{2mc}\right), \tag{5}$$

where $\vec{p}$ is the momentum operator.

This nonrelativistic approximation is valid for light atoms. For heavy atoms relativistic effects become important, since the PN force acts in a region of space where the Coulomb potential Ze^2/r is large compared to the electron rest mass. Relativistic corrections to Eq. (5) have been discussed at length by Bouchiat and Bouchiat.[5] We shall employ Eq. (3) and not Eq. (5).

We consider the case of a parity-allowed transition which is purely magnetic dipole ($M1$), and a PN one which is purely electric of the same order, $E1$. We neglect electric quadrupole effects because they are small for cases of interest to us. By perturbation theory, which is certainly applicable for the weak interactions in lowest order, we find

$$\langle f|E1|i\rangle = \sum_n \frac{\langle f|H_{\rm PN}|n\rangle\langle n|E1|i\rangle}{E_f - E_n} + \sum_{n'} \frac{\langle f|E1|n'\rangle\langle n'|H_{\rm PN}|i\rangle}{E_i - E_{n'}}\,. \tag{6}$$

The first term is due to PN admixing in the final state, whereas the second one arises from PN in the initial one. The PN signals we shall consider, e.g., photon circular polarization and optical rotation, are proportional to the ratio of the interference term between $M1$ and $E1$ and the sum of the squared matrix elements:

$$\text{Signal} \propto \frac{\langle f|M1|i\rangle^*\langle f|E1|i\rangle}{|\langle f|M1|i\rangle|^2 + |\langle f|E1|i\rangle|^2} \simeq \frac{\langle f|E1|i\rangle}{\langle f|M1|i\rangle}\,. \tag{7}$$

It is the evaluation of these matrix elements which is our primary concern here. For $\langle f|E1|i\rangle$, we use a Green's function technique.[5] We define

$$|I\rangle = \sum_n \frac{|n\rangle\langle n|E1|i\rangle}{E_f - E_n}, \quad \langle F| = \sum_n \frac{\langle f|E1|n\rangle\langle n|}{E_i - E_n}, \tag{8}$$

where $|I\rangle$ satisfies the equation

$$(H - E_f)|I\rangle = -E1|i\rangle\,, \tag{9}$$

with H the atomic parity-conserving Hamiltonian. A similar equation holds for $\langle F|$.

In the calculation presented here we assume an independent particle (Hartree-Fock) model for the electronic wave functions. No correlations are included for the core electrons. Tl has only one valence electron; Bi has three. In the case of Bi, some correlations of the valence electrons are included through the use of intermediate coupling.

Electron wave functions and effective potentials were kindly provided by R. D. Cowan.[9] The radial wave functions are solutions of a Schrödinger-like equation,

$$\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + V_{\rm eff}(r; n, l, u, \epsilon) - \epsilon\right) u_{nl}(r) = 0\,, \tag{10a}$$

where atomic units ($\hbar = m = e = 1$) are used, except that the energy is measured in rydbergs, $\frac{1}{2}me^4/\hbar^2$. The effective potential contains the "principal" exchange and relativistic effects, but no spin-orbit coupling. It is cast in the form

$$V_{\rm eff} = V(r; n, l) - \frac{\alpha^2}{4}(\epsilon - V)^2 - \delta_{l,0}\,\frac{\alpha^2}{4}\left(1 + \frac{\alpha^2}{4}(\epsilon - V)\right)^{-1} \times \frac{dV}{dr}\left(\frac{1}{u_{nl}}\,\frac{du_{nl}}{dr} - \frac{1}{r}\right), \tag{10b}$$

where $V(r; n, l)$ is computed self-consistently by the method described by Cowan.[9]

