Electron scattering from atoms in the presence of a laser field. II*

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In the first paper of this series a formal theory of atomic scattering of electrons in the presence of an intense electromagnetic field was given. Cross sections, near the forward direction, between atomic states (modified by the laser) were obtained. However, it was assumed that the atom could not emit spontaneous radiation. In this paper the effect of spontaneous radiation is included, and it is shown that in most cases the measured cross section will be a weighted average of the two different cross sections starting from the two different initial states which are those atomic states resonantly linked by the laser. The Born approximation for the scattering is obtained, and it is shown that for the simplest description of the spontaneous radiation field the effect of the laser on the cross sections is simply to multiply them by a factor which depends upon the laser detuning.

I. INTRODUCTION

The phenomenon of electron scattering by atoms in the presence of an intense electromagnetic field is complicated in detail by the occurrence of a number of parameters. These may be summarized by a set of time parameters which are as follows:

(1) The collision time

$$t_c = x/V = 2\times 10^{-17}(x/a_0)(\mathcal{R}/E_k)^{1/2}\ \text{sec}\,, \tag{1.1}$$

where x is some relevant distance in the collision and E_k is the kinetic energy of the electron (a_0 being the Bohr radius and $\mathcal{R}$ the Rydberg constant).

(2) The period of the laser or of the atom. These are essentially equal, since the laser is assumed to resonantly excite the atom,

$$\tau = 2\pi/\omega = 3\times 10^{-16}(\mathcal{R}/\Delta E)\ \text{sec}\,. \tag{1.2}$$

(3) The time for a laser-induced transition in the atom,

$$t_L = \frac{\hbar}{|e\vec{\mathrm{E}}a_0|} = 5\times 10^{-17}\left(\frac{2\times 10^{16}\ \mathrm{W/cm^2}}{I}\right)^{1/2}\ \text{sec}\,, \tag{1.3}$$

where I is the laser intensity and $\vec{\mathrm{E}}$ is the associated electric field.

(4) The natural decay time

$$t_d \cong 10^{-8}\text{–}10^{-9}\ \text{sec}\,. \tag{1.4}$$

(5) The time interval T between the instant that the laser first illuminates the atom and the time of the collision.

It is assumed that the laser is turned on adiabatically on the time scale of the atom τ, since it would be difficult to do otherwise. In most cases T is by far the longest of these times, so that the atom emits many spontaneously radiated photons (SRP's) under the influence of the laser before the collision takes place. In a previous paper,[1] the opposite condition, where no SRP's are emitted, was treated. This may apply to situations such as plasmas illuminated by a laser, but not to the usual collision experiment.[2] However, the formalism introduced there will be of use here.

For electrons with an energy above a rydberg or so, the collision time t_c is likely to be the shortest by far of the five times. In that case there are no SRP's emitted during the collision, so that this interaction may be neglected in describing the collision proper. It is also tempting to argue that since the laser period is long compared to the collision time, the effect of the laser can also be neglected in describing the collision. This would not be correct, since the laser period is comparable to the period of the atom. Hence the straightforward use of time-independent scattering theory would be wrong, since a time of the order of a few atomic periods must elapse for the atom to settle into a stationary state. During that time the laser varies in intensity and can transfer energy to or from the atom. Then, instead of using the bare atomic states to describe the target (appropriate to time-independent scattering theory), it is more appropriate to use the states of the atom interacting with the laser and to calculate the transitions among those states. This was the procedure followed in Ref. 1. We extend that work by including the SRP.

In Sec. II we obtain the cross section for electron scattering for a definite final atomic (plus laser) state. If no observation is made of the SRP's, the result is given in terms of cross sections calculated in the absence of the SRP's. In Sec. III the details of electron-hydrogen scattering in the Born approximation are given. The exchange part of the cross sections requires some discussion.

II. KINEMATICS OF THE SPONTANEOUS RADIATION FIELD

The first task is to calculate the state of the atom-laser system at the time of collision. The laser is idealized as a single highly occupied mode of the electromagnetic field which is described separately from all of the other modes. The atom is idealized as a two-state system; thus the atom plus laser states are denoted by $|0,N\rangle$ or $|1,N'\rangle$, where 0 is the ground state and N ($\gg 1$) is the occupation number of the laser mode. The state of the remaining modes of the electromagnetic field are described by the set of numbers $[q]_n$, which is an infinite set of non-negative integers, one for each mode, with the constraint

$$\sum_i q_i = n , \tag{2.1}$$

which says that there are exactly n SRP's in the state described by $[q]_n$.

Obtaining the states of the atom plus laser is a formidable task, even in the two-state approximation,[3] which we shall not attempt here. Instead, we will be satisfied with a rotating-wave approximation,[4] which is a good one when the laser is sufficiently near resonance and not too intense. In that case, the states are given by

$$\phi_\pm(N) = \frac{1}{\sqrt{2}}\left[\left(1 \pm \frac{x}{2\mathcal{E}}\right)^{1/2}|0,N\rangle \pm \frac{\Lambda^*}{\mathcal{E}}\left(1 \pm \frac{x}{2\mathcal{E}}\right)^{-1/2}|1,N-1\rangle\right] \tag{2.2}$$

and the energies are given by

$$\begin{aligned} E_\pm(N) &= \tfrac{1}{2}(E_0 + E_1 - \omega) + N\omega \pm \mathcal{E} , \\ \mathcal{E} &= (|\Lambda|^2 + (\tfrac{1}{2}x)^2)^{1/2} , \\ x &= \omega - (E_1 - E_0) = \omega - \Delta E , \end{aligned} \tag{2.3}$$

where the $E_{0,1}$ are the energies of the bare atomic states and Λ is the coupling matrix element in the dipole approximation linking the states:

$$\Lambda = \frac{e}{m}\vec{P}_{01}\cdot\hat{\epsilon}\left(\frac{2\pi c^2}{\omega V}(N-1)\right)^{1/2} = \frac{e}{2m}\frac{\vec{P}_{01}\cdot\vec{E}}{\omega} . \tag{2.4}$$

Here $\vec{P}_{01}$ is the momentum matrix element, $\hat{\epsilon}$ the polarization vector of the laser, and V the quantization volume. The last step defines the amplitude E of the equivalent classical electric field. This classical approximation is well known to be a good one when $N \gg 1$ and the laser mode is never significantly depleted. This is the situation described here.

The wave function for the system, including the SRP, is taken to be

$$\Psi = \sum_{n,[q]_n}\{b_+([q]_n,t)\phi_+(N-n)\,e^{-iE_+(N-n)t} + b_-([q]_n,t)\phi_-(N-n)e^{-iE_-(N-n)t}\} \times|[q]_n\rangle e^{-iW([q]_n)t} , \tag{2.5}$$

where

$$W([q]_n) = \sum_i q_i\,\omega_i \tag{2.6}$$

is the energy of the SRP. It should be noted that Eq. (2.5) is not the most general form of the wave function of the SRP. Instead, a "super-rotating-wave approximation" has been made for the wave function in that only terms which approximately conserve energy are kept. This is so since each of the SRP's will have very nearly the laser photon energy. (It is known[5] that the fluorescent spectrum will be centered about the atomic transition frequency and will be very narrow on the scale of the laser frequency.) Further, the Rabi frequency is assumed to be very small compared to ω. The initial condition, where $\Lambda = 0$, is $\Psi = |0,N\rangle$, which implies that

$$b_+([q]_n) = \delta([q]_n,[0]), \quad b_-([q]_n) = 0, \quad \text{for } x > 0 , \tag{2.7a}$$

or

$$b_+([q]_n) = 0, \quad b_-([q]_n) = \delta([q]_n,0), \quad \text{for } x < 0 . \tag{2.7b}$$

If Eq. (2.5) is now substituted into the Schrödinger equation an equation is obtained for the set $b_\pm([q]_n,t)$ which can be solved subject to one of the above initial conditions; then the wave function will be known (at least in principle) after an elapsed time T. This serves as the initial condition for the scattering event. For each different state of the SRP we have a different initial wave function, which we write

$$\chi^n = \hat{b}_+([q]_n,T)\,\phi_+(N-n) + \hat{b}_-([q]_n,T)e^{2i\mathcal{E}T}\phi_-(N-n) , \tag{2.8}$$

which has been normalized so that

$$\hat{b}_\pm([q]_n,T) = b_\pm([q]_n,T)(|b_+([q]_n,T)|^2 + |b_-([q]_n,T)|^2)^{-1/2} . \tag{2.9}$$