The solutions to Eqs. (10) have the correct relativistic behavior at short distances for electrons in s states of a Coulomb field:

$$u_{n0}(r) \propto r^\gamma\,, \quad \gamma = (1 - Z^2\alpha^2)^{1/2}\,.$$

However, the p-state functions behave like

$$u_{n1}(r) \propto r^\beta\,, \quad \beta = \tfrac{1}{2} + (\tfrac{5}{4} - Z^2\alpha^2)^{1/2}\,,$$

whereas β should be j dependent; that is, for the states (j, l),

$$\beta_j = [(j + \tfrac{1}{2})^2 - Z^2\alpha^2]^{1/2}\,,$$

independent of l. Thus, the $s_{1/2}$ and $p_{1/2}$ states should have the same radial behavior close to the nucleus. Therefore, we do not rely on the small-r behavior of the solutions to Eqs. (9) and (10).

Because the inhomogeneous term of Eq. (9) is weighted by the radial distance r, we can use the solutions of Eqs. (10) $\langle\vec{r}|i\rangle$ for the inhomogeneous term in Eq. (9). We also use the quasirelativistic $V_{\rm eff}$, Eq. (10b), to find $\langle\vec{r}|I\rangle$. However, we need the wave functions $\langle\vec{r}|I\rangle$ and $\langle\vec{r}|i\rangle$ (as well as $\langle F|\vec{r}\rangle$ and $\langle f|\vec{r}\rangle$) at small distances, in order to evaluate the electric dipole matrix element:

$$\langle f|E1|i\rangle=\langle f|H_{\rm PN}|I\rangle+\langle F|H_{\rm PN}|i\rangle\,. \tag{11}$$

The small-r behaviors of $\langle\vec{r}|I\rangle$, $\langle F|\vec{r}\rangle$, $\langle\vec{r}|i\rangle$, and $\langle f|\vec{r}\rangle$ are obtained by solving the homogeneous Dirac equation in the field of the nucleus, with the potential given by

$$\begin{aligned} V &= V(R;n,l)+Z\left(\frac{r^2}{R^3}-\frac{1}{R}\right), \quad r\le R\,,\\ &= V(r;n,l), \quad r\ge R\,. \end{aligned} \tag{12}$$

The integration is carried out outwards with regular boundary conditions at the origin. For $|I\rangle$ and $\langle F|$ the energy could be chosen to be zero or the correct eigenvalue with little effect. For these states which in our case have s-wave electrons, the normalization constant N is found by matching the density of the Dirac wave function with the quasirelativistic wave function close to the edge of the nucleus, $r\simeq R$, e.g.,

$$G_{Is}^2(R)+F_{Is}^2(R)=\int|\langle\vec{R}|I\rangle|^2\,d\Omega\,. \tag{13}$$

For the atoms that we consider here, the states $|i\rangle$ and $\langle f|$ correspond to p waves. The density of the Dirac solution was scaled to match the quasirelativistic one at radii large compared with the nucleus, but still small compared with the atomic size. In practice, the two densities could be matched over the first four or five maxima with a variation in the ratios

$$\int|\langle f|\vec{r}\rangle|^2\,d\Omega/(G_{fp}^2+F_{fp}^2)=X_{fp}$$

and

$$\int|\langle\vec{r}|i\rangle|^2\,d\Omega/(G_{ip}^2+F_{ip}^2)=X_{ip}$$

of less than 5%.

In terms of Eqs. (3), (8), and (11), we can write the ratio $\langle f|E1|i\rangle/\langle f|M1|i\rangle$, which is required to compute PN effects in the atoms we treat, as

$$\frac{\langle f|E1|i\rangle}{\langle f|M1|i\rangle}=\frac{G}{\sqrt{2}}\,\frac{Q}{2}\,\frac{\alpha}{4\pi}\,\frac{1}{\sqrt{3}}\,\frac{\{E1\}}{\{M1\}}\,. \tag{14}$$

Since only the ratio of $\langle f|E1|i\rangle$ and $\langle f|M1|i\rangle$ is required, we have chosen to evaluate $E1$ and $M1$ for the operator $E1(z)$ and $M1(z)$. The quantities $\{E1\}$ and $\{M1\}$ for a single-electron orbital then are given by

$$\{E1\}=-i\int r^{-2}(G_{Is}F_{fp}-F_{Is}G_{fp})\rho_A 4\pi r^2\,dr$$

$$-i\int r^{-2}(G_{Fs}F_{ip}-F_{Fs}G_{ip})\rho_A 4\pi r^2\,dr\,, \tag{15a}$$

$$\{M1\}=-\langle f|L_z+\sigma_z|i\rangle\,. \tag{15b}$$

We assume a uniform nuclear density, $\rho_A(\vec{r})=(\frac{4}{3}\pi R^3)^{-1}$, where R is the nuclear radius, in order to carry out the integration in Eq. (15a).

III. APPLICATIONS

A. $^{205}_{81}$Tl (λ = 12 832 Å)

The low-lying level structure of Tl is shown in Fig. 1.[10] The transition of interest is that to the first excited state at 0.071 Ry, which is primarily $M1$ with a small admixture of $E1$. The latter matrix comes about through the PN mixture of $|ns_{1/2}\rangle$ states in the $\frac{1}{2}^-$ ground state, as shown in Fig. 1. The first excited state is essentially parity pure. We neglect electron-electron correlations, core polarization, and hyperfine structure effects.

The matrix $M1(z)$ required in Eq. (15b) is readily evaluated to be, for the state with $j_z=\frac{1}{2}$,

$$\{M1\}=-\langle 6p_{3/2}^{1/2}|L_z+\sigma_z|6p_{1/2}^{1/2}\rangle=\tfrac{1}{3}\sqrt{2}\,. \tag{16}$$

This value agrees with the transition rate given by Garstang.[11]

In the solution of Eq. (9), or rather $\langle F|(H-E_i)=-\langle f|E1$, which is required to find $\{E1\}$, we have not used the single-particle energy computed by Cowan (−0.361) but rather the separation energy of −0.449. Cowan's value is an average of the $6p_{1/2}$ and $6p_{3/2}$ separation energies, whereas we need the energy of the $6p_{1/2}$ level. It is the separation of the latter state from $ns_{1/2}$ states which is required in Eq. (8). However, we have checked the sensitivity and found that it is small. The shift from −0.449 to −0.361 (20%) produces a change in $\{E1\}$ of ~3%. We find for $\{E1\}$ evaluated with $R=(1.35\times10^{-13})A^{1/3}$ cm $=1.5\times10^{-4}a_0$, $\{E1\}=i4090$. The value of $\{E1\}$ is a slowly varying function of R. By means of Eq. (14) we readily obtain the PN ratio $\langle f|E1|i\rangle/\langle f|M1|i\rangle$, with $Q=Q_W$ for $\sin^2\theta_W=0.33$, $R/a_0=1.5\times10^{-4}$:

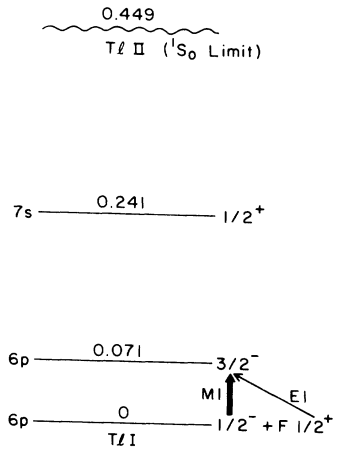

FIG. 1. Low-lying level structure of Tl I and series limit. The PN transition is shown explicitly.