Now suppose that we can calculate the scattering amplitudes for the problem of electron plus atom plus laser with these initial atomic states. The amplitudes will be calculated into some (as yet unspecified) set of final states, ψ_ξ. The cross sections for each different initial state, χ^n, add incoherently, since it is, in principle, possible to observe

the state of the SRP. We call these differential cross sections

$$\frac{d\bar{\sigma}_\nu}{d\Omega}(\xi, n; \vec{p}_f, \vec{p}_i), \tag{2.10}$$

where $\vec{p}_i$ and $\vec{p}_f$ are the initial and final momenta of the electron and ν is an integer which describes[1] the inelasticity of the electron,[6]

$$p_i^2/2m = p_f^2/m - \nu\omega. \tag{2.11}$$

The index n describes the state of the SRP during the collision and the index ξ denotes the final state of the atom plus laser. The state of the SRP will usually not be measured; therefore we sum over all possible states weighted with the probability that each will occur:

$$\frac{d\bar{\sigma}_\nu}{d\Omega}(\xi; \vec{p}_f, \vec{p}_i) = \sum_{n[q]_n} \frac{d\bar{\sigma}_\nu}{d\Omega}(\xi, n; \vec{p}_f, \vec{p}_i) \times [|b_+([q]_n, T)|^2 + |b_-([q]_n, T)|^2]. \tag{2.12}$$

This depends upon the interval T which is, in general, not observed; therefore we must average over it. For example, if the electrons arrive in a pulse of duration $\Delta T \sim 10^{-6}$ sec, then this is the interval over which T must be averaged. Returning to Eq. (2.8) we note that T enters through $b_\pm$ and through the phase factor $e^{2i\mathcal{E}T}$. We anticipate that the dependence of $b_\pm$ upon T will be slow and for the numbers considered here $e^{2i\mathcal{E}T}$ will average to zero.

It is clear that the n dependence of $\phi_\pm(N-n)$ is negligible because of the approximation that the laser state is undepleted, i.e., that the number of SRP's is much less that the number of laser photons. The initial states then simplify to

$$\chi^n = \hat{b}_+([q]_n, T)\,\phi_+(N) + \hat{b}_-([q]_n, T)e^{2i\mathcal{E}T}\phi_-(N), \tag{2.13}$$

in which the state of the SRP now appears only in the $b_\pm$. The scattering amplitudes to any final state will be a linear functional of these initial states and the cross sections on the right-hand side of Eq. (2.12) will be a bilinear functional of them. The average over T will then eliminate the cross terms between the b_+ and b_-, due to the fact that $e^{2i\mathcal{E}T}$ averages to zero. Hence Eq. (2.12) will become

$$\frac{d\bar{\sigma}_\nu}{d\Omega}(\xi; \vec{p}_f, \vec{p}_i) = P_+ \frac{d\sigma_\nu}{d\Omega}(\xi, +; \vec{p}_f, \vec{p}_i) + P_- \frac{d\sigma_\nu}{d\Omega}(\xi, -; \vec{p}_f, \vec{p}_i), \tag{2.14}$$

where $P_\pm$ is the probability that the atom plus laser will be in the (+) or (−) state sector of the total wave function (2.5) with no observation of the SRP:

$$P_\pm = \sum_{n[q]_n} |b_\pm([q]_n, T)|^2_{\text{ave}}. \tag{2.15}$$

The average is the average over T described above. The cross sections appearing on the right-hand side in Eq. (2.14) are those calculated with initial states given by $\phi_\pm$, Eq. (2.2), with no mention of the SRP. These are the ones obtained approximately in Ref. 1. Thus our result is that the effect of the SRP is to give a cross section which is a weighted average of those from initial states ϕ_+ or ϕ_-. The weighting is simply the probability that the electron will encounter the system in ϕ_+ or ϕ_- at the instant of collision and there is no interference.

An additional word about this averaging is in order here, since it provides the central result of this paper, Eq. (2.14). The essential assumption made has been that the experiment does not observe the phase of the Rabi cycle at which the collision occurs. That is, the frequency at which the atom is pumped between the states u_0 and u_1, which is $2\mathcal{E}$, is high, so that the observation of the scattered electrons cannot determine the instant in the cycle at which the electron is scattered. In that case the interference in the $\phi_\pm$ basis vanishes. If this condition is not met the interference is present and the $\phi_\pm$ are no longer a useful basis in which to work.