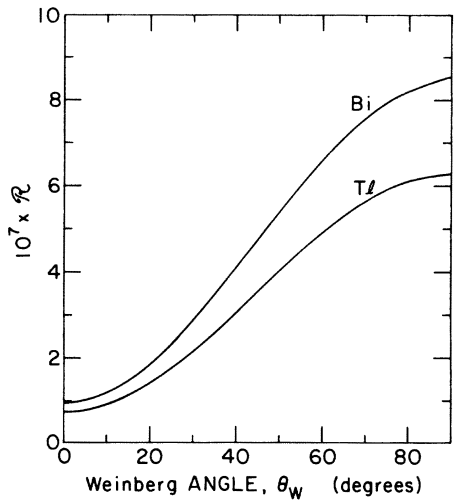

FIG. 2. Dependence of PN ratio $\mathcal{R}$ on Weinberg angle θ_W for Tl and Bi.

$$\mathcal{R} \equiv -\frac{\mathrm{Im}\langle f|E1|i\rangle}{\langle f|M1|i\rangle} = 2.57\times 10^{-7}. \tag{17}$$

In Fig. 2 we plot the ratio $\mathcal{R}$ as a function of θ_W, the Weinberg angle, for $R/a_0 = 1.5\times 10^{-4}$. The value obtained in Eq. (17) agrees in order of magnitude with estimates made by methods developed by Bouchiat and Bouchiat.[5]

B. $^{209}_{83}$Bi ($\lambda = 8757$ Å)

The case of Bi is more complex than that of Tl because there are (at least) three active (valence) electrons to consider, and the effects of the residual interactions among these electrons are important. The low-lying level structure of Bi I and some other levels of concern to our computation are shown in Fig. 3. We are particularly interested in the transition between ground and first excited states, for which a PN signal is being sought at the University of Washington.[8] We shall use this transition to illustrate our method. Since the PN interaction is a single-electron one, we reduce the problem and associated Green's function to this case. However, because the splitting of the $6p^2 7s$ states (see Fig. 3) is large compared to their excitation energies, we treat these states more accurately.

In order to carry out the reduction to a one-electron problem, we require the configuration of the ground and first excited states. This decomposition has been considered by Condon and Shortley,[12] by Landman and Lurio,[13] and by others. We use the decomposition of Landman and Lurio in L-S coupling, which differs slightly from that of Condon and Shortley,

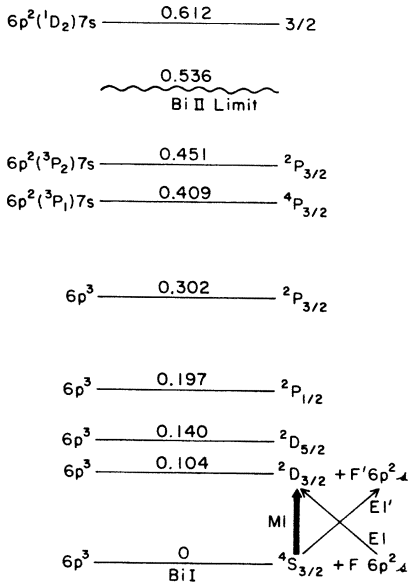

FIG. 3. Partial level structure of Bi I and Bi II. The PN transition is shown explicitly.

$$|\alpha, \tfrac{3}{2}, m\rangle = \sum_{L,S} a_{LS}|LSm\rangle$$
$$= \sum_{i,j,k} b^m_{ijk}\,\phi_i(1)\phi_j(2)\phi_k(3), \tag{18}$$

where the sum $\sum_{LS}$ is over the three states $^4S_{3/2}$, $^2P_{3/2}$, $^2D_{3/2}$, and $\sum_{ijk}$ is over single-particle states. In the sums $\sum_n$ and $\sum_{n'}$ required for Eq. (6b), only the electron affected by the single-particle operators H_{PN} and $E1$ is excited. Thus, the overlap integral for the other two electrons can readily be carried out. The problem which arises is that the energies in the denominator are not single-particle energies. For all but the $6p^2 7s$ states we neglect the energy splittings of the various $6p^2 ns$ states, for a fixed n. Consistent with this neglect, we assume that the corrections to the single-particle energies $2\epsilon'(6p) + \epsilon(ns)$ are the same for all n and can be obtained from those for the $6p^2 7s$ states. For these states we use a weighted [by $(2j+1)$] average, corresponding to an excitation of 0.437. That is, we write