The appropriate set of atom plus laser states for describing the situation after collision are again the eigenstates of the combined system. In the same approximation as the one describing the initial states, these are the set $\phi_\pm$ and the bare states

$$\phi_j(N) = |j, N\rangle, \quad j = 2, 3, \ldots. \tag{2.16}$$

That is, it is a good approximation to neglect the effect of the laser on the atomic states which are not almost resonantly coupled together. The cross section to final states of Eq. (2.16) are individually observable. For example, if $|0\rangle$ is an S state and $|1\rangle$ is a $P, m=0$ state coupled by a linearly polarized laser, then the scattering can create a $P, m=1$ final state which can be observed by its circularly polarized decay photon. On the other hand, final states ϕ_+ or ϕ_- are not individually observable, since they are coupled together by the laser after the collision event. The only observable then is the elastic cross section σ_{el}, which is the sum of the two:

$$\frac{d\bar{\sigma}_v}{d\Omega}(\sigma_{el};\vec{p}_f,\vec{p}_i)$$
$$=\frac{d\bar{\sigma}_v}{d\Omega}(+;\vec{p}_f,\vec{p}_i)+\frac{d\bar{\sigma}_v}{d\Omega}(-;\vec{p}_f,\vec{p}_i)$$
$$=P_+\left(\frac{d\sigma_v}{d\Omega}(+,+;\vec{p}_f,\vec{p}_i)+\frac{d\sigma_v}{d\Omega}(-,+;\vec{p}_f,\vec{p}_i)\right)$$
$$+P_-\left(\frac{d\sigma_v}{d\Omega}(+,-;\vec{p}_f,\vec{p}_i)+\frac{d\sigma_v}{d\Omega}(-,-;\vec{p}_f,\vec{p}_i)\right). \quad (2.17)$$

The calculation of the weighting factors $P_\pm$, Eq. (2.15), requires a knowledge of the dynamics of the SRP. That is, the coefficients $|b_\pm([q]_n)|^2$ must be explicitly known. Their equations of motion can be obtained from the wave function (2.5) and the Schrödinger equation. We shall not write them in detail here but merely note that the time derivative of $b_+([q]_n)$ is driven by four terms. The first two contain $b_+([q]_n+1_k)$ and $b_+([q]_n-1_k)$, respectively. These describe transitions within the ϕ_+ manifold due to absorption or emission of the photon k. They are not important for our purposes, since they describe only a spreading of the distribution of SRP's within the manifold and do not contribute to changes in $P_\pm$. The last two terms in the equation for $b_+([q]_n)$ contain terms $b_-([q]_n+1_k)$ and $b_-([q]_n-1_k)$. These represent transitions from the ϕ_- manifold to the ϕ_+ manifold due to photon absorption or emission, respectively. They drive changes in $P_\pm$. A similar equation can be written for $b_-([q]_n)$.

A result that emerges from these equations without explicit solution is that near resonance, when

$$x/2|\Lambda| = (\omega-\Delta E)/|\Lambda| \ll 1, \quad (2.18)$$

the equations are symmetric and after many SRP's are emitted the probabilities $P_\pm$ will be equal. Coupled with the normalization, this yields

$$P_+ = P_- = \tfrac{1}{2}. \quad (2.19)$$

We shall not attempt to pursue these equations here but shall rely on a crude evaluation based on the equation

$$P_+/P_- = w_-/w_+, \quad (2.20)$$

where w_- (w_+) is the total transition rate out of the ϕ_- (ϕ_+) manifold into the ϕ_+ (ϕ_-) manifold. w_- is made up of two terms, the emission rate of a SRP from a state ϕ_- to a state ϕ_+ plus the absorption rate of a SRP with a change of state from ϕ_- to ϕ_+. As a crude approximation we shall neglect the absorption rate. This is the approximation that once a SRP photon is emitted it escapes. That is, virtual transitions are neglected. In that case the ratio in Eq. (2.20) can be calculated by a simple application of the "golden rule" as an expansion in $|\Lambda|/\mathcal{E}$,

$$P_+/P_- = w_-/w_+ = e^{-4\mu}, \quad (2.21)$$

where

$$\tfrac{1}{2}x = |\Lambda| \sinh\mu \quad (2.22)$$

or

$$P_\pm = \tfrac{1}{2}e^{\mp 2\mu}/\cosh 2\mu. \quad (2.23)$$

The inclusion of the absorption processes which were neglected in obtaining Eq. (2.23) complicates the results by inclusion of details of the fluorescent spectrum. These may actually be directly obtainable from experiments on the spectra, but we shall not pursue this here.

III. BORN APPROXIMATION FOR A HYDROGEN TARGET

The development above treated the laser as a single highly occupied mode of the electromagnetic field. It is well known[7] that the limits $N \gg n$, $N \gg 1$ allow an equivalent development in which the laser is treated as a classical electromagnetic field whose amplitude, in terms of N, can be obtained from Eq. (2.4). It is simpler to describe the electron scattering with this latter method; thus we now make that transition. It is accomplished by going to the interaction representation of the laser and then "replacing" the potentials by their classical value. In that case it can be shown that the rotating-wave-approximation wave functions (2.2) correspond to the time-dependent wave functions

$$\phi_+ \to \psi_0 = (2\cosh\mu)^{-1/2}[e^{\mu/2}u_0 e^{-i(E_0-x/2)t} + (\Lambda^*/|\Lambda|)e^{-\mu/2}u_1 \times e^{-i(E_1+x/2)t}]e^{-i\delta t}, \quad (3.1a)$$

$$\phi_- \to \psi_1 = (2\cosh\mu)^{-1/2}[e^{-\mu/2}u_0 e^{-i(E_0-x/2)t} - (\Lambda^*/|\Lambda|)e^{\mu/2}u_1 \times e^{-i(E_1+x/2)t}]e^{i\delta t}, \quad (3.1b)$$

where the detuning parameter μ is defined by Eq. (2.22).

The S matrix has a direct and an exchange part. In the Born approximation[8] the direct part is given by

$$S^D(f,\vec{p}_f;i,\vec{p}_i) = -i\int dt\, d^3r_1\, d^3r_2\, \chi_f^*(\vec{r}_1,t)\,\phi_f^*(\vec{r}_2,t) \times V(\vec{r}_1,\vec{r}_2)\,\chi_i(\vec{r}_1,t)\,\phi_i(\vec{r}_2,t), \quad (3.2)$$

where the functions χ_λ describe the electron interacting only with the laser,

$$\chi_\lambda(r,t) = \exp i\{\vec{p}_\lambda\cdot[\vec{r}-\vec{\alpha}(t)] - i(p_\lambda^2/2m)t\}, \quad (3.3)$$

with

$$\vec{\alpha}(t) = \frac{e}{m}\int_0^t dt'\,\vec{A}(t') = \frac{e}{m}\frac{\vec{E}}{\omega^2}\sin\omega t\,. \tag{3.4}$$

The atomic states are given by Eqs. (3.1) and

$$\phi_n(\vec{r},t) = u_n(\vec{r})e^{-iE_n t},\quad n = 2, 3\pm\,. \tag{3.5}$$

If the small energy shifts due to the laser are neglected compared to the energy of the bare atom, then S^D may be rewritten

$$S^D(f,\vec{p}_f;\,i,\vec{p}_i) = -2\pi i\sum_\nu \delta(p_f^2 - p_i^2 - \nu\omega)\,T_\nu^D(f;\,i;\,\vec{q})\,, \tag{3.6}$$

where $\vec{q} = \vec{p}_f - \vec{p}_i$ is the momentum transfer of the electron and ν is the integer describing the inelasticity of the electron in units of laser photon energies. The values of the direct T matrix for initial states ϕ_0 or ϕ_1 and final states ϕ_0 and ϕ_1, where $2 = 2S$, $3\pm = 2P$, $m = \pm 1$, are listed here:

$$T_\nu^D(0,0) = (-1)^\nu\beta^2[J_\nu(e^{\mu}\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,0\rangle + e^{-\mu}\langle\vec{p}_f\,1|\,V|\vec{p}_i\,1\rangle) - J_{\nu-1}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,1\rangle - J_{\nu+1}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|\,V|\vec{p}_i\,0\rangle]\,,$$

$$T_\nu^D(1,0) = (-1)^\nu\beta^2[J_\nu(\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,0\rangle - \langle\vec{p}_f\,1|\,V|\vec{p}_i\,1\rangle) - J_{\nu-1}e^{-\mu}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,1\rangle + J_{\nu+1}e^{\mu}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|\,V|\vec{p}_i\,0\rangle]\,,$$