$$\overline{E}(6p^2 7s) = 2\epsilon'(6p) + \epsilon(7s) + \Delta' = 0.437,$$
$$E_i(6p^3) = 3\epsilon_p + \Delta_i = 0,$$
$$E_i - E_{7s} = 3\epsilon_p - 2\epsilon'_p + \Delta_i - \Delta' - \epsilon_\beta = 0.437.$$

From the calculated value $\epsilon_p - \epsilon_{7s} = -0.525 + 0.227 = -0.298$, we find

$$\delta_i \equiv 2(\epsilon_p - \epsilon'_p) + \Delta_i - \Delta' = -0.437 + \epsilon_{7s} - \epsilon_p$$
$$= -0.437 + 0.298 = -0.139,$$

$$E_i - E_{ns} = \epsilon_p - \epsilon_{ns} + \delta_i = -0.525 - 0.139 - \epsilon_{ns}$$

$$= \epsilon_i(\text{eff}) - \epsilon_{ns} = -0.664 - \epsilon_{ns} .$$

This is close to the weighted [by $(2j+1)$] average separation energy of the $6p$ electron, -0.734 eV. Although we use the latter value, we shall consider the dependence of the results on ϵ_i(eff). We take into account the actual splitting between the $^2D_{3/2}$ and $^4S_{3/2}$ levels of 0.104; that is, we take $\epsilon_f(\text{eff}) = \epsilon_i(\text{eff}) + 0.104$.

We are now in a position to evaluate $\langle f|E1|i\rangle$. In the case of the transition in Bi which we are studying, parity mixing occurs in both initial and final states (see Fig. 3), since $6p^2ns$ states can admix with both $^2D_{3/2}$ and $^4S_{3/2}$ states. When integrated over the nuclear density, we find

$$\{E1\} = -4\pi i \left[0.0559 \int (G_{Fs} F_{ip} - F_{Fs} G_{ip}) \rho_A \, dr + 1.537 \int (G_{Is} F_{fp} - F_{Is} G_{fp}) \rho_A \, dr \right] , \quad (19)$$

The multiplicative constants arise from the decomposition of the three electron states into single-particle ones. PN mixing in the ground and excited states thus reinforce each other, but the $E1$ matrix due to mixing in the excited state is negligible ($\lesssim 5\%$).

Because the splitting of $6p^27s$ states due to non-central and spin-orbit effects is large compared to their separation energies from the ground state (see Fig. 3), we apply a correction for these terms. We treat the $6p^27s$ terms by considering their experimental energies and evaluating the dipole matrix elements directly. We write $\langle f|E1|i\rangle$ as

$$\langle f|E1|i\rangle = \langle 6p|V_{\text{PN}}|Is\rangle + \langle Fs'|V_{\text{PN}}|6p\rangle - \frac{\langle 6p|V_{\text{PN}}|7s\rangle\langle 7s|E1|6p\rangle}{E_f - E_{7s}} - \frac{\langle 6p|E1|7s\rangle\langle 7s|V_{\text{PN}}|6p\rangle}{E_i - E_{7s}}$$
$$+ \sum_n \frac{\langle f|V_{\text{PN}}|(6p)^2 7s; n, \tfrac{3}{2}\rangle\langle (6p)^2 7s; n, \tfrac{3}{2}|E1|i\rangle}{E_f - E_{n,3/2}} + \sum_n \frac{\langle f|E1|(6p)^2 7s; n, \tfrac{3}{2}\rangle\langle (6p)^2 7s; n, \tfrac{3}{2}|V_{\text{PN}}|i\rangle}{E_i - E_{n,3/2}} , \quad (20)$$

where the sum n is over the three $j = \frac{3}{2}$ states located at excitation energies $E_n = 0.409$, 0.451, and 0.612. For the configuration of the $|(6p)^2 7s; n, \frac{3}{2}\rangle$ states, we solve the secular equations on the basis of matrix elements given by Condon and Shortley,[12] to obtain

$$\begin{aligned} |\text{high}\rangle &= |(6p)^2 7s;\ ^2D_{3/2}\rangle \\ &= -0.169|\alpha\rangle + 0.910|\beta\rangle - 0.379|\gamma\rangle , \\ |\text{med}\rangle &= |(6p)^2 7s;\ ^2P_{3/2}\rangle \\ &= 0.162|\alpha\rangle + 0.405|\beta\rangle + 0.900|\gamma\rangle , \\ |\text{low}\rangle &= |(6p)^2 7s;\ ^4P_{3/2}\rangle \\ &= 0.972|\alpha\rangle + 0.091|\beta\rangle + 0.215|\gamma\rangle . \end{aligned} \quad (21a)$$

These values do not (quite) agree with those found by Breit and Wills,[14] with the phase convention of Condon and Shortley,

$$\begin{aligned} |\text{high}\rangle &= 0.347|\alpha\rangle + 0.825|\beta\rangle - 0.446|\gamma\rangle , \\ |\text{med}\rangle &= 0.283|\alpha\rangle + 0.362|\beta\rangle + 0.889|\gamma\rangle , \\ |\text{low}\rangle &= 0.894|\alpha\rangle - 0.436|\beta\rangle - 0.111|\gamma\rangle . \end{aligned} \quad (21b)$$

Here α, β, γ are pure $^4P_{3/2}$, $^2P_{3/2}$, and $^2D_{3/2}$ states, respectively.

The $7s$ state correction was evaluated for both solutions. It is very small; it increases the $\{E1\}$ matrix by ~1% for the Breit-Wills case and by ~2% for our solution. One reason that the correction is so small is that the contribution of the $6p^27s$ excited states to the dipole sum in Eq. (6b) is only ~ 20%.[15] Furthermore, the splitting is taken into account in a reasonable manner by the Green's function. Indeed, the smallness of the correction gives us confidence in the result obtained with the single-particle Green's function.

For $R/a_0 = 1.51 \times 10^{-4}$ or $R = 8$ fm $= 1.35A^{1/3}$ and the Weinberg model with $\sin^2\theta_W = 0.33$ [with a $\frac{3}{2}\%$ correction for the $(6p)^2 7s$ states]

$$\{E1\} = i\,7630 .$$

The magnetic dipole matrix element, Eq. (15b), can be evaluated in a straightforward manner; for the wave function of Landman and Lurio,[13] we obtain

$$\{M1\} = 0.669 .$$

Substitution into Eq. (17) gives

$$\mathcal{R} = 3.46 \times 10^{-7} .$$

The dependence of the ratio on R and θ_W are shown in Figs. 4 and 2, respectively. The dependence on the choice ϵ_i(eff) is shown in Fig. 5. We used $\epsilon_i(\text{eff}) = -0.734$ eV.

Our result for $R/a_0 = 1.51 \times 10^{-4}$ and $\sin^2\theta_W = 0.33$ ($Q_W = -153$) can be compared with the results obtained by Brimicombe, Loving, and Sandars.[7]

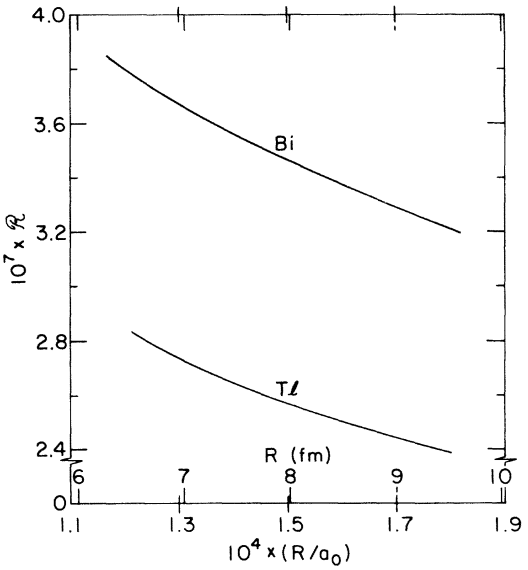

FIG. 4. Dependence of PN ratio $\mathcal{R}$ on nuclear radius R for Tl and Bi.