$$T_\nu^D(n,0) = (-1)^\nu\beta[J_\nu e^{-\mu/2}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,n|\,V|\,\vec{p}_i\,1\rangle - J_{\nu+1}e^{\mu/2}\langle\vec{p}_f\,n|\,V|\,\vec{p}_i\,0\rangle]\,, \tag{3.7}$$

$$T_\nu^D(0,1) = (-1)^\nu\beta^2[J_\nu(\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,0\rangle - \langle\vec{p}_f\,1|\,V|\,\vec{p}_i\,1\rangle) + J_{\nu-1}e^{\mu}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|\,V|\vec{p}_i\,1\rangle - J_{\nu+1}e^{-\mu}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|\,V|\,\vec{p}_i\,0\rangle]\,,$$

$$T_\nu^D(1,1) = (-1)^\nu\beta^2[J_\nu(e^{-\mu}\langle\vec{p}_f\,0|\,V|\,\vec{p}_i\,0\rangle + e^{\mu}\langle\vec{p}_f\,1|\,V|\,\vec{p}_i\,1\rangle) + J_{\nu-1}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|\,V|\vec{p}_i\,1\rangle + J_{\nu+1}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|\,V|\vec{p}_i\,0\rangle]\,,$$

$$T_\nu^D(n,1) = (-1)^\nu\beta^2[-J_\nu e^{\mu/2}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,n|\,V|\vec{p}_i\,0\rangle - J_{\nu+1}e^{-\mu/2}\langle\vec{p}_f\,n|\,V|\vec{p}_i\,0\rangle]\,,$$

where

$$\beta = (2\cosh\mu)^{-1/2}\,. \tag{3.7a}$$

The reason for limiting the calculation to these states is that $1S$ and $2P_0$ are coupled by the laser and the states $2S$ and $2P_{\pm1}$ are degenerate with $2P_0$ and are therefore strongly coupled by the scattering. The matrix elements appearing in these T matrices are defined by

$$\langle\vec{p}_f\,n|\,V|\vec{p}_i\,n'\rangle = \int d^3r_1\,d^3r_2\,e^{-i\vec{p}_f\cdot\vec{r}_1}u_n^*(\vec{r}_2)\times\left(\frac{-2}{r_1}+\frac{2}{r_{12}}\right)e^{i\vec{p}_i\cdot\vec{r}_1}u_{n'}(\vec{r}_2)\,, \tag{3.8}$$

and the Bessel functions all have arguments

$$\vec{\alpha}_0\cdot\vec{q} = (e/2m)(\vec{E}/\omega^2)\cdot\vec{q}\,. \tag{3.9}$$

The exchange S matrix cannot be written unambiguously even in the Born approximation because of the lack of orthogonality of the states χ_λ and ϕ_n. There are several forms which are known[9] to give the same results at sufficiently high collision energy but which give very different results at intermediate energy. We shall use the simplest form here, even though it is not the best:

$$S^X(f,\vec{p}_f;\,i,\vec{p}_i) = -i\int dt\,d^3r_1\,d^3r_2\,\chi_f^*(\vec{r}_1,t)\,\phi_f^*(\vec{r}_2,t)\times V(\vec{r}_1,\vec{r}_2)\,\phi_i(\vec{r}_1,t)\chi_i(\vec{r}_2,t)\,. \tag{3.10}$$

It may also be written

$$S^X(f,\vec{p}_f;\,i,\vec{p}_i) = -2\pi i\sum_\nu\delta(p_f^2 - p_i^2 - \nu\omega)\times T_\nu^X(f,\vec{p}_f;\,i,\vec{p}_i)\,, \tag{3.11}$$