They find $R = 3.0 \times 10^{-7}$ by a method which differs from ours, as indicated earlier. They solve the relativistic versions of Eqs. (9) and (10) in an analytic potential with parameters that are adjusted to give a best fit to the energies of the $6p$ and $7s$ shells. The agreement is good and indicates that the approximation schemes used to compute the parity nonconserving $E1$ matrix are reasonable. Since neither of us includes core excitation or core polarization effects, the comparison does not touch on these aspects. However, we expect these effects to be small. It thus appears to us that the theoretical estimate is accurate to better than about 20%, and that good tests of the neutral current theories are feasible in atoms. Our calculations can readily be extended to other atoms.

IV. EXPERIMENTS

The experiment being carried out at the University of Washington consists of a search for the optical rotation of laser light tuned to frequencies close to the resonant one, 8755 Å, in Bi. The angle of rotation for a given length l of vapor traversed by the beam is given by

$$\phi = \tfrac{1}{2} k l \, \mathrm{Re}(n_L - n_R)$$

$$= 2Nl \, \mathrm{Im}(\langle f|E1|i\rangle \langle f|M1|i\rangle^*) \frac{\omega - \omega_0}{(\omega - \omega_0)^2 + (\Gamma/2)^2} ,$$

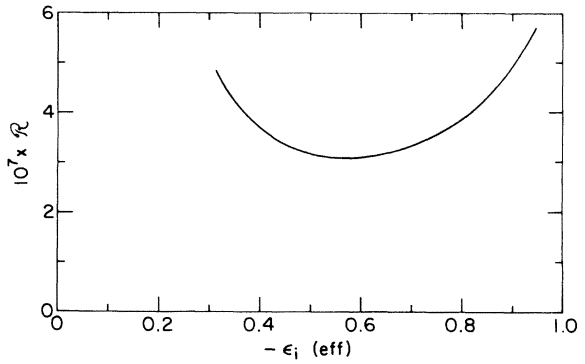

FIG. 5. Dependence of PN ratio $\mathcal{R}$ in Bi on the effective single-particle energy ϵ_i(eff).

where $\omega = k = \lambda^{-1}$, N is the number density of atoms, ω_0 is the resonance circular frequency, and Γ is the resonance width. The absorption coefficient is

$$\alpha = 2N|\langle f|M1|i\rangle|^2 \frac{\Gamma/2}{(\omega - \omega_0)^2 + (\Gamma/2)^2} ,$$

so that the angle of rotation in one mean free path is (with $\langle f|M1|i\rangle$ real)

$$\varphi = \frac{\phi}{\alpha l} = \frac{\mathrm{Im}\langle f|E1|i\rangle}{\langle f|M1|i\rangle} \frac{\omega - \omega_0}{\Gamma/2} .$$

For the Weinberg theory we predict that ϕ is positive for $\omega < \omega_0$, for both Bi and Tl if $G > 0$. The precision that the group at the University of Washington[8] expects to be able to achieve is better than 10^{-7} for φ so that a signal should be observed if the Weinberg theory is correct.

One can also measure the circular polarization of light emitted in the transitions which we have investigated. The circular polarization of the electromagnetic radiation is given by

$$P_\gamma = 2 \frac{\mathrm{Im}\langle f|E1|i\rangle}{\langle f|M1|i\rangle} .$$

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