where the various exchange T matrices are given by

$$
\begin{aligned}
T_\nu^X(0,\vec{p}_f;0,\vec{p}_i) &= (-1)^\nu\beta^2[J_\nu(e^\mu\langle\vec{p}_f\,0|V|0\,\vec{p}_i\rangle+e^{-\mu}\langle\vec{p}_f\,1|V|1\,\vec{p}_i\rangle)-J_{\nu-1}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|V|1\,\vec{p}_i\rangle\\
&\quad -J_{\nu+1}(\Lambda/|\Lambda|)\langle\vec{p}_f\,0|V|0\,\vec{p}_i\rangle],\\
T_\nu^X(1,\vec{p}_f;0,\vec{p}_i) &= (-1)^\nu\beta^2[J_\nu(\langle\vec{p}_f\,0|V|0\,\vec{p}_i\rangle-\langle\vec{p}_f\,1|V|1\,\vec{p}_i\rangle)-J_{\nu-1}e^{-\mu}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|V|1\,\vec{p}_i\rangle\\
&\quad +J_{\nu+1}e^{\mu}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|V|0\,\vec{p}_i\rangle],\\
T_\nu^X(n,\vec{p}_f;0,\vec{p}_i) &= (-1)^\nu\beta[J_\nu e^{-\mu/2}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,n|V|1\,\vec{p}_i\rangle-J_{\nu+1}e^{\mu/2}\langle\vec{p}_f\,n|V|0\,\vec{p}_i\rangle],\\
T_\nu^X(0,\vec{p}_f;1,\vec{p}_i) &= (-1)^\nu\beta^2[J_\nu(\langle\vec{p}_f\,0|V|0\,\vec{p}_i\rangle-\langle\vec{p}_f\,1|V|1\,\vec{p}_i\rangle)+J_{\nu-1}e^{\mu}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|V|1\,\vec{p}_i\rangle\\
&\quad -(\Lambda/|\Lambda|)e^{-\mu}J_{\nu+1}\langle\vec{p}_f\,1|V|0\,\vec{p}_i\rangle],\\
T_\nu^X(1,\vec{p}_f;1,\vec{p}_i) &= (-1)^\nu\beta^2[J_\nu(e^{-\mu}\langle\vec{p}_f\,0|V|0\,\vec{p}_i\rangle+e^{\mu}\langle\vec{p}_f\,1|V|1\,\vec{p}_i\rangle)+J_{\nu-1}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,0|V|1\,\vec{p}_i\rangle\\
&\quad +J_{\nu+1}(\Lambda/|\Lambda|)\langle\vec{p}_f\,1|V|0\,\vec{p}_i\rangle],\\
T_\nu^X(n,\vec{p}_f;1,\vec{p}_i) &= (-1)^\nu\beta[-J_\nu e^{\mu/2}(\Lambda^*/|\Lambda|)\langle\vec{p}_f\,n|V|1\,\vec{p}_i\rangle-J_{\nu+1}e^{-\mu/2}\langle\vec{p}_f\,n|V|0\,\vec{p}_i\rangle].
\end{aligned}
\tag{3.12}
$$

Again the arguments of the Bessel functions are given by Eq. (3.9), and the matrix elements are

$$
\langle\vec{p}_f\,n|V|n'\,\vec{p}_i\rangle=\int d^3r_1\,d^3r_2\,e^{-i\vec{p}_f\cdot\vec{r}_1}u_n^*(\vec{r}_2)\times\left(\frac{-2}{r_1}+\frac{2}{r_{12}}\right)u_{n'}(\vec{r}_1)e^{i\vec{p}_i\cdot\vec{r}_2}\,.
\tag{3.13}
$$

The cross sections (in units of a_0^2) are obtained from the expression

$$
\frac{d\sigma_\nu}{d\Omega}(n,n';\vec{p}_f,\vec{p}_i)\times\left|\frac{1}{2\pi}[T_\nu^D(n,\vec{p}_f;n',\vec{p}_i)\pm T_\nu^X(n,\vec{p}_f;n',\vec{p}_i)]\right|^2,
\tag{3.14}
$$

where the plus (minus) sign gives the singlet (triplet) cross sections.

We shall not continue with the cataloging of the various cross sections here. We note instead that if no observation is made on the final state of the target then the unpolarized cross section is given by

$$
\frac{d\bar{\sigma}_\nu}{d\Omega}(\vec{p}_f;\vec{p}_i)=\sum_n\left(P_+\frac{d\sigma_\nu}{d\Omega}(n,0;\vec{p}_f,\vec{p}_i)+P_-\frac{d\sigma_\nu}{d\Omega}(n,1;\vec{p}_f,\vec{p}_i)\right),
\tag{3.15}
$$

where the sum runs over the final states $n=0$, $1,2,3\pm$ and the weighted average of the singlet and triplet cross sections. All of these final states yield final electron energies given by Eq. (2.11). Other final target states will not. Equation (3.15) can be evaluated by substitution of the T matrices and the matrix elements, all of which can be evaluated analytically. Explicit inclusion of the result here would be prohibitively complicated, but numerical evaluation for a particular experiment is not difficult.

Instead we note that the argument of the Bessel functions is a small number, except for very-high-intensity lasers, and hence an expansion can be made. Keeping only the leading term we get

$$
J_{\nu+\sigma}\to\delta_{\nu,-\sigma},
\tag{3.16}
$$

so that the T matrices of Eqs. (3.7) and (3.12) can be greatly simplified. The result for the observed cross sections still depends on the dynamics of the SRP through the factors $P_\pm$ in Eq. (3.15). If we are content with the approximation for these factors given in (2.23), then the cross sections simplify. After some algebra the result is

$$
\frac{d\bar{\sigma}_0^{(\pm)}}{d\Omega}=\beta'^2\left(|\langle\vec{p}_f,0|V(1\pm X_{12})|\vec{p}_i,0\rangle|^2+\sum_m|\langle\vec{p}_f,m|V(1\pm X_{12})|\vec{p}_i,1\rangle|^2\right),
\tag{3.17}
$$

$$
\frac{d\bar{\sigma}_1^{(\pm)}}{d\Omega}=\beta'^2(1+\omega/p_i^2)^{1/2}\,|\langle\vec{p}_f,0|V(1\pm X_{12})|\vec{p}_i,1\rangle|^2,
\tag{3.18}
$$

$$
\frac{d\bar{\sigma}_{-1}^{(\pm)}}{d\Omega}=\beta'^2\left(1-\frac{\omega}{p_i^2}\right)^{1/2}\sum_m|\langle\vec{p}_f,m|V(1\pm X_{12})|\vec{p}_i,0\rangle|^2
\tag{3.19}
$$

where

$$
\beta'^2=(2\cosh2\mu)^{-1/2}\,.
$$

Here the $(\pm)$ now indicate spin symmetry: the $(+)$ is associated with the singlet and the $(-)$ with the triplet states; X_{12} is the electron exchange operator. The subscript on σ is the index ν defining

the inelasticity of the electron by Eq. (2.11). The m sums in Eqs. (3.17) and (3.19) now run over the entire $n=2$ manifold of hydrogen, the $2S$ and all three $2P$ states treated symmetrically. The magnitude of the final momentum occurring in each of these equations is determined by Eq. (2.11), but we note that for $\nu=0$ only elastic matrix elements occur. For $\nu=1$ (superelastic scattering) only the one possible superelastic matrix element enters, and for $\nu=-1$ (inelastic scattering) all possible inelastic matrix elements enter symmetrically. We see from these results that the only remaining mention of the laser is through the overall factor β'^2. Then, to the extent that Eq. (2.23) is to be believed and the Born approximation is to be trusted, the effect of the laser on the cross sections is minimal.

Finally, we should point out that some of the cross sections for $\nu=0$ (elastic scattering) in the forward direction have a weak divergence due to the neglect of the energy shifts in the wave functions given in Eq. (3.1). This neglect can be remedied. It removes the divergence but no longer allows us to write the S matrices [Eqs. (3.6) and (3.11)] with a single energy δ function. This was discussed more fully in Ref. 1.

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[1]J. I. Gersten and M. H. Mittleman, Phys. Rev. A 13, 123 (1976).

[2]I. V. Hertel and W. Stoll, J. Phys. B 7, 570 (1974); J. Macek and I. V. Hertel, J. Phys. B 7, 2173 (1974).

[3]F. T. Hioe and E. W. Montroll, J. Math Phys. 16, 1259 (1975).

[4]See for example, M. Sargent III, M. O. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, Mass., 1974).

[5]See for example, C. R. Stroud, Jr., Phys. Rev. A 3, 1044 (1971).

[6]We can write (2.11) only in the approximation that the laser photon energy is exactly resonant with the atomic transition. Otherwise there are other final electron energies which are possible.

[7]See, for example, N. Kroll and K. Watson, Phys. Rev. A 13, 1018 (1976).

[8]Rydberg units are used.

[9]C. Joachain and M. H. Mittleman, Phys. Rev. 151, 7 (1966